A Contribution to the Study on the Effects of Variation for the Slot Rotor Inclination - The Three-Phase Induction Motor Case.

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***Abstract* - The main proposal in this work is the theoretical-experimental analysis of the three-phase induction motor operation under different slot rotor inclination. The linear mathematical model[2] for the motor takes in consideration space harmonics of magnetomotive force (MMF) distribution. The motor feeding is done through a PWM inverter with sinusoidal modulation (PWSM), which means that the time harmonics must be also considered. This study allow us to draw some conclusions on how the slot rotor inclination can change the induction motor behavior.**

I – INTRODUCTION

Nowadays the electrical motors design, despite of its importance, is an area that is not under severe scrutiny. The comprehension of the methodology, the objectives that lead to a motor design, and the optimization of existing designs, it is not an easy but necessary task. Such necessity is very modern. It is easy to realize that recently the driving systems philosophy have changed dramatically, and the induction motors importance grew enormously in the electrical system.

Induction motors are not driven anymore only for sinusoidal sources, but also through rich harmonics converter sources. The interest in such equipment, was raised also by its importance in conservation and quality of energy programs around the world, requiring more efficient machines and driving systems.

This work presents a study about an specific item on the induction motor design: the rotor slot inclination[1]. Laboratory tests and computational simulation have been done using the ordinary rotor inclination obtained from design and other inclinations with a diversity of values.

II - MATHEMATICAL MODEL

While doing this work a three-phase induction motor linear model has been used. This model takes into consideration the space harmonics of stator distributed MMF[2]. With this model is possible to obtain the influence of winding distribution on phase voltage and current waveforms.

A. Magnetomotive Force Spatial Distribution

When currents begin circulate in induction machine windings, the magnetomotive force spatial distribution (MMF) build up in the air-gap of the machine. The distribution of MMF can be discribed [3] considering only one coil from winding mentioned above with coil pitch equal $\beta\pi$ and for a generic phase "j".

$$MMF_{jh}(a) = \left(\frac{2}{p}\right) N_j N_j \sum_{h=1}^{\infty} \frac{1}{h} sen q_h \cos q_b \tag{1}$$

where;

$$q_h = hb\left(\frac{p}{2}\right) \tag{2}$$

$$q_b = h(a - a_b) \tag{3}$$

i_j = current of the phase "j"
N_j = number of turns for phase "j"
α = air-gap angular position from a fixed reference
α_b= position of the coil "b"

B. Magnetic Flux Density

The magnetic field density spatial distribution B_{jh} caused by MMFjh is obtained from Ampere Law.

$$B_{jh} = \frac{m_0}{d} MMF_{jh} \tag{4}$$

where;

μ_0 = air magnetic permeability
δ = air-gap width

Now considering the effects of whole winding of generic phase "j" located at an angle α_j, it can write;

$$B_{jh}(a) = \frac{2}{p}\frac{m_0}{d} N_j q_j Kp_{jh} Kd_{jh} K' \tag{5}$$

where;

$$K' = \frac{i_j}{h}\cos[h(a - a_j)] \tag{6}$$

q_j = distributed coil number
Kp_{jh} = pitch factor for the harmonic order "h"
Kd_{jh} = breadth factor for the harmonic order "h"

The linkage flux in a coil of order "b" of phase winding "i" caused by field of the phase "j" and considering spatial harmonic of order "h" is given by;

$$l_{bjh} = 2pN_i \int_{a_b - b_i\frac{p}{2}}^{a_b + b_i\frac{p}{2}} B_{jh} LRda \tag{7}$$

where;

L = magnetic length of rotor
R = air-gap average radius
2p = pole number of induction machine

Solving equation (7) it can be written;

$$l_{ijh} = K_1 Kw_{ih} Kw_{jh} K'' \tag{8}$$

where;

$$K_1 = 4\frac{2pLRm_0 q_i q_j N_i N_j}{pd} \tag{9}$$

$$K'' = \frac{i_j}{h^2}\cos[h(a_i - a_j)] \tag{10}$$

Kw_{ih} = winding factor of the phase "i"
Kw_{jh} = winding factor of the phase "j"
q_i = distributed coil number of phase "i"
q_j = distributed coil number of phase "j"
N_i = turns number of phase "i"
N_j = turns number of phase "j"

C. Electrical and Electromagnetic torque equations

Basic voltage equations of the phase "i" at an induction motor is given by;

$$v_i = R_i i_i + \frac{d\,l_i}{dt} \tag{11}$$

where;

R_i = winding resistance of phase "i"
i_i = current of the phase "i"
λ_i = total linkage flux of phase "i"

The total magnetic flux linkage is given by;

$$l_i = Lm_i i_i + \sum_h \sum_j Lm_{ijh} i_j \tag{12}$$

where;

