THREE-PHASE INDUCTION GENERATOR FED BY SINGLE PHASE SOURCE - TIME DOMAIN MATHEMATICAL MODEL

Roberlam G. de Mendonça,MSc.* *Rui V. R. Silva,MSc.***
Samuel C. M. de Paula,MSc.* *Luciano M. Neto,Dr.***

* CEFETGO – Jataí Decentralized Unity - Electrotechnical Coordination
Jataí – GO – Brazil – 76300 – 000
** Electrical Machines Lab. , Electrical Engineering Dept., Universidade Federal de Uberlândia - Campus Santa Mônica - Uberlândia – MG – Brazil – 38400-902
e.mail: **roberlam@alunos.ufu.br**

Abstract

This paper presents a time domain mathematical model of a three phase induction generator feeding a single-phase electrical distribution system including space harmonic effects. The objective in this work is to analyze the three-phase induction generator when working in the conditions above described. From such model it is possible to make a more precise analysis, since some problems should be easily observed using the time domain technique, such as unbalancing of magnetomotive forces *(mm*f) at the machine inner magnetic circuit. This is impossible to be visualized when using frequency domain modeling. A 2 HP three-phase induction generator model is used on digital simulation. Through the results it is easy to show that the proposed model is very efficient.

Keywords: induction generator, single-phase, distribution system, space harmonic effects.

1. Introduction

Countries with a large territorial extension, where electrical energy consumers are characterized by:
- small monthly kWh expenditure;
- small number of consumers per km of distribution network;
- small maximum simultaneous demands;
- and also a small amount of financial resources to be spent in rural electrification programs.

Having the Brazilian scenery as our example, the electrical energy authorities turned to single-phase (one wire with earth return) as a less expensive option to electrical distribution systems in rural areas. In the production side, due to a constant pressure of the technology to become more efficient and to increase the maximum demand, the rural consumer becomes locked to the limitations of the single-phase system. However, in some regions where small hydroelectric potentials are available, there's a possibility to generate energy with the use of three-phase induction machines, with squirrel cage rotor connected to a single-phase distribution system in order to supply typical rural three-phase loads. Some studies have already been done about the three-phase induction generator connected to a single-phase energy system. These studies were of great importance but developed through frequency domain mathematical models[1]. The use of frequency domain techniques doesn't allow the analysis for the behavior of the three-phase induction generator concerning the electromagnetic unbalances inside the machine. So, aware of this matter this paper presents a time domain mathematical model [2][3][4], who allows such studies to be made. These studies show the electromagnetic behavior in the three-phase induction generator inner electromagnetic circuit.

2. Induction Generator

The proposed three-phase induction generator is connected to a single phase electrical distribution system. It is an ordinary induction machine with a squirrel cage rotor and stator phases displaced spatially by 120 o and same number turns/phase. An adequate capacitance Cap is coupled between phases B and C, as in Figure 1. The objective is to obtain a reliable operating point for this unbalanced configuration, where the generator is delivering nominal power under nominal voltage. As described in reference [1], a balanced situation between generator phases can be obtained, and this balance is of fundamental importance concerning the connection of a three-phase induction generator in parallel with a single-phase voltage source. Through the frequency domain mathematical modeling proposed in [1], it is clear the good behavior for the proposed generator, in terms of "rms" values, where the voltage unbalance is not so accentuated. However, an interesting point not brought in picture in [1] is the behavior of electromagnetic torque inside the generator, for small voltage unbalancing the internal electromagnetic torque shows a swinging behavior. Those swings are related to the current unbalance in the generator phases as part of a unbalanced three-phase system. As a consequence this generator has different magnetomotive forces *(mmf)* in each phase. This unbalancing allows the decomposition of the *mmf* distribution in such a way to obtain static and rotary magnetic field distribution, where the latter are of positive and negative sequence. The positive sequence magnetic field, BR1, under the three-phase symmetrical induction machine working principles, create indirectly a rotating magnetic field, BS1, of same sequence, in the machine's rotor. The same happens to the negative sequence magnetic field, BR2, creating another BS2. The above is illustrated in Figure 2, and can be seen clearly that:

- The magnetic fields (BS1, BR1) and (BS2, BR2) show a constant angular displacement between them, therefore the resulting aligning electromagnetic torque is constant;
- The angular position between magnetic fields (BS1, BR1) and (BS2, BR2) changes periodically, therefore the electromagnetic torque is an oscillating value.

