A Study of Composite Resonance in AC/DC Converters

Sreeramulu Raghuram Naidu and Antonio Marcus Nogueira Lima

Department of Electrical Engineering - DEE
Centro de Ciências e Tecnologia - CCT
Universidade Federal da Paraíba - UFPB
58109-970 Campina Grande, Paraíba, BRAZIL
naidu@dee.ufpb.br - amnlima@dee.ufpb.br

Abstract - **A time domain technique has been used to simulate single-phase and 3-phase converters. The technique allows the computation of converter impedances or admittances viewed from the AC or DC bus. At any operating point, a linear circuit represents the converter and harmonic magnification factors are calculated. Composite resonance has been investigated using a small signal comprising several harmonics. The cross-frequency effects of composite admittances are therefore included. Response of the linear circuit to the signal has been used to locate the frequencies for which there is significant distortion in the AC-bus voltage or the DC current.**

I. INTRODUCTION

Waveform distortion in AC/DC converters has been very difficult to quantify and consequently, has not been well understood. It has been observed that AC/DC converters with low short-circuit ratio experience high levels of waveform distortion. This has been attributed to the high AC system impedance, whose inductance may resonate with the capacitors and filters installed at the converter's AC side. It is believed that a resonance of this nature may lead to harmonic instability.

The AC/DC converter is the interconnection of the AC and DC systems via the static converter. The AC system impedance interacts through the converter characteristics to present entirely different impedance to the DC side. This gives rise to resonance frequencies, which depend on the AC system impedance, the DC system impedance and the switching of the converter. The resonance is 'composite', implying its dependence on all elements of the AC/DC converter.

Several contributions have appeared in the literature on harmonic instability but very few have directly addressed the phenomenon of composite resonance. In fact, the concept of impedance seen from the converter terminals has not yet been clearly defined.

In one study [1], the linear relationships between integer harmonics on both sides of the converter have been obtained numerically from the converter simulation. The converter control system has been included in the simulation and the overall harmonic impedances at both the AC and DC terminals are derived. The harmonic impedance has been used to predict lightly or negatively damped integer harmonics.

In another contribution [2], a frequency domain analysis has been used to obtain a set of simultaneous equations, which, after considerable manipulation, reduce to a matrix equation relating the AC and DC harmonics. The converter impedance seen by the DC system is derived from this equation. At this stage, the expression for the composite impedance becomes complicated. An equivalent RLC network is derived which matches the composite impedance at its resonant frequency. The composite resonance damping is taken to be identical to the damping factor of the equivalent RLC circuit.

Composite impedance is essentially a matrix quantity. This is true for the composite impedances seen on the AC and DC sides of the converter. Any single harmonic component of current flowing into a composite impedance produces a multitude of voltage harmonics. Some means for identifying the resonance frequencies becomes necessary. Amplification factors have been used [3] to isolate the resonant frequencies and these factors are defined as the transfer functions from a fictitious voltage source placed in series with the converter to the voltage across the dc filter.

In this paper, a time-domain approach for determining the steady-state responses of AC/DC converters is described. The simulation procedure is general and can include the frequency-dependence of parameters. Only uncontrolled converters have been simulated in this study. However, the inclusion of control systems is straightforward. At any operating point, the linear equivalent circuit of the converter is derived. A small voltage signal comprising several harmonic frequency components has been used as the excitation for the linear circuit to determine the harmonic frequency for which there could be significant increases in the AC-bus voltage harmonics or DC current harmonics.

II. TIME-DOMAIN IMPEDANCES

A $n-$vector of equidistant samples may represent a periodic band-limited waveform. If $[x]$ is the vector of equidistant samples representing a waveform, then

$$[X] = (1/n) \cdot fft([x]) \qquad (1)$$

is the phasor representation of the waveform.

