Parametrization Methodology of Polyphase Brushless Dc-Machines Aiming a Complete Dynamical Analysis and Modeling Approach

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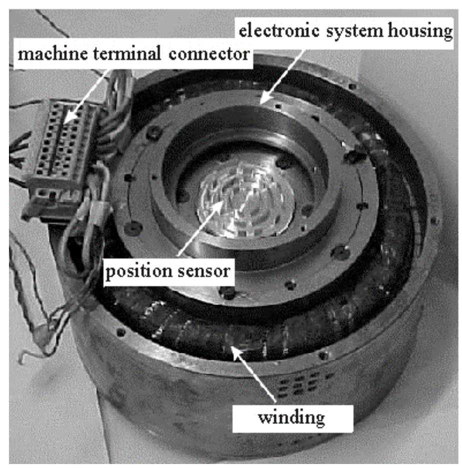
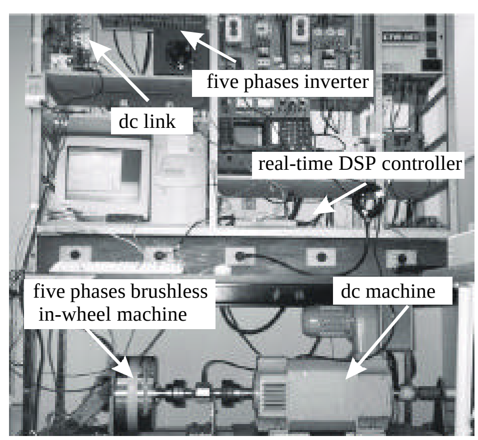
Abstract - The main objective of mathematical modeling in this paper was to obtain the most realistic simulation study of machine current. Most people take for grant ideal rectangular current waveforms for dc machines. However, it was found that distortions do exist in real machines, and neglecting such imperfections can lead to erroneous control strategy development. This work is concerned with the analysis made to put together the modeling with the expected behavior concerned with mutual inductance and armature reaction effect, so as to have simulation results verified by the experimental results. The papers will present an approach of linear equivalent circuits instantaneously switched by the machine rotor position, aiming a dynamic model. A complete parametrization of self-inductance, mutual inductance, armature reaction and relationship of displacement of optical disc encoder with timing of inverter switching was fundamental to achieve a meaningful and complete dynamical system which was implemented with Simulink/MATLAB. A permanent magnet five-phases brushless-dc machine has been designed, analyzed, constructed, simulated and evaluated for this work. Coherent and consistent results were observed by comparing experimentation and simulation. The authors believe that this approach can be useful for machine designers who want to consider transient response effects in their electromagnetic design.

I. INTRODUCTION

Selection of electrical machines for electric vehicles applications is motivated with the increasing need and requirements for reduction of oil dependence and environmental restrictions on pollution and emissions. Electric vehicles are restricted in autonomy, in addition for an optimized performance it is required to reduce losses. In-wheel embedded machines reduce mechanical losses and gearbox weight, on the other hand dimension and interaction with the vehicle suspension are stringent. After a successful design under such restrictions a detailed modeling and analysis are of paramount importance for putting together a driving approach for a torque control loop. Torque is an inner control variable and therefore optimized time response is required in order to have good current waveforms improving the overall electrical behavior. A permanent magnet five-phases brushless-dc machine [1] [2] has been designed, analyzed, constructed, simulated and evaluated for this work . The main objective of mathematical modeling [3] in this paper was to obtain the most realistic simulation study of machine current. Most people take for grant ideal rectangular current waveforms for brushless-dc machines. However, it was found that distortions do exist in real machines, and neglecting such imperfections can lead to erroneous control strategy development [4]. In order to demonstrate the obtained theoretical results for the constructed motor a workbench shown in Fig. 1(a) allowed the dynamical and electronic experimentations. Dynamical impress is possible by an off-the-shelf dc machine with four-quadrant control emulating the required load on the common shaft. The photography shows such structure and all the interlocking required for protection. In addition, sensors for torque, speed and temperature provide system's monitoring. The electronic system is composed by a five-phases inverter commanding the permanent magnet five-phases brushless-dc machine, a power interlocking system, a dc-link with dynamic braking capabilities and a personal computer with a DSP system for control, indicated in Fig. 1(b). Such experimental apparatus allowed comparison with the machine modeling, helping in fine-tuning the overall system observations.