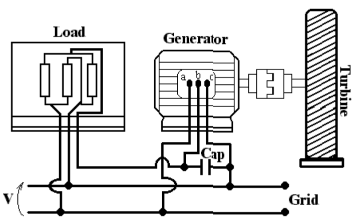

Figure 1 – Connection diagram for three-phase squirrel-cage induction generator. Connected to a single-phase distribution system.

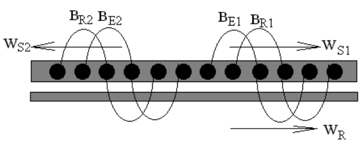

Figure 2 – Demonstration of the internal rotating magnetic field inside a three-phase induction generator connected to a single-phase distribution system.

3. Mathematical Model

For the sake of simplicity, the development of mathematical equations will be made from a generic winding consisting of two phases "i" and "j". In the following paragraphs this concept will be extended to three-phase induction generator connected to a single-phase distribution system. Machine equations can be written based in Figure 3, which shows a transverse cut of machine stator windings.

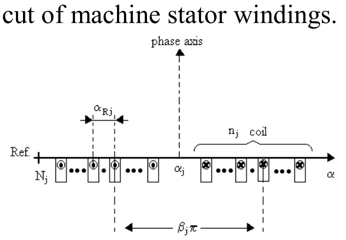

Figure 3 – Coil distribution for the phase "j" windings.

Taking in consideration two generic generator “i” e “j” the following equations can be written:

$$v_i = -r_i.i_i - \frac{d\lambda_i}{dt} \quad (1)$$

$$v_j = -r_j.i_j - \frac{d\lambda_j}{dt} \quad (2)$$

$$\lambda_i = L_{ii}.i_i + L_{ij}.i_j + L_{di}.i_i \quad (3)$$

$$\lambda_j = L_{jj}.i_j + L_{ij}.i_i + L_{dj}.i_j \quad (4)$$

where:

v_i , v_j - voltages at phases “i” and “j”;

i_i , i_j - currents at phases “i” and “j”;

λ_i , λ_j - magnetic fluxes of phases “i” and “j”;

r_i , r_j - resistance for phases “i” and “j”;

L_{ii} , L_{jj} - self inductances for phases “i” e “j”, without leakage;

L_{ij} - mutual inductances between phases “i” e “j", and;

L_{di} , L_{dj} - leakage inductances for phases “i” and “j” , respectively.

The magnetomotrice force of phase “j” is given by:

$$Fmm_{jh}(\alpha) = \frac{2}{\pi}.N_j.n_j.k_{pjh}.k_{djh}.i_j.\frac{1}{h}.\cos\left[h.\left(\alpha - \alpha_j\right)\right] \quad (5)$$

where:

k_{pjh} , k_{djh} - step and distribution factors, respectively, for the h^{th} harmonics, given equations (6) and (7).

$$k_{pjh} = \mathrm{sen}\left(h.\beta_j.\frac{\pi}{2}\right) \quad (6)$$

$$k_{djh} = \frac{\mathrm{sen}\left(h.n_j.\frac{\alpha_{Rj}}{2}\right)}{n_j.\mathrm{sen}\left(h.\frac{\alpha_{Rj}}{2}\right)} \quad (7)$$

The magnetic field density distribution B_{jh} produced by $Fmm_{jh}(\alpha)$ is obtained through the Ampere's Law and the result is given by equation (8). In this case the iron's magnetic circuit reluctance is neglected when compared with air-gap's reluctance, considered uniform.

$$B_{jh}(\alpha) = \frac{2}{\pi}.\frac{\mu_o}{\sigma}.N_j.n_j.k_{pjh}.k_{djh}.i_j.\frac{1}{h}.\cos\left[h.\left(\alpha - \alpha_j\right)\right] \quad (8)$$

where μ_o is the air's magnetic permeability and σ is the air-gap's radial length.