The first element of $[X]$ is the DC component of $[x]$. The next m elements are the phasors representing the $'m'$ harmonic components of $[x]$, where $m = n/2 - 1$. The phasor component at the highest frequency is $X(m+2)$. The subsequent m elements from $X(m+3)$ to $X(n)$ are the conjugates of $X(2)$ to $X(m+1)$ flipped upside down. In Matlab notation, the vector $[X]$ is written as:

$$[X] = \begin{bmatrix} X_{DC} \\ X(2:m+1) \\ X(m+2) \\ \mathrm{flipud}(\mathrm{conj}(X(2:m+1))) \end{bmatrix}. \qquad (2)$$

Let $[Z]$ be the complex impedance matrix representing a linear circuit. It is a diagonal matrix and its diagonal of harmonic impedances has the same structure as that given by (2). Let $[e]$, $[i]$ be periodic voltage and current waveforms associated with the impedance. Then the phasor representations $[E]$,$[I]$ are related by:

$$[E] = [Z] \cdot [I]. \qquad (3)$$

From (3), we obtain

$$\begin{aligned} [e] &= [ifft][Z][fft] \cdot [i] \\ &= [z] \cdot [i], \end{aligned} \qquad (4)$$

where $[z]$ is the impedance matrix in the time domain. It is a real, full matrix. The computation of the impedance matrix requires only the inverse fast-Fourier transform of the diagonal of $[Z]$.

In contrast to the impedance matrix of a linear circuit, the time domain conductance matrix of a thyristor switch is diagonal and the elements on the diagonal are the equidistant samples of the thyristor's conductance over a fundamental period. When transformed to the harmonic domain, the complex conductance matrix is a full matrix, with the off-diagonal elements representing the cross-frequency coupling admittances. The elements on the diagonal are the harmonic self-admittances.

III. CONVERTER ANALYSIS

A. Single-phase AC/DC converter

Fig. 1 shows a single-phase AC/DC converter. The thyristor pairs are switched simultaneously and they are assumed to operate identically. An equidistant switching scheme is used for the thyristor pairs.

Let $[e_a]$,$[e_d]$ be the terminal voltage waveforms on either sides of the thyristor bridge. Then the thyristor voltages are given by

$$\begin{aligned} [v_{s1}] &= [v_{s4}] = [e_1] = 0.5 \cdot \{[e_d] - [e_a]\}, \\ [v_{s2}] &= [v_{s3}] = [e_3] = 0.5 \cdot \{[e_d] + [e_a]\}. \end{aligned} \qquad (5)$$

The thyristor voltages and the switching instants enable the calculation of the thyristor conductance variations from the nonlinear characteristics. Let the diagonal conductance matrices be

$$\begin{aligned} [g_{s1}] &= [g_{s4}] = [g_1], \\ [g_{s2}] &= [g_{s3}] = [g_3]. \end{aligned} \qquad (6)$$

Applying Kirchhoff's laws to the converter circuit,

$$\begin{aligned} &[c_{sc}] - [ya] \cdot [e_a] - [g_3] \cdot [e_3] + [g_1] \cdot [e_1] = 0 \\ &[yd] \cdot \{[e_d] - [e_{DC}]\} + [g_3] \cdot [e_3] + [g_1] \cdot [e_1] = 0, \end{aligned} \qquad (7)$$

where $[c_{sc}]$ is the short-circuit current on the AC side of the thyristor bridge, $[e_{DC}]$ is the emf of the DC system, $[ya]$ and $[yd]$ are the AC and DC system admittances. The solutions $[e_a]$, $[e_d]$ of equations (7) are computed iteratively using the Newton-Raphson procedure.