(a)



(b)

Fig. 1. Photography showing (a) five-phases brushless dc machine, (b) workbench

II. SELF INDUTANCE BASED MODELING

Modeling a polyphase brushless dc-machine starts with the assumption of balanced phases. Therefore, parameters like resistances, inductances, mutual inductances and back-emf are considered symmetrically distributed . Commutator is electronically embedded in the machine; it is a disc that addresses the sequence of turning on and off the machine phases. Therefore, our view to model such behavior was to use the absolute rotor position to enable an equivalent circuit model. Fig. 2 shows an example of the inverter switches determining an equivalent circuit for that state. By describing such equivalent circuits mathematically one can have the operation of the machine in both clockwise and counterclockwise direction. As an example of how this idea works one can see the current flow in the machine for state I represented by (1) to (5) where only the self-inductance was considered.

$$\frac{d\ i_a}{d\ t}=\frac{1}{4\ L}\left(-4\ R\ i_a-3\ e_a+e_b+e_d+e_e+2\ V_S-4\ V_Q\right) \quad (1)$$

$$\frac{d\ i_b}{d\ t}=\frac{1}{4\ L}\left(-4\ R\ i_b+e_a-3\ e_b+e_d+e_e+2\ V_S-4\ V_Q\right) \quad (2)$$

$$i_c=0 \quad (3)$$

$$\frac{d\ i_d}{d\ t}=\frac{1}{4\ L}\left(-4\ R\ i_d+e_a+e_b-3\ e_d+e_e-2\ V_S+4\ V_Q\right) \quad (4)$$

$$\frac{d\ i_e}{d\ t}=\frac{1}{4\ L}\left(-4\ R\ i_e+e_a+e_b+e_d-3\ e_e-2\ V_S+4\ V_Q\right) \quad (5)$$

Equation (6) adds a little bit of more complexity, however required for incorporating the armature reaction by considering the mutual inductance as well.

$$\frac{d}{dt}\left[I_{adcde}\right]=\left\{\left[I\ \right]-\frac{M}{4\ L}\cdot\left[M_{abcde}\right]\right\}^{-1}\cdot\frac{1}{4\ L}\left\{-4\ \left[R\ \right]\cdot\left[I_{abcde}\right]+\begin{bmatrix}-3 & 1 & 1 & 1\\ 1 & -3 & 1 & 1\\ 1 & 1 & -3 & 1\\ 1 & 1 & 1 & -3\end{bmatrix}\cdot\left[E_{abcde}\right]+\left[V_{TD}\right]\right\} \quad (6)$$

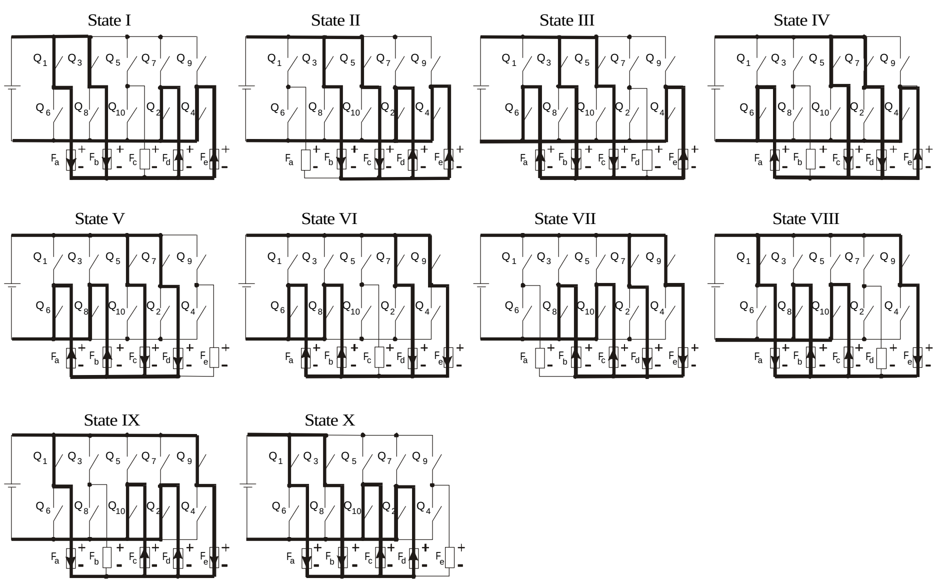


Fig. 2. Equivalent circuits for each commutation state

In addition to such forced excitation it is also required to consider the commutation effect where the current established in one state continues to circulate during next state, like indicated in Fig. 3. Such conditions can be considered by three equations (7) to (9). Therefore, a complete five phases model is composed by a set of equations that is turned on or off in accordance to the instantaneous position dictated by the encoder [5]. Figure 4 shows the on/off controlling function here denominated as Matrix Switching (MS). If a pulse-width-modulation (PWM) is used for modulating the current, such MS will determine the appropriate circuits to be turned on and off. Figure 4 also shows that from voltages, torque and speed are related to the differential equations that determine the current behavior.