In order to obtain the magnetic flux of phase "j" in phase "i", λ_{ijh}, must be obtained the phase "j" magnetic flux who embraces phase "i". Therefore, starting from equation (8), can be obtained the mentioned magnetic flux and then the flux coupling between phases λ_{ijh} , given by equation (9).

$$\lambda_{ijh} = k_1.N_i.N_j.i_j.\frac{k_{wih}.k_{wjh}}{h^2}\cos\left[h.\left(\alpha_i - \alpha_j\right)\right] \quad (9)$$

$$k_1 = 4.\frac{2.p.L.R.n_i.n_j.\mu_o}{\pi.\sigma} \quad (10)$$

$$k_{wih} = k_{pih}.k_{dih} \quad (11)$$

$$k_{wjh} = k_{pjh}.k_{djh} \quad (12)$$

where:
L - rotor cylinder length; R - air-gap radial length; 2p - machine pole number; n_i , n_j - coil number of phases “i” and “j” , respectively; N_i , N_j - turns number of phases “i” and “j” , respectively; k_{dih} , k_{djh} - distribution factor of phases “i” e “j” , respectively; k_{pih} , k_{pjh} - step factor of phases “i” e “j” , respectively.

It is important to mention that λ_{ijh} doesn't include the phase mutual leakage flux.

The harmonic inductance of order h between phases “i” e “j”, L_{ijh} , can be obtained through equation (13).

$$L_{ijh} = \frac{\lambda_{ijh}}{i_j} \qquad (13)$$

Therefore, substituting (9) in (13) the following expression can be obtained:

$$L_{ijh} = k_1.N_i.N_j.\frac{k_{ijh}}{h^2}.\cos\left[h\left(\alpha_i - \alpha_j\right)\right] \qquad (14)$$

$$k_{ijh} = k_{wih}.k_{wjh} \qquad (15)$$

The total magnetic flux coupling of one phase, "i" as an example, λ_i, which can be split in the sum of contributions of magnetic flux coupling which embraces stator and rotor, λ_i’, and the dispersion flux, λ_{di} . Therefore, we have:

$$\lambda_i = \lambda_{di} + \lambda_i' \qquad (16)$$

$$\lambda_{di} = L_i.i_i \qquad (17)$$

where:
L_i - dispersion inductance of phase “i”.
The dispersion inductance can be supposed constant and therefore can be written:

$$L_i = k_{di}.N_i^2 \qquad (18)$$

where:
k_{di} - magnetic circuit dispersion permeance.

Accordingly with reference [11], the flux coupling λ_i’ can be obtained through the superposition of coupling harmonic components of all phases λ_{ijh}. Therefore, can be written:

$$\lambda_i' = \sum_j \sum_h \lambda_{ijh} \qquad (19)$$

From equations (13) and (19) we have:

$$\lambda_i' = \sum_j \sum_h L_{ijh}.i_j \qquad (20)$$

from equations (16), (17) and (20) we obtain equation (21):

$$\lambda_i = L_i.i_i + \sum_h \sum_j L_{ijh}.i_j \qquad (21)$$

Finally, substituting equation (21) in equation (1) can be obtained the full voltage equation

$$v_i = -r_i.i_i - L_i.\frac{di_i}{dt} - \sum_h \sum_j \left[L_{ijh}.\frac{di_j}{dt} + i_j.\frac{dL_{ijh}}{dt}\right] \qquad (22)$$

Once defined the magnetic flux coupling general equation and voltage equation, those can be extended to the three-phase machine. In the matrix form we have:

$$[\lambda] = [L].[I] \qquad (23)$$

where:

$$[\lambda] = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda'_A \\ \lambda'_B \\ \lambda'_C \end{bmatrix} \quad (24) \qquad [I] = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I'_A \\ I'_B \\ I'_C \end{bmatrix} \quad (25)$$

$$[L] = k.\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} & L_{16} \\ L_{12} & L_{22} & L_{23} & L_{24} & L_{25} & L_{26} \\ L_{13} & L_{23} & L_{33} & L_{34} & L_{35} & L_{36} \\ L_{14} & L_{24} & L_{34} & L_{44} & 0 & 0 \\ L_{15} & L_{25} & L_{35} & 0 & L_{44} & 0 \\ L_{16} & L_{26} & L_{36} & 0 & 0 & L_{44} \end{bmatrix} \qquad (26)$$

where:

$$L_{11} = \sum_h \frac{k_{aah}}{h^2} + K_L ; L_{22} = \sum_h \frac{k_{bbh}}{h^2} + K_L ;$$