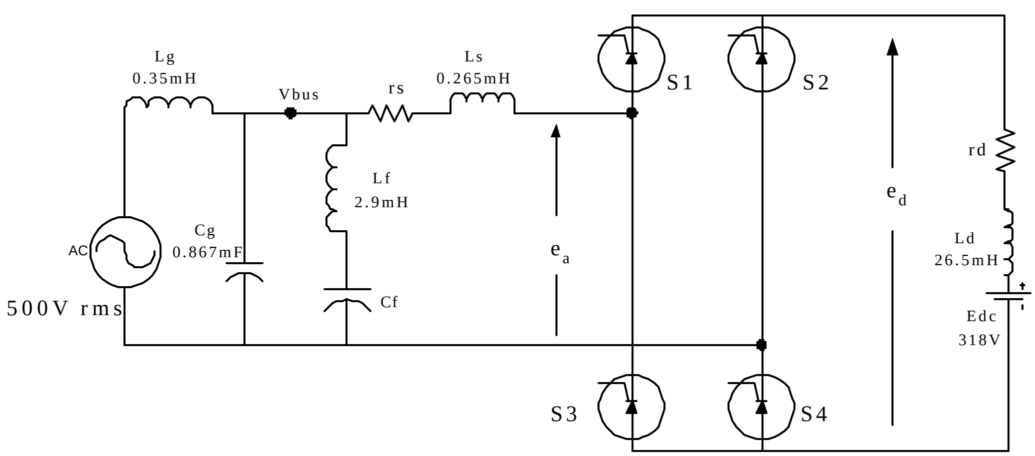


Fig. 1. Single-phase converter

B. AC side composite resonance

Using circuit analysis techniques, it may be shown that the converter admittance on the AC side of the bridge is given by

$$[ycon_{ac}] = [g_a] - [g_b] \cdot \{[g_a] + [yd]\}^{-1} \cdot [g_b], \qquad (8)$$

where $[g_a] = \frac{1}{2} \cdot \{[g_3] + [g_1]\}$ and $[g_b] = \frac{1}{2} \cdot \{[g_3] - [g_1]\}$. We now suppose that the AC source is modulated by a test signal $[e_t]$ and that it is required to determine the effect of the modulation on the AC-bus voltage harmonics. Let $[v_0]$ be the open-circuit voltage at the input terminals of the converter when the test signal $[e_t]$ is the excitation. Then the AC-bus voltage due to the test signal is

$$\begin{aligned}[e_c] &= \{[ya]+[ycon_{ac}]\}^{-1} \cdot [ya] \cdot [v_0] \\ &= [af_{ac}] \cdot [e_t].\end{aligned} \tag{9}$$

If any of the harmonic amplitudes of $[e_c]$ is significantly higher than the corresponding harmonic amplitude of the test signal, then there is the possibility of composite resonance. The matrix $[af_{ac}]$ is the transfer function from the test signal to the AC-bus voltage. It is not a diagonal matrix and therefore, each harmonic component of $[e_c]$ is the summation of the contributions from all the harmonic components of $[e_t]$.

C. Test Signal

In order to include the cross-frequency effects, a test signal has been constructed by adding together a range of harmonic components from the 2nd to 16th, each component having the same amplitude but with a quadratic phase shift [4]. The test signal is of the form:

$$[e_t] = \sum_{k=2}^{16} A \cdot \sin k\omega(t + k^2 \Delta t), \tag{10}$$

where $\Delta t = T/n = 1/(f \cdot n)$. Only low order harmonic interaction is of interest and therefore, the highest harmonic order included in the test signal is limited to 16. The quadratic phase shift ensures that there are no large spikes in the test signal. The amplitudes of the harmonic components may be chosen so that the total harmonic distortion (THD) of the source voltage when modulated by the test signal is 1%. The low THD guarantees that the switch-off instants of the thyristors are not affected by the modulation. Fig. 2 shows the test signal used in this study.

Finally, composite resonance on the DC side may be studied by reducing the converter to an equivalent impedance matrix $[ycon_{dc}]$ when viewed from its output terminals.

