

# A Comparative Analysis Between Phase-Shift Transformers for Application in Multi-Level Converters

Angelo J.J. Rezek; J.P.G. Abreu; Valberto F. Silva; J.A. Cortez;  
Carlos R. Borges; Adriana A.S. Izidoro

Federal Scholl of Engineering of Itajubá – Electrical Engineering Institute  
EFEI/IEE/DET/DON  
P.O.Box:50 - Zip Code:37500-903 Tel: +553536291377 Fax: +553536291187  
Itajubá, MG- Brazil - e-mail [rezek@iee.efei.br](mailto:rezek@iee.efei.br)

**Abstract** - The aim of this paper is to make a comparative analysis between the transformers with the secondaries connected in zig-zag and extended delta, which are adequate to obtain some necessary angular displacements, typical of 12,24,48-pulse multiconveter systems.

For 12-pulse converters, the star and delta connections could be used. In this case, the angular displacement is thirty degrees. For obtaining 24 and 48-pulse converters, secondary connections in zig-zag or extended delta should be employed, in order to obtain the adequate angular displacements, fifteen and seven and half degrees, respectively.

In this paper a comparative analysis in terms of copper expent, winding power and insulation and harmonic contents will be made, taking into account the use of zig-zag and extended delta connections.

## I. INTRODUCTION

Harmonic mitigation can be achieved when the converter's pulse number is increased. However, special transformers should be used, in order to obtain the suitable angular displacements of fifteen and seven and half degrees between the supply secondary voltage of 24 and 48 pulse converters, respectively. For this application, the secondary connection of the transformers can be zig-zag or extended delta. The

secondary taps of such zig-zag and extended-delta connections will be determined in order to obtain the angular displacements of  $0^\circ$ ;  $7.5^\circ$ ;  $15^\circ$ ;  $22.5^\circ$ ;  $30^\circ$ ;  $37.5^\circ$ ;  $45.0^\circ$ ;  $52.5^\circ$ . A recent application of these transformers is, for instance, in multi-level 48-pulse inverters, to be employed in Static Var Generation (SVG)[13] and in 24-pulse inverters to be used in Adjustable Speed Drives (ASD). These ASD offer significant advantages in fan, pump and process control applications, in terms of high efficiencies and high performance, with major reliability in critical process areas, such as petroleum pumping [14].

## II. TRANSFORMERS CONNECTED IN ZIG-ZAG

Figure 1 shows the star-zig/zag transformer. The taps will be calculated to obtain the angular displacements of  $0^\circ$ ;  $7.5^\circ$ ;  $15^\circ$ ;  $22.5^\circ$ ;  $30^\circ$ ;  $37.5^\circ$ ;  $45.0^\circ$ ;  $52.5^\circ$ . The taps  $N_2$  and  $N_3$  will be determined. Figure 2 shows the phasor diagram of the star-zig/zag transformer (YZ:30°). In this case the value of taps  $N_2$  and  $N_3$  is 100%.

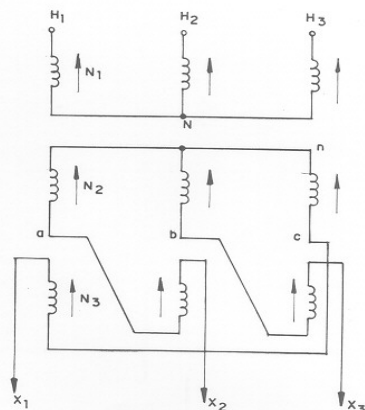


Figure 1: Star-zig/zag transformer

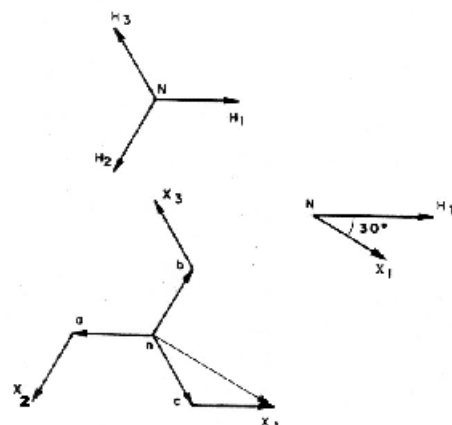


Figure 2: Phasor diagram (YZ:30°) transformer

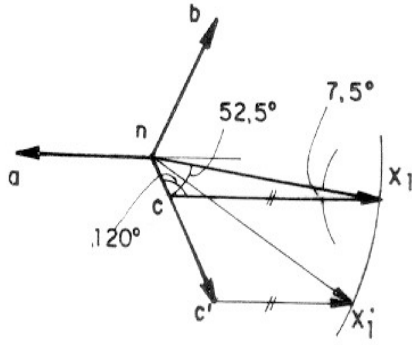


Figure 3: Phasor diagram (YZ 7.5°) transformer

Figure 3 shows the phasor diagram, for obtaining the angular phase shift of 7.5°, by keeping constant the output voltage.