$$L_{33} = \sum_h \frac{k_{cch}}{h^2} + K_L ; L_{44} = \sum_h \frac{k_{RRh}}{h^2}\left[1 - \cos\left(\frac{2\pi}{3}\right)\right] + K'_L ;$$

$$L_{12} = \sum_h \frac{k_{abh}}{h^2}.\cos\left(h.\frac{2\pi}{3}\right); L_{13} = \sum_h \frac{k_{ach}}{h^2}.\cos\left(h.\frac{2\pi}{3}\right);$$

$$L_{14} = \sum_h \frac{k_{aRh}}{h^2}.\cos(h.\theta);$$

$$L_{15} = \sum_h \frac{k_{aRh}}{h^2}.\cos\left[h\left(\theta + \frac{2\pi}{3}\right)\right];$$

$$L_{16} = \sum_h \frac{k_{aRh}}{h^2}.\cos\left[h\left(\theta - \frac{2\pi}{3}\right)\right];$$

$$L_{23} = \sum_h \frac{k_{bch}}{h^2}.\cos\left(h.\frac{2\pi}{3}\right); L_{24} = \sum_h \frac{k_{bRh}}{h^2}.\cos\left[h\left(\theta - \frac{2\pi}{3}\right)\right];$$

$$L_{25} = \sum_h \frac{k_{bRh}}{h^2}.\cos(h.\theta); L_{26} = \sum_h \frac{k_{bRh}}{h^2}.\cos\left[h\left(\theta + \frac{2\pi}{3}\right)\right];$$

$$L_{34} = \sum_h \frac{k_{cRh}}{h^2}.\cos\left[h\left(\theta + \frac{2\pi}{3}\right)\right]; L_{35} = \sum_h \frac{k_{cRh}}{h^2}.\cos\left[h\left(\theta - \frac{2\pi}{3}\right)\right];$$

$$L_{36} = \sum_h \frac{k_{cRh}}{h^2}.\cos(h.\theta)\cdot$$

where:
k – machine's magnetic circuit constant;

For the three-phase induction generator connected to a single-phase distribution system, can be observed the presence of an additional capacitor between terminals "B" and "C", with the task to solve the machine's starting problem, and improve the performance in nominal steady-state conditions, as can be seen in Figure 1.
Through a complete analysis of electrical and mechanical equations for the machine, based on well established equations for a generic three-phase induction machine, we have:

$$v_a = -r_a . i_a - \frac{d\lambda_a}{dt} \tag{27}$$

$$v_b = -r_b . i_b - \frac{d\lambda_b}{dt} \tag{28}$$

$$v_c = -r_c . i_c - \frac{d\lambda_c}{dt} \tag{29}$$

With Figure 1 as our reference, the following can be written:

$$v = v_a - v_c \tag{30}$$

$$i_{Cap} = Cap.\frac{d(V_b - V_c)}{dt} \tag{31}$$

$$i_c = -(i_a + i_b) \tag{32}$$

Mathematical manipulation of equations from (27) to (32), give us:

$$v = -(r_a + r_c).i_a - r_c.i_b - \frac{d\lambda_{ac}}{dt} \tag{33}$$

where:

$$\lambda_{ac} = \lambda_a - \lambda_c$$
$$\lambda_{bc} = \lambda_b - \lambda_c$$

For the short-circuited rotor we have:

$$0 = -r'_R . i'_A - \frac{d\lambda'_A}{dt} \tag{34}$$

$$0 = -r'_R . i'_B - \frac{d\lambda'_B}{dt} \tag{35}$$

$$0 = -r'_R . i'_C - \frac{d\lambda'_C}{dt} \tag{36}$$

In order to solve the system of equations from (33) to (36) it is necessary to relate the magnetic flux coupling with stator and rotor currents. For an machine, the following can be written:

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_A' \\ \lambda_B' \\ \lambda_C' \end{bmatrix} = [L].\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_A' \\ i_B' \\ i_C' \end{bmatrix} \tag{37}$$

where $[L]$ is the inductance matrix from equation 26.
From equations (32) and (37) can be obtained equation (38).