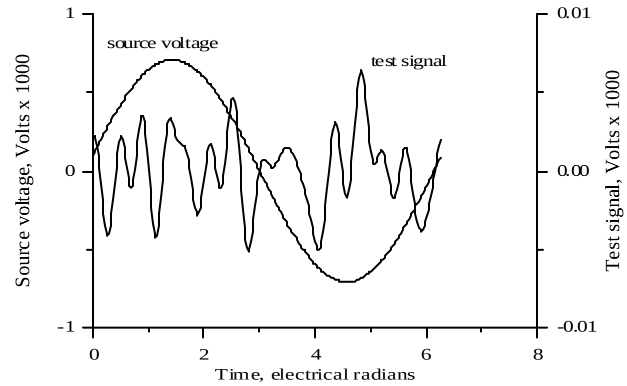
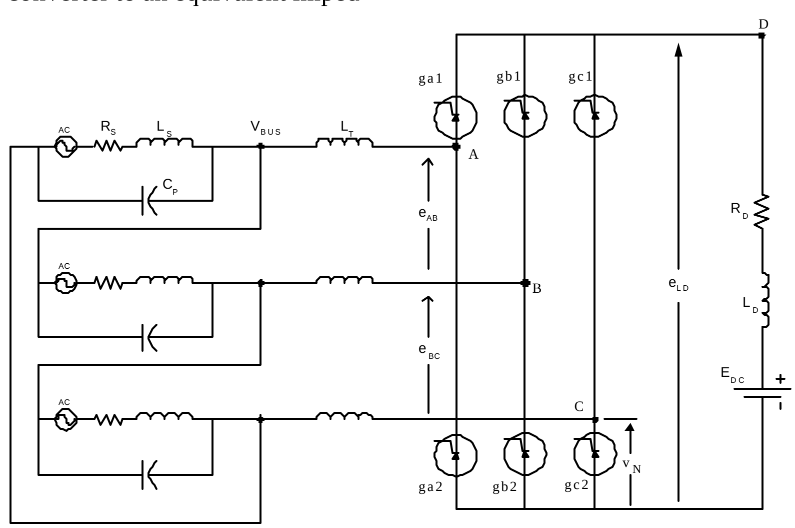


Fig. 2. Test signal waveform

IV. PHASE AC/DC CONVERTER

A 3-phase, 6-pulse AC/DC converter is shown in Fig. 3. The AC system consists of a source, its internal impedance, a shunt capacitance and the transformer's leakage inductance between the AC-bus and the converter input terminals. There are 4 unknown voltage waveforms to determine. These voltages are $[e_{ab}], [e_{bc}], [e_{ld}]$ at the input and output terminals of the bridge and the voltage $[v_n]$ of one of the thyristor valves.

The equations for the unknown variables are obtained by applying KCL at the nodes A, C and D. The fourth equation is obtained by stipulating that the sum of the currents in the upper row of thyristors is equal to the sum of the currents in the lower



$R_S = 21\Omega$, $L_S = 0.558H$

$L_T = 0.0936H$, $R_D = 0.005\Omega$

Fig. 3. Three-phase AC/DC converter

row of thyristors. These equations are nonlinear because the thyristor conductance matrices are dependent on the unknown voltages and the switching instants. The equations are solved iteratively using the Newton-Raphson procedure. The computations are listed below.

Compute the admittance matrices $[yf], [yd]$ of the AC and DC systems respectively and the short-circuit currents at the input terminals of the thyristor bridge. The short-circuit currents are $[clab1], [clbc1], [clca1]$ for the phases AB, BC and CA. The matrix $[yf]$ includes the leakage inductance of the transformer.

1. Assume the waveforms $[e_{AB}], [e_{BC}], [e_{LD}]$ and $[v_N]$.
2. Calculate the DC current: $[c_{LD}] = [yd] \cdot [e_{LD}]$.
3. Using the DC current waveform obtained in step 2, obtain the switching instants from the current-controller.
4. Compute the diagonal conductance matrices for the thyristors.
5. Apply Kirchhoff's current law as described previously. Since the waveforms in step 1 are solution estimates, KCL will not be satisfied but will give the residual currents:

$$\begin{aligned}&(clab1 - clca1) - yf \cdot e_{AB} - yf \cdot (e_{AB} + e_{BC})\\&- ga2 \cdot e_{A2} + ga1 \cdot e_{A1} = \delta j_{AB}\end{aligned}, \tag{11}$$

$$\begin{aligned}&(clbc1 - clca1) - yf \cdot e_{BC} - yf \cdot (e_{AB} + e_{BC})\\&+ gc2 \cdot e_{C2} - gc1 \cdot e_{C1} = \delta j_{BC} - \delta j_N\end{aligned}, \tag{12}$$