Lm_i = leakage inductance in phase "i"
Lm_{ijh} = h^{th} order harmonic between phases "i and j"
Considering Lm_i constant and using equations (11) and (12) it has;

$$v_i = R_i i_i + Lm_i \frac{di_i}{dt} + \sum_h \sum_j \left[Lm_{ijh} \frac{di_j}{dt} + i_j \frac{dLm_{ijh}}{dt} \right] \tag{13}$$

The electromagnetic torque is obtained from magnetic co-energy variation of the system (W_c) relatively to the electrical displacement of the rotor (θR).

$$Tel = p\frac{\partial W_c}{\partial q_R}\Big|_{I_{const}} \tag{14}$$

The magnetic co-energy related with hth order harmonic inductance Lm_{ijh} [ ] and phase currents is given by;

$$W_c = \frac{1}{2}\sum_h \sum_i \sum_j Lm_{ijh} i_i i_j \tag{15}$$

where;

$$Lm_{ijh} = \frac{l_{ijh}}{i_j} \tag{16}$$

D. Slot rotor inclination effect

Figure (1) shows the slot rotor inclination and its effect[3] can be obtained from the following equation:

$$q_j . a_{Rj} = \frac{a_s}{2} \tag{17}$$

Where;

α_s = rotor slot inclination angle.
α_{Rj} = angular displacement between slots

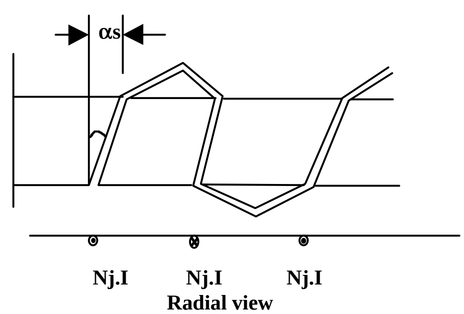

Figure 1- Equivalent coil with inclination αs

or

$$sen\left(\frac{hqa_{Rj}}{2}\right)\cos\left(\frac{hqa_{Rj}}{2}\right) = sen\left(\frac{ha_s}{2}\right)\cos\left(\frac{ha_s}{2}\right) \qquad (18)$$

knowing that α_{Rj} is an angle between slots and when it is close to zero would have the ideal condition to induction machine because so it have most uniformity;

$$\lim_{a_{Rj}\to 0}\left[q_j sen\left(\frac{ha_{Rj}}{2}\right)\right] = \frac{qha_{Rj}}{2} = \frac{has}{4} \qquad (19)$$

Using equations (1), (18) and (19), it obtains;

$$Fmm_{jh}(a) = \left(\frac{4}{p}\right)\left(\frac{N_j I}{2h}\right)\frac{sen\left(\frac{ha_s}{4}\right)\cos\left(\frac{ha_s}{4}\right)}{\frac{ha_s}{4}} \qquad (20)$$

However;

$$sen\left(\frac{ha_s}{4}\right)\cos\left(\frac{ha_s}{4}\right) = \frac{1}{2}sen\left(\frac{ha_s}{2}\right) \qquad (21)$$

Equation (20) becomes;

$$Fmm_{jh}(a) = \left(\frac{4}{p}\right)\left(\frac{N_j I}{2h}\right)K_{sh} \qquad (22)$$

Where the variable;

$$K_{sh} = \frac{sen\left(\frac{ha_s}{2}\right)}{\frac{ha_s}{2}} \qquad (23)$$

is called inclination factor to order harmonic "h". Considering Z as an arc established between slots, it can be written;

$$a_s = \frac{Z}{t_p}p \qquad (24)$$

where;

τ_p = polar step measured on stator surface

so, K_{sh} can be written;

$$K_{sh} = \frac{sen\left[h\left(\frac{Z}{t_p}\right)\frac{p}{2}\right]}{h\left(\frac{Z}{t_p}\right)\frac{p}{2}} \qquad (25)$$

and finally, it obtains;

$$Fmm_{jh}(a) = \frac{4}{p}\frac{N_j I}{2h}K_{ph}K_{dh}K_{sh} \qquad (26)$$

III - RESULTS

One can observes the digital computer simulation results using figures (2-7). Figures (2-4) show the stator current and its harmonic spectrum and electromagnetic torque when induction motor is into operation with ordinary slot inclination 7^0 (mechanical degrees).