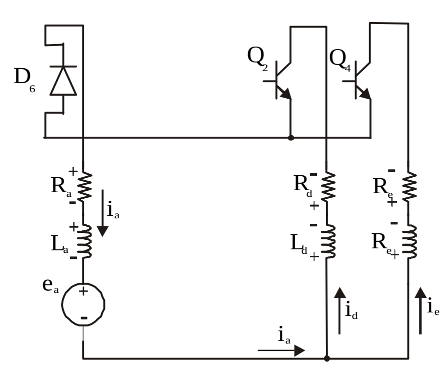


Fig. 3. Equivalent circuit for the freewheeling current between state I and state II

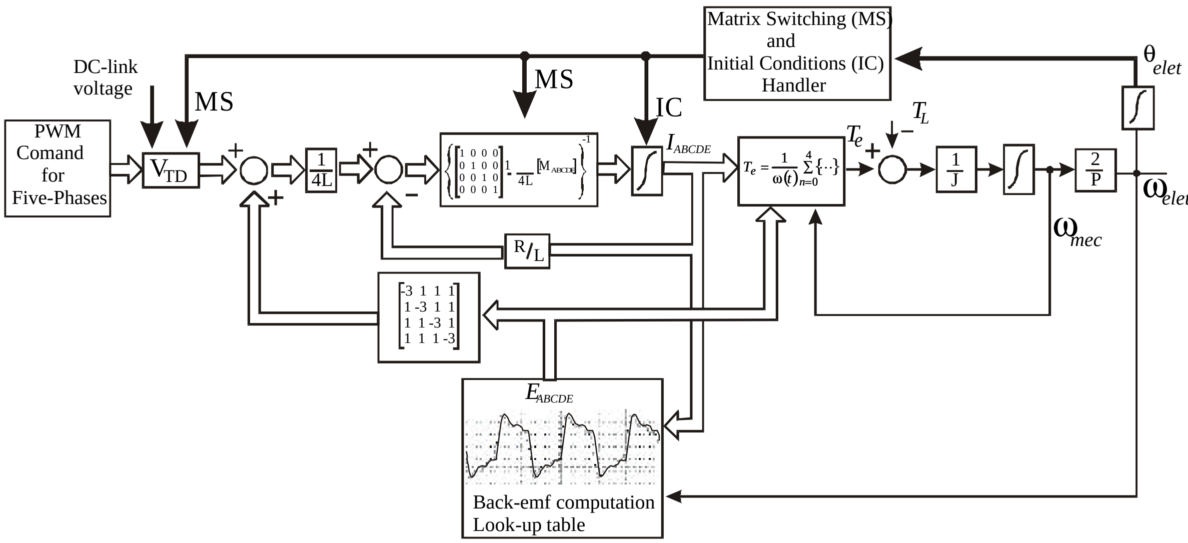


Fig. 4. Block diagram showing the mathematical model equation flow

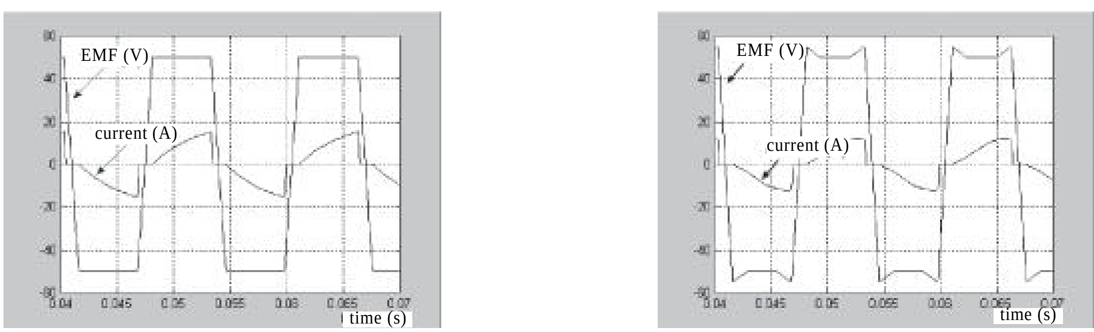
$$\frac{d\ i_a}{d\ t} = \frac{1}{L}\left(-\frac{2}{3}e_a - R\ i_a\right) \tag{7}$$

$$\frac{d\ i_d}{d\ t} = \frac{1}{L}\left(\frac{1}{3}e_a - R\ i_d\right) \tag{8}$$

$$\frac{d\ i_e}{d\ t} = \frac{1}{L}\left(\frac{1}{3}e_a - R\ i_e\right) \tag{9}$$