$$\begin{aligned}&ga1 \cdot e_{A1} + gb1 \cdot e_{B1} + gc1 \cdot e_{C1}\\&+ yd \cdot (e_{LD} - E_{DC}) = \delta j_{LD}\end{aligned}, \tag{13}$$

$$\begin{aligned}&ga1 \cdot e_{A1} + gb1 \cdot e_{B1} + gc1 \cdot e_{C1} - ga2 \cdot e_{A2}\\&- gb2 \cdot e_{B2} - gc2 \cdot e_{C2} = \delta j_N\end{aligned}, \tag{14}$$

where

$$\begin{aligned}&e_{A2} = e_{AB} + e_{BC} + v_N; \quad && e_{A1} = e_{LD} - e_{A2};\\&e_{B2} = e_{BC} + v_N; && e_{B1} = e_{LD} - e_{B2};\\&e_{C2} = v_N; && e_{C1} = e_{LD} - e_{C2}.\end{aligned}$$

Note that equations (11)-(14) are matrix equations. These equations are written in the form:

$$\begin{bmatrix} clab1 - clca1 \\ clbc1 - clca1 \\ -yd \cdot E_{DC} \\ 0 \end{bmatrix} - [DG] \cdot \begin{bmatrix} e_{AB} \\ e_{BC} \\ e_{LD} \\ v_N \end{bmatrix} = \begin{bmatrix} \delta j_{AB} \\ \delta j_{BC} \\ \delta j_{LD} \\ \delta j_N \end{bmatrix}. \tag{15}$$

6. The corrections $\delta e_{AB}, \delta e_{BC}, \delta e_{LD}, \delta v_N$ that should be added to the assumed waveforms to minimize the residual currents are:

$$\begin{bmatrix} \delta e_{AB} \\ \delta e_{BC} \\ \delta e_{LD} \\ \delta v_N \end{bmatrix} = [DG]^{-1} \cdot \begin{bmatrix} \delta j_{AB} \\ \delta j_{BC} \\ \delta j_{LD} \\ \delta j_N \end{bmatrix}. \tag{16}$$

7. Return to step 1 if the maximum norm of the residuals is above a specified tolerance.

A. AC-side composite resonance

Removing the AC and DC system excitations and substituting the thyristors by their respective conductance matrices gives the linear circuit representing the AC/DC converter at a given operating point. The impact of a small negative-sequence, test-signal distortion in the AC system voltages may be estimated by using the linear converter circuit. It is assumed that the distortion in the AC system excitation will not affect the switching of the thyristors.

Consider the linear circuit of the converter at an operating point and let the excitation be a 3-phase, negative-sequence test-signal of amplitude δv_{NS}. The test-signal in each phase is the composition of harmonic voltages as given in equation (10). Let $[\delta V_{AB}]$ be the phasor representation of the AC-bus voltage waveform between the terminals A and B. The AC-side harmonic magnification factors are defined as:

$$[m_{ac}] = (1/\delta v_{NS}) \cdot abs([\delta V_{AB}]) \tag{17}$$

Of particular interest is the 4th element of $[m_{ac}]$ which is the magnification factor for the 3rd harmonic. High values of this element indicate AC-side composite resonance at the 3rd harmonic.

B. DC-side composite resonance

Consider again, the linear circuit of the converter excited by the 3-phase, negative-sequence test-signal. Let $[j_0]$ be the short-circuit current at the DC terminals of the linear circuit with the negative-sequence voltage as the AC-system excitation. Let $[j_C]$ be the DC current when the DC system is connected to the converter. The amplification factor at the kth harmonic is defined as the ratio of the amplitudes of the kth harmonic of $[j_C]$ to the corresponding amplitude of $[j_0]$.

The magnification factor depends on the relative phase of the negative-sequence voltage, which is random. Only the maximum magnification factor as the phase varies, is considered. A high magnification factor implies the possibility of composite resonance on the DC-side.

V. RESULTS AND DISCUSSION

Jalali has previously studied the single-phase converter shown in Fig. 1 [4]. The switching sequence for the converter was first adjusted so that the overlap angle in the steady state was 22.85°. Two solutions were obtained, the first with an undistorted AC source and the second with the AC source modulated by the test signal.