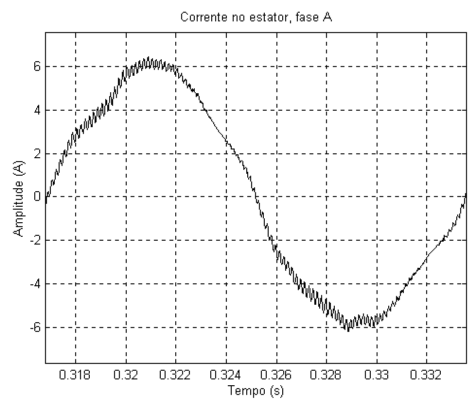

Fig. 2 – Phase current of the induction motor for a 7 mechanical Degrees rotor slot inclination.

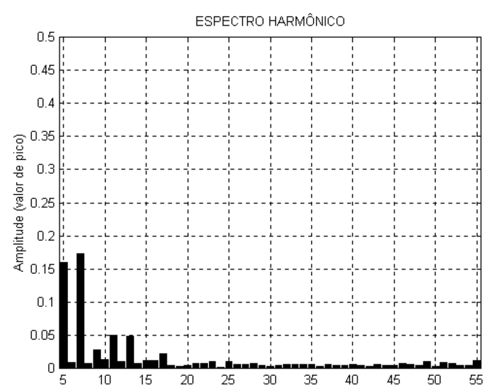

Fig. 3 – Phase current harmonic spectrum of the induction motor For a 7 mechanical degrees rotor slot inclination

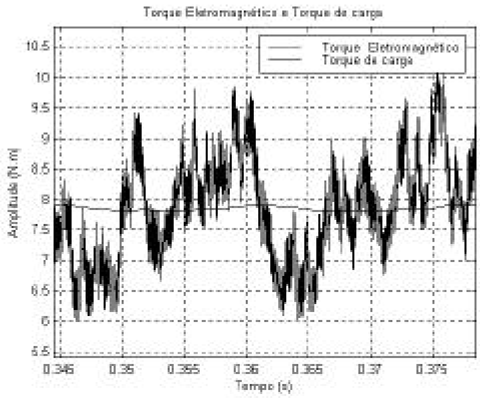

Fig. 4 – Electromagnetic torque of the induction motor for a 7 mechanical degrees rotor slot inclination .

Figures (5-7) show the stator current and its harmonic spectrum and electromagnetic torque when induction motor is into operation with ordinary slot inclination 13^0 (mechanical degrees).

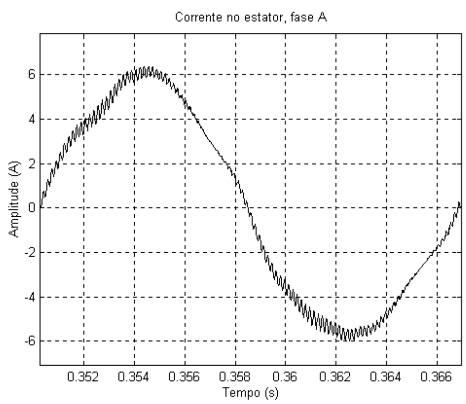

Fig. 5 – Phase current of the induction motor for a 13 mechanical Degrees rotor slot inclination.

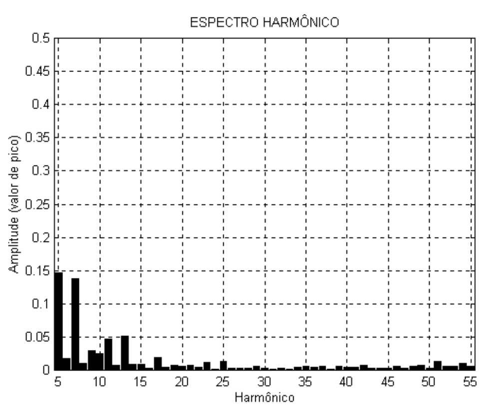

Fig. 6 – Phase current harmonic spectrum of the induction motor For a 13 mechanical degrees rotor slot inclination

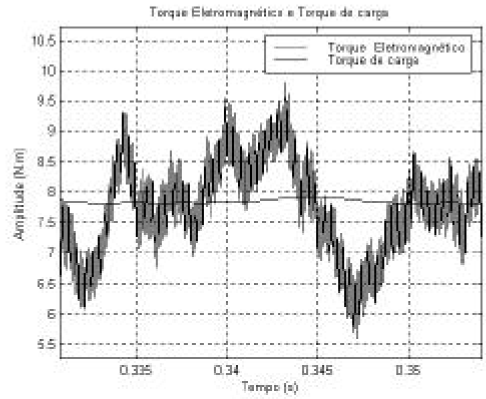

Fig. 7 – Electromagnetic torque of the induction motor for a 13 mechanical degrees rotor slot inclination .

The following figures are obtained from laboratory tests where two situations were taken into consideration: induction motor operation with ordinary slot inclination (7 mechanical degrees) and (13 mechanical degrees). Figures (8-11) show the stator current and its harmonic spectrum.