The mechanical equation relates the machine torque, dependent on the instantaneous position, current and back-emf, and the net torque resulting from subtracting the load torque accelerates the load. Equation (10) describes the torque relating to the current, back-emf and armature reaction. Figure 5(a) shows the initially considered waveform for the back-emf, however, realistic current waveforms were obtained by considering the waveform indicated in Fig. 5(b) with a look-up table commanded by the instantaneous position.

$$T_{elet} = \frac{1}{w(t)}\sum_{n=0}^{4}\left\{\left(K_V\left|w(t)\right|\Gamma_w\left(t-\frac{n}{5}2p\right)+K_a\left|i_n(t)\right|\Lambda_c\left(t-\frac{n}{5}2p\right)\right)i_n(t)\right\} \tag{10}$$



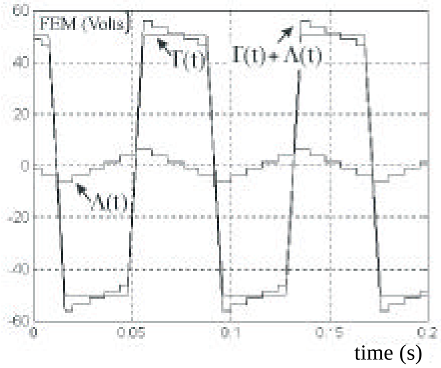
(a) Current obtained by theory back-emf

(b) Current obtained by experimental back-emf

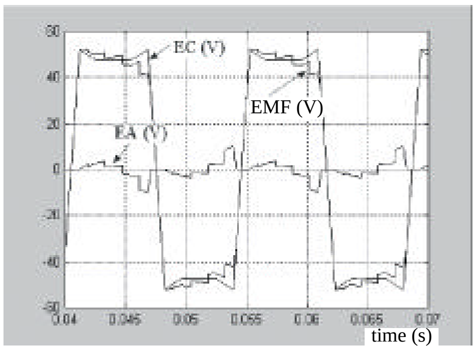
Fig. 5. Current waveforms without considering the mutual inductance and armature reaction

III. MUTUAL INDUTANCE AND ARMATURE REACTION BASED MODELING

Mutual inductance was integrated to the mathematical model based on (6). The block diagram depicted in Fig. 4 shows the mutual inductance incorporated into a single block to facilitate the observation of such effects in the simulation which can be compared with the results on figures Fig. 5(b) and Fig. 7(a). One can observe that the current amplitude for the case considering mutual inductance is greater than for the case considering self-inductance only because the magnetic fluxes have a coupling effect [6]. Consequently the air-gap flux is dependent on such contribution due the mutual inductance. The variable motional amplitude resulting from the rotation of the magnets is represented by function $\Gamma_\omega(t)$ and the armature current distortion due to the armature reaction by the function $\Lambda_c(t)$; $\Gamma_\omega(t)$ is proportional to the speed and $\Lambda_c(t)$ is proportional to the armature current producing a current plotted in Fig. 7(b) [7], [8].