The difference between the AC current waveforms is shown in Fig. 4. This is almost identical to the AC current waveform obtained for the linear circuit with the test signal as the excitation. Therefore, the linear equivalent of the converter circuit in the neighborhood of an operating point is valid.

Fig. 5 shows the harmonic amplitudes of the AC-bus voltage when the test signal is the excitation and it may be observed that there is significantly high 5^{th} harmonic amplitude. It may also be noted that there is a difference in the harmonic amplitudes when the 5^{th} harmonic component of the test signal alone is used as the excitation. This is a demonstration of the cross-frequency effect.

The harmonic amplitudes of the AC-bus voltage obtained from the solutions with and without the test signal modulation are shown in Fig. 6. Though the change in the 5^{th} harmonic amplitude is nearly 20 times the amplitude of the test signal, it is swamped by the inherent 5^{th} harmonic component of the AC-bus voltage waveform.

The parameter values for the 6-pulse converter are shown in Fig. 3. The thyristors are switched using an equidistant firing scheme. The firing instants are adjusted so that the DC current is approximately 60% of the current corresponding to $\alpha = 0$. The primary concern of this study is the impact of a 3-phase, negative-sequence test-signal distortion in the AC system voltages on the AC-bus voltage and DC current harmonics.

Fig. 7 shows the harmonic magnification factors for the AC-bus voltages when there is a 0.5% THD, negative-sequence test-signal distortion in the AC system voltages. A very high gain at the 3^{rd} harmonic may be noted. Fig. 8 shows one of the AC-bus voltage waveforms. The waveform is significantly changed by the small distortion, implying composite resonance at the 3^{rd} harmonic. The figure also shows the AC-bus voltage waveform when the distortion consists of the fundamental frequency, negative-sequence alone. In this case, the deviation in the waveform is smaller and this is a demonstration of the cross-frequency coupling. Fig. 9 shows the harmonic amplitudes of the AC-bus voltage waveforms.

Fig. 10 shows the DC-side harmonic magnification factors when there is a 1% THD, negative-sequence, test-signal distortion in the AC-system voltages. A reasonable 2^{nd} harmonic gain may be noted though this is not as high as the AC-side, 3^{rd} harmonic gain. Fig. 11 shows the DC current waveforms and the deviations due the distortions in the AC system voltages may be observed. The DC current harmonic amplitudes are shown in Fig. 12.