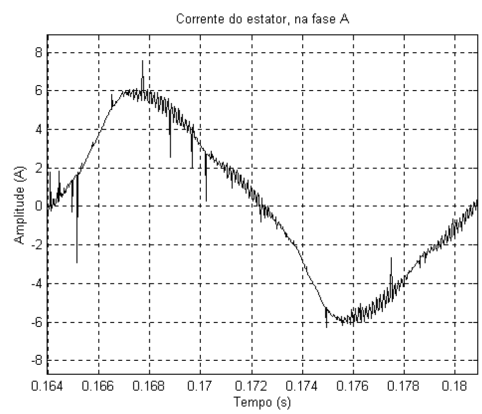

Fig. 8 – Phase current of the induction motor for a 7 mechanical Degrees rotor slot inclination.

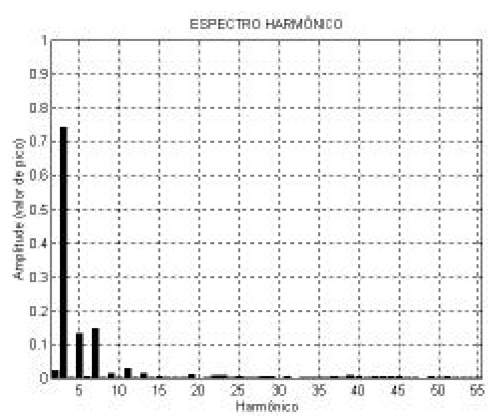

Fig. 9 – Phase current harmonic spectrum of the induction motor For a 7 mechanical degrees rotor slot inclination

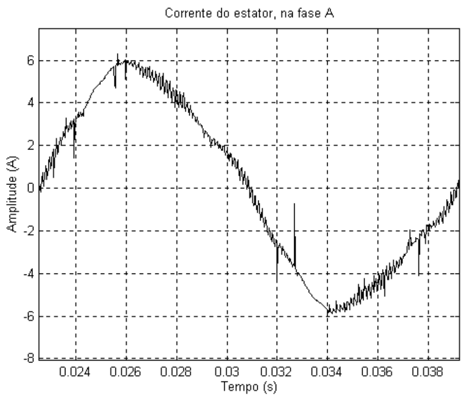

Fig. 10 – Phase current of the induction motor for a 7 mechanical Degrees rotor slot inclination.

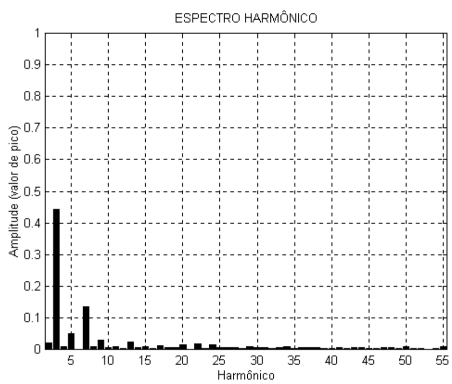

Fig. 11– Phase current harmonic spectrum of the induction motor For a 7 mechanical degrees rotor slot inclination

IV - CONCLUSIONS

Through the analysis of computer simulations and laboratory results, can be observed an improvement on the motor phase current performance for the operating condition where the rotor has its slots inclined of 13 mechanical degrees. This fact becomes evident through the phase current harmonic spectrum show in figures (3) and (6) in simulation results and figures (9) and (11) in laboratory tests, where a decrease can be seen in the 3^{rd}, 5^{th}, 7^{th} harmonic. One can notice that 3^{rd} harmonic do not appear in simulation results and can be seen in laboratory tests. That is because non-linear behavior of the induction machine do not be represented in the mathematical model used in the simulations.

Electromagnetic torque can be watched in figures (4) and (7) where it can observed a better torque behavior when the slot inclination has 13 mechanical degrees than it has 7 mechanical degrees.

V – REFERENCES

[1] R.B. Lehmann, A Contribution to the Study of the Rotor Slot Inclination Effects on the Three-Phase Induction Motor, MSc Dissertation, Universidade Federal de Uberlândia, Electrical Engineering Department, August 1999 (In Portuguese).

[2] Neto, L.M.; Camacho, J.R.; Salerno, C.H. & Alvarenga, B.P.; Analysis of a Three-Phase Induction Machine Including Time and Space Harmonic Effects: The A, B, C Reference Frame; PES-IEEE Transactions on Energy Conversion, Volume 14, Nr. 1, March 1999. Article no.: PE-154-EC-0-10-1997.

[3] LIPO, T.A. ; Introduction to AC Machine Design, Vol. I, Madison, Wisconsin, University of Wisconsin, U.S.A.,1996.