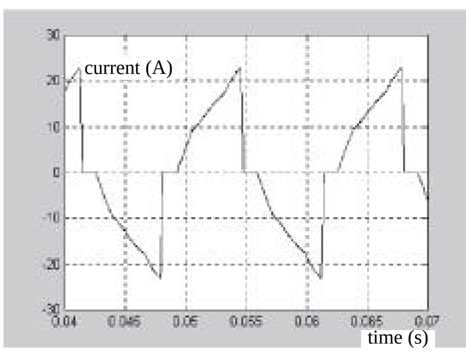


(a) Back EMF abstract considering the armature reaction

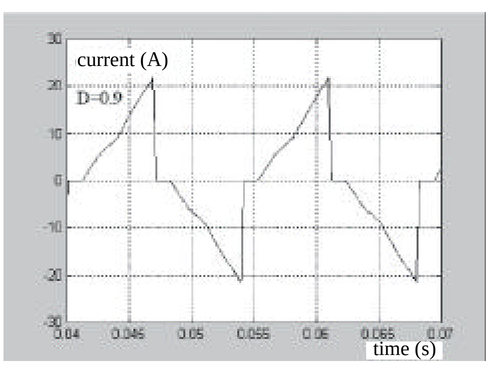


(b) Simulation result using experimental back-emf

Fig. 6. Back EMF waveforms composition considering armature reaction



(b) Considering armature reaction

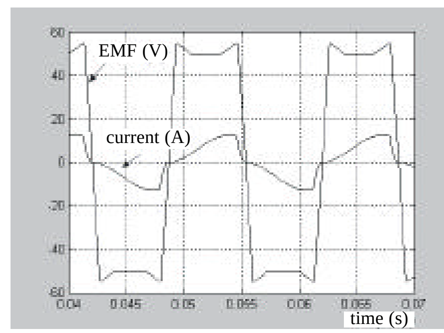


(a) Considering mutual inductance

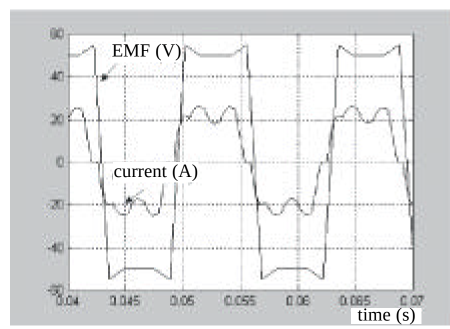
Fig. 7. Armature current waveforms for 0,9 duty cycle

IV. OPTICAL DISC DISPLACEMENT EFFECT

Because such machine has high inductance, the nominal value does not establish its peak value for high speed conditions. Therefore, a technique used for this situation was to phase shift the firing of the inverter switches in order to improve the motor efficiency contributing for a higher slope and sharper current rise time. That was also observed by mechanically displacing the optical disc, shown in Fig. 1(a). Fig 8 shows the current waveforms for such displacement in electrical degrees and a comparison with our simulation methodology was done in order to corroborate our method. Fig. 8 shows the current waveform by simulation with mutual inductance. A complete system can be considered with mutual inductance, armature reaction and the optical disc displacement as results in Fig. 9. Our modeling efforts drove us to implement a PI closed loop system like the block diagram depicted in Fig. 4. The back-emf for a step torque response are shown in Fig. 10 where it is clearly seen the FEM distortion due to current variation.



(a) 3 electrical degrees displacement

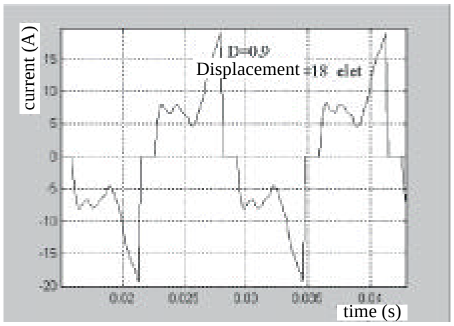


(b) 27 electrical degrees displacement

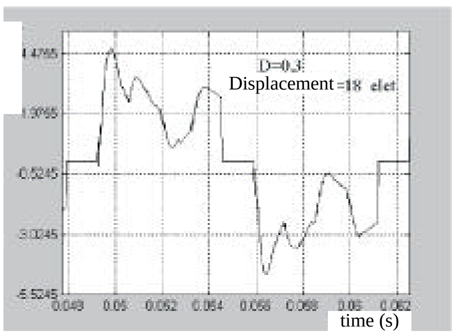
Fig. 8. Current waveforms for some displacement in electrical degrees

V. MODEL ADJUSTMENT EXPERIMENTALLY

In order to adjust the parameters, the torque and speed constants, as well as the armature reaction effects had to be incorporated. Two assay sets for back-emf were required: (1) open-circuit back-emf in the whole speed operating range and (2) full-loaded machine to obtain the armature reaction. The first assay was done by rotating the five phases BPM machine like a generator in open-circuit (with a dc-motor connected on the shaft) and the trapezoidal [9] back-emf waveform was acquired, to generate a look-up table and determine the velocity constant constant K_v. Connecting resistive loads on the five-phases BPM was the next assay; the BPM machine was rotated for the full-range of the operating speed. The trapezoidal waveform was by its turn distorted like Fig. 11, the amplitude of such distortion was proportional to the current and the constant K_a was obtained. Fig. 12 shows the current due to the optical disc displacement.