$$\begin{bmatrix} \lambda_{ac} \\ \lambda_{bc} \\ \lambda'_A \\ \lambda'_B \\ \lambda'_C \end{bmatrix} = \begin{bmatrix} \\ \\ L_1 \\ \\ \\ \end{bmatrix}.\begin{bmatrix} i_a \\ i_b \\ i'_A \\ i'_B \\ i'_C \end{bmatrix} \tag{38}$$

where:

$$[L_1] = k.\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{12} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{13} & A_{23} & A_{33} & 0 & 0 \\ A_{14} & A_{24} & 0 & A_{33} & 0 \\ A_{15} & A_{25} & 0 & 0 & A_{33} \end{bmatrix} \tag{39}$$

The terms in matrix $[L_1]$ are:
$A_{11} = L_{11} - 2.L_{13} + L_{33}$;
$A_{12} = L_{12} - L_{23} - L_{13} + L_{33}$;
$A_{13} = L_{14} - L_{34}$; $A_{14} = L_{15} - L_{35}$;
$A_{15} = L_{16} - L_{36}$;
$A_{22} = L_{22} - 2.L_{23} + L_{33}$; $A_{23} = L_{24} - L_{34}$;
$A_{24} = L_{25} - L_{35}$; $A_{25} = L_{26} - L_{36}$; $A_{33} = L_{44}$.
Where the values of L_{ij} are the terms of the matrix in equation (26).
The machine's mechanical equations are introduced as in the following equations:

$$Tb - Tm = J.\frac{dW_R}{dt} \tag{40}$$

$$w_R = \frac{d\theta_R}{dt} \tag{41}$$

where:
J - inertia moment of rotating parts;
w_R - machine's angular speed;
θ_R - angular displacement, in mechanical degrees;
Tm - electromagnetic torque;
Tb - turbine torque.

The electromagnetic torque is given by:

$$Tm = \frac{p}{4}.\begin{bmatrix} i_a & i_b & i_c & i'_A & i'_B & i'_C \end{bmatrix}\left[\frac{d[L]}{d\theta}\right].\begin{bmatrix} i_a \\ i_b \\ i_c \\ i'_A \\ i'_B \\ i'_C \end{bmatrix} \tag{42}$$

where:
p - pole number;
[L] - inductance matrix in equation (26);
θ - angular displacement in electrical degrees.

From the union of electrical and mechanical equations, can be obtained an equation system which represents the three-phase induction generator connected to a single-phase distribution system. The resulting matrix equation system is shown below:

$$\frac{d[I']}{dt} = [L']^{-1}.\left\{-[V'] - \left([R'] + \left[\frac{d[L']}{dt}\right]\right)[I']\right\} \qquad (43)$$

where:

[I’] - represents the current matrix (stator, rotor and load);

[L’] - represents the inductance matrix (stator, rotor and load);

[V’] - represents the voltage matrix (stator, rotor and load);

[R’] – represents the resistance matrix (stator, rotor and load).

4. Digital Simulation

Once having all the induction generator equations, digital simulations have been done in order to confirm the electromagnetic torque oscillations. Therefore, the benchmark was the three-phase induction motor used in [1], a three-phase induction machine with similar characteristics was taken, a 2 HP, 4 poles, 380/220 Volts, squirrel cage rotor, with equivalent circuit parameters given by Table 1.

TABLE 1. THREE-PHASE INDUCTION MACHINE EQUIVALENT CIRCUIT PARAMETERS.

Stator Resistance	*3,80±0,03Ω*
Rotor Resistance (referred to the stator)	3,01±0,03Ω
Locked Rotor Reactance (referred to the stator)	3,10±0,03Ω
Phase Magnetization Reactance	75,15±0,7Ω

The following simulations are obtained with respectively capacitors of 50 μF in order to have an idea of its effect on the electromagnetic torque behavior inside the generator as well as the influence of the space harmonic. As load is used a resistance bank three-phase in the value of 65Ω. The space harmonic are considered up to 25 order.

For simulation effect it is applied a torque of turbine of 7.5 N.m and speed of 1880 rpm.

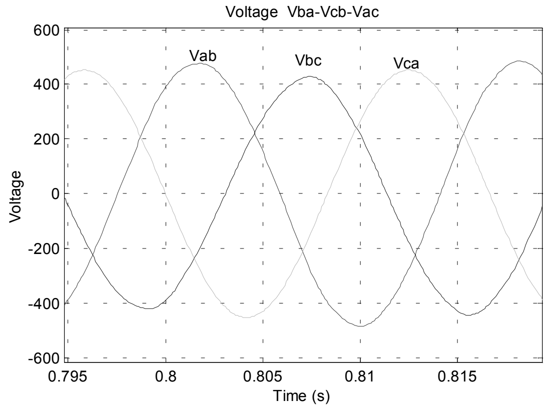

Figure 4 – Generator output voltages – Vab, Vbc, Vca.