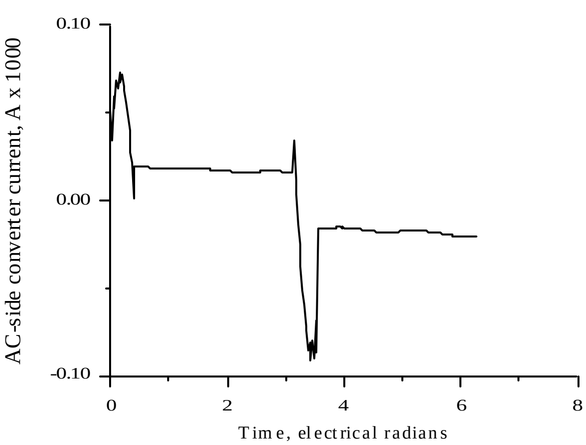


Fig.4. AC-side current for test-signal excitation

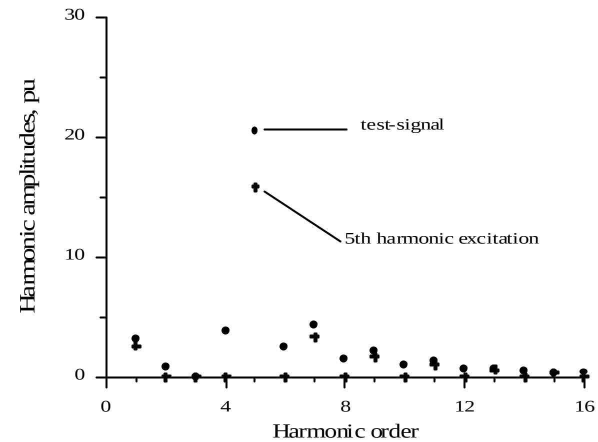


Fig.5. AC-bus voltage harmonic amplitudes for test-signal excitation

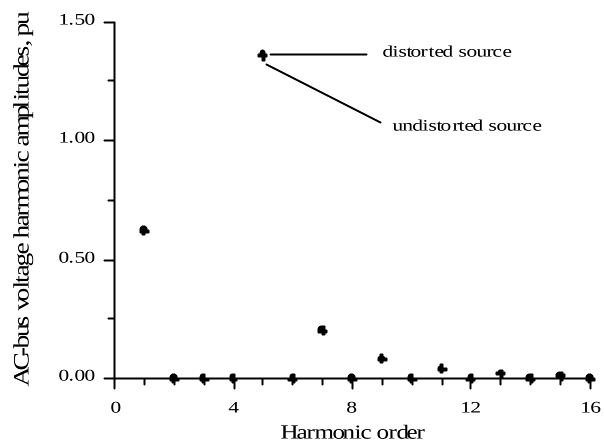


Fig.6. 1-phase converter: AC-bus voltage harmonic amplitudes

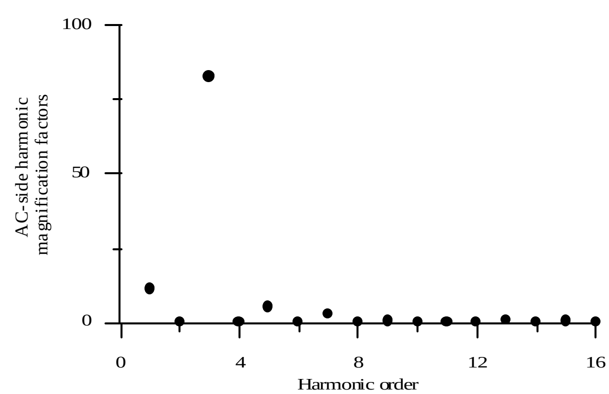


Fig. 7. AC-side harmonic magnification factors

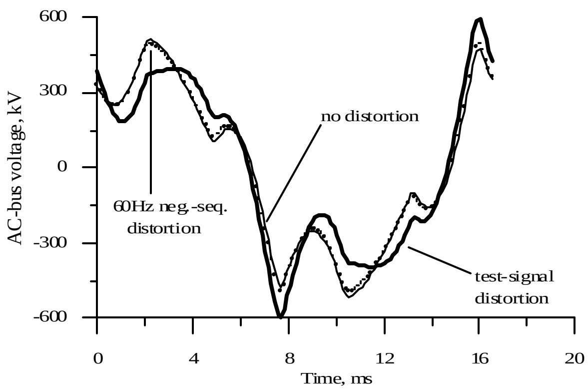


Fig. 8. 3-phase converter: AC-bus voltage waveforms

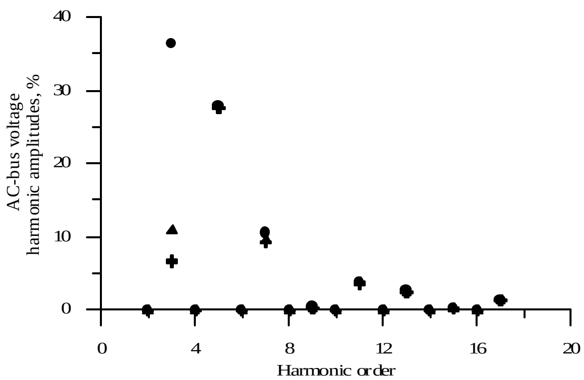


Fig. 9. AC-bus voltage harmonic amplitudes

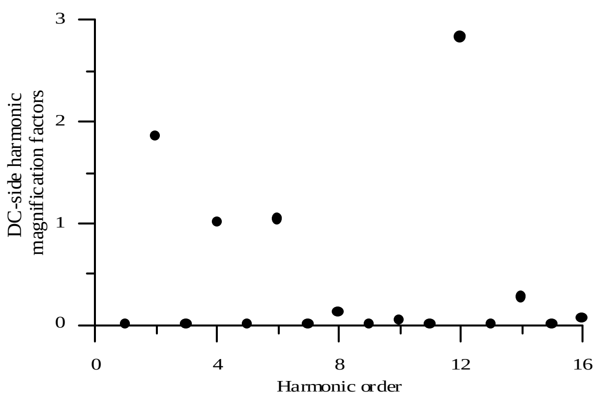


Fig. 10. DC-side harmonic magnification factors

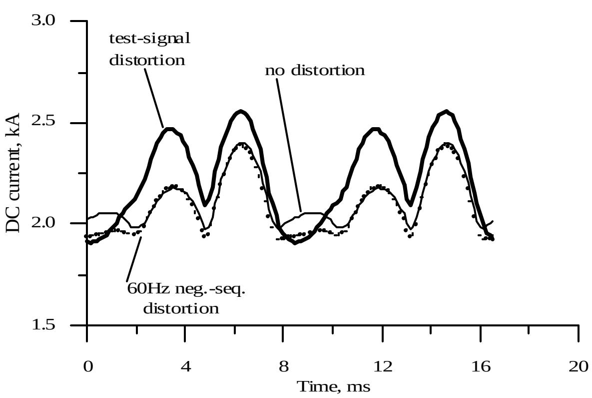


Fig. 11. 3-phase converter: DC current waveforms

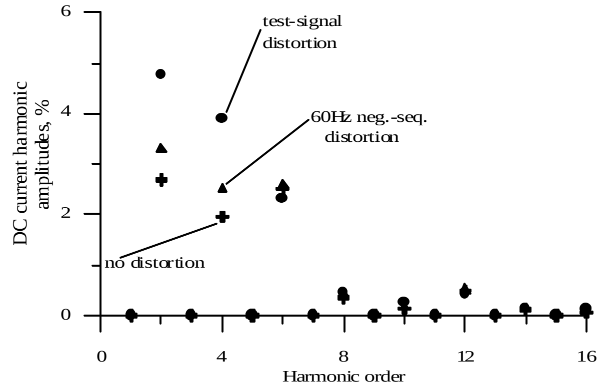


Fig. 12. DC current harmonic amplitudes

VI. CONCLUSIONS

The admittances presented by a converter at the AC and DC terminals depend on the operating conditions of the circuit such as the firing angle delay and filter circuit parameters. These admittances are instrumental in amplifying the harmonics in the DC current and the AC-bus voltage.