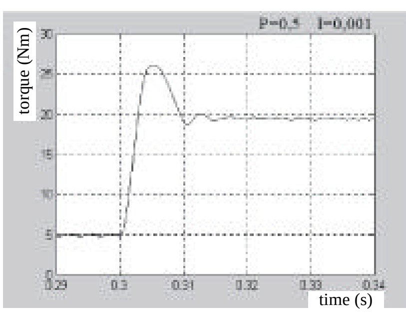


(a) Current for 0,9 duty cycle

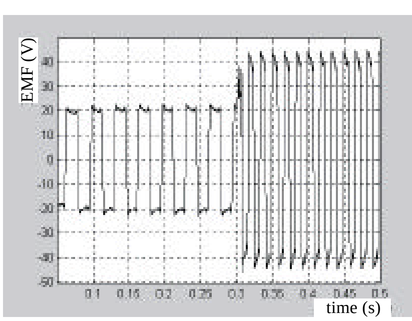


(b) Current for 0,3 duty cycle

Fig. 9. Current waveform considering mutual inductance, armature reaction and the optical disc displacement

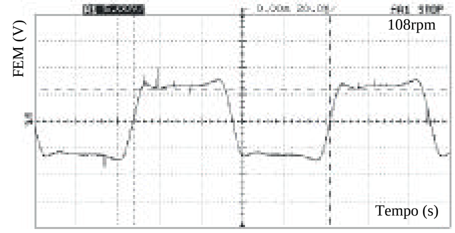


(a) Torque response

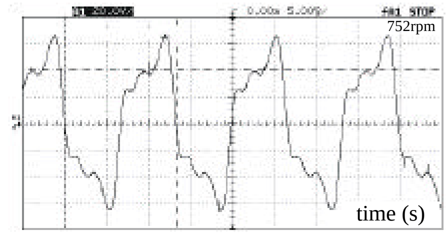


(b) The back-emf for a step torque response

Fig. 10. Waveforms for a PI torque control

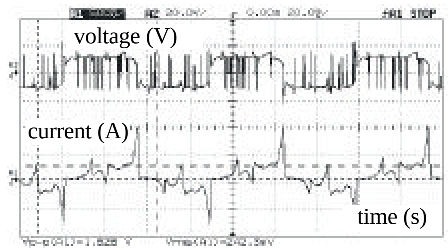


(a) Distortion by 108 rpm

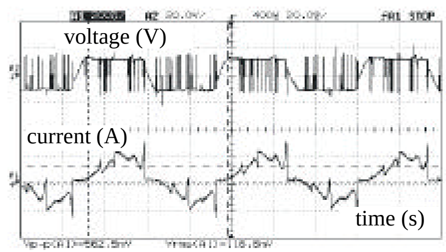


(b) Distortion by 752 rpm (full range).

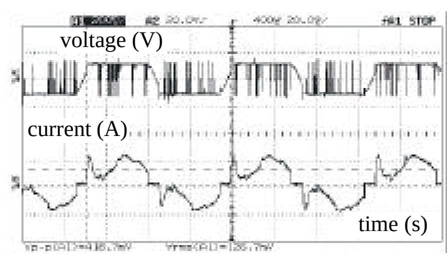
Fig. 11. Experimental Back-emf waveform distortion considering armature reaction



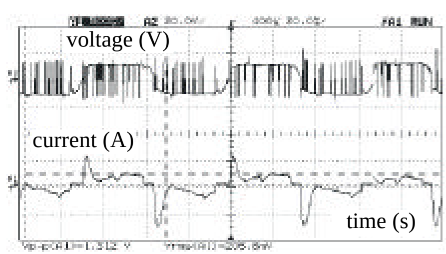
(a) Displacement 1



(b) Displacement 2



(c) Displacement 3



(d) Displacement 4

Fig. 12. Experimental current waveform due to the disc optical mechanic displacement

VI. CONCLUSION

Our concluding remarks are that the mathematical modeling efforts are very more complex to fully incorporated the real machine waveform distortions observed in practice. This work embedded the mutual inductance and armature reactions effects into a dynamical model framework, through equivalent circuits instantaneously switched by the rotor position. A complete parametrization of self-inductance, mutual inductance, armature reaction and relationship of displacement of optical disc encoder with timing of inverter switching was fundamental to achieve a meaningful and complete dynamical system which was implemented with Simulink/MATLAB. Coherent and consistent results were observed by comparing experimentation and simulation. The authors believe that this approach is useful for machine designers who want to consider transient response effects in their electromagnetic design and to control system designers to encompass a space vector controller capable of predicting the actual current vector for improved pulse modulation.

VII. REFERENCES

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