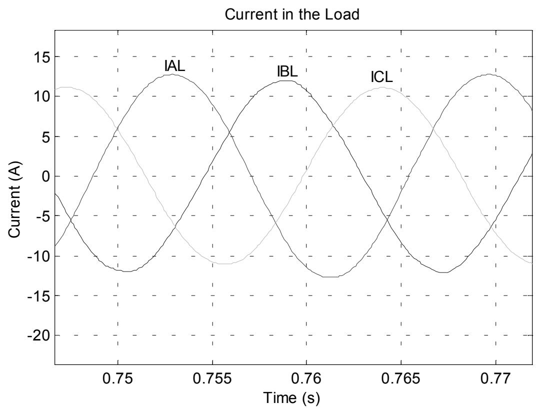

Figure 5 – Current in the load – IAL, IBL, ICL.

The figures 4 and 5 show the generated voltages and the applied line currents the load resisitive three-phase. It can be noticed an unbalance among the values of the generated tensions as well as an unbalance in the currents of the load due to the values of the voltages.

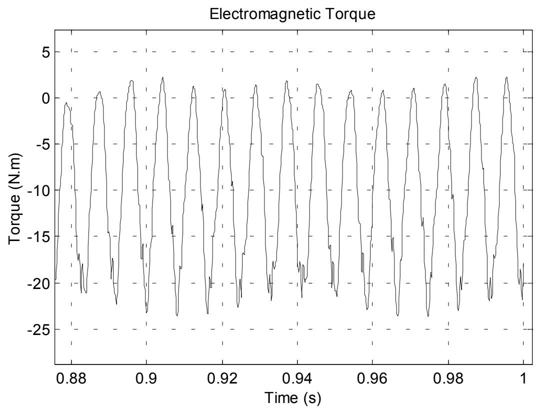

Figure 6 – Electromagnetic torque.

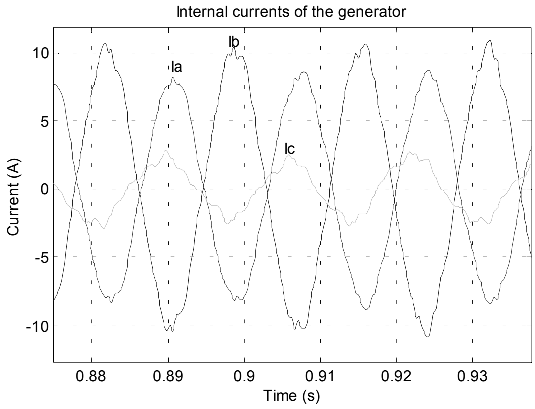

Figure 7 – Internal currents in the generator.

The figures 6 and 7 show the electromagnetic torque and the internal currents in the generator. The unbalanced currents inside the generator promote electromagnetic torque oscillations. These illustrations still show the distortions in the forms of wave of the electromagnetic torque and of the currents provoked by the space harmonic of the machine.

5. Conclusions

From the figures previously presented can be clearly observed the existence of heavy electromagnetic torque oscillations inside the generator; oscillations mainly due to the internal electromagnetic unbalance in the machine. It is still noticed the distortions in the forms of wave of the internal currents of the generator and coming of the space harmonic of the machine electromagnetic torque. This unbalance can be very damaging, in the mechanical point of view, to the generator performance, since they are the cause of highly oscillating rotational speed.

With the use of frequency domain modeling it is not possible to see clearly these effects. Therefore, it is evident that the mathematical model presented here, for a three-phase induction generator connected to a single-phase system, give us conditions and resources for the instantaneous analysis for the electromagnetic unbalance present inside the generator as well as the influence of the space harmonic. In another words it is possible to make the analysis of generated voltage at the machine's terminals. This model still give us conditions to evaluate a more adequate capacitor for the generator operating under such restrictions. So the mathematical model become a valuable tool for the design of such capacitor.

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