A single-phase converter and a 3-phase, 6-pulse converter have been simulated using a time-domain technique. A suitable test signal, which is the composition of several harmonic frequencies, has been used to pinpoint the frequencies for which there could be significant harmonic distortion in the AC-bus voltage or the DC current. Linearization of the converter circuit in the neighborhood of an operating point has been validated and the cross-frequency effect of composite impedances has been demonstrated.

The converters have been simulated without any closed-loop control. However, the inclusion of control systems and the investigation of harmonic instability are straightforward.

VII. REFERENCES

[1]. E. V. Larsen, D. H. Baker, J. C. McIver, " Low-order harmonic interactions on AC/DC systems", IEEE Trans. on Power Delivery, vol.4 (1), Jan.1989, pp.493-501.

[2]. A. R. Wood, J. Arrillaga, "Composite resonance; a circuit approach to the waveform distortion dynamics of an HVDC converter", IEEE Trans. on Power Delivery, vol.10 (4), Oct.1995, pp.1882-1888.

[3]. M. P. Bahrman, G. C. Brownell, K. J. Peterson, P. R. Shockley, R. H. Lasseter, "DC system resonance analysis", IEEE Trans. on Power Delivery, vol.2 (1), Jan.1987, pp.156-164.

[4]. Xiao Jiang, A. M. Gole, "A frequency scanning method for the identification of harmonic instabilities in HVDC systems", IEEE Trans. on Power Delivery, vol.10 (4), Oct.1995, pp.1875-1881.