According to the sine law for the triangle  $ncX_1$  results:

$$\frac{cn}{\sin 7.5} = \frac{X_1 c}{\sin 52.5} = \frac{nX_1}{\sin 120} \quad (1)$$

$$cn = \frac{nX_1}{\sin 120} \times \sin 7.5$$

$$cn = (nX_1) 0.1507 \quad (2)$$

$$X_1 c = \frac{nX_1}{\sin 120} \times \sin 52.5 \rightarrow X_1 c = (0.9161) \times nX_1 \quad (3)$$

For tap at 100% (YZ:30°)

$$nX_1 = \sqrt{3} \times cn \quad (4)$$

$$nX_1 = \sqrt{3} \times X_1 c \quad (5)$$

The taps will be:

$$\text{tap } N_2 \quad cn = N_2 = 0.1507 \times \sqrt{3}$$

$$\text{tap } N_2 = 26.10\% \quad (6)$$

$$\begin{aligned} \text{tap } N_3 \quad X_1 c = N_3 &= 0.9161 \times \sqrt{3} \\ \text{tap } N_3 &= 158.67\% \end{aligned} \quad (7)$$

For other displacements, the procedure is analogous. Table 1 resumes the required taps for obtaining the angular displacements 0°; 7.5°; 15°; 22.5°; 30°; 37.5°; 45.0°; 52.5°.

Table 1: Taps  $N_2$  and  $N_3$  for (YZ) transformer

Displacement	tap $N_2$	tap $N_3$
7.5°	26.10%	158.67%
15°	51.76%	141.42%
22.5°	76.54%	121.74%
30°	100%	100%
37.5°	121.74%	76.54%
45°	141.42%	51.76%
52.5°	158.67%	26.10%

For obtaining transformer voltage ratio 1:1 the tap 100% corresponds to:

$$N_2 = N_3 = 0.5773 N_1 \quad (8)$$

The transformer with angular displacement (0°) will be star-star YY(0°).

### III. TRANSFORMERS CONNECTED IN EXTENDED-DELTA

Figure 4 shows the star- extended/delta transformer.

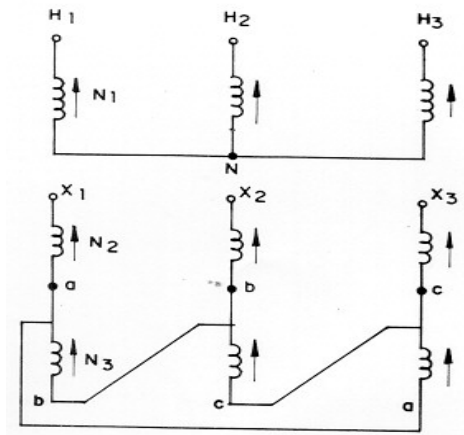


Figure 4: Star-Extended/Delta transformer

Figure 5 shows the phasor diagram for obtaining the angular displacement of 7.5°.

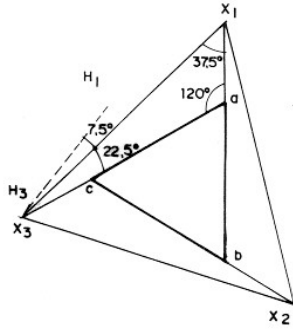


Figure 5: Phasor diagram (Y-ED:7.5°) transformer

By applying the sine law in the triangle  $X_1X_3a$ , results:

$$\frac{aX_1}{\sin 22.5} = \frac{X_1X_3}{\sin 120} = \frac{aX_3}{\sin 37.5} \quad (9)$$

$$aX_1 = X_1X_3 \sin \frac{22.5}{\sin 120} = X_1X_3 \times 0.4419 \quad (10)$$

$$aX_3 = X_1X_3 \sin \frac{37.5}{\sin 120} = X_1X_3 \times 0.7029 \quad (11)$$

$$\text{Considering the tap } X_1X_3 = 100\% \quad (12)$$

Results:

$$aX_1 = 44.9\% \quad (13)$$

$$ac = aX_3 - cX_3$$

$$ac = aX_3 - aX_1$$

$$ac = 0.7029 - 0.4419 = 0.2610 \quad (14)$$

$$\text{tap } N_2 = 44.19\% \quad (15)$$

$$\text{tap } N_3 = 26.10\% \quad (16)$$

The other taps can be calculated. Table 2 resumes.

Table 2: Necessary taps for obtaining 48-pulse converter with star-extended/delta transformers

Displacement	tap $N_2$	tap $N_3$	Primary Phase Sequence
-22.5°	15.07%	76.53%	Inverse
-15°	29.88%	51.76%	Inverse
-7.5°	44.19%	26.10%	Inverse
+7.5°	44.19%	26.10%	Direct
+15°	29.88%	51.76%	Direct
+22.5°	15.07%	76.53%	Direct
30°	0%	100%	Direct

For obtaining transformer voltage ratio 1:1, the tap 100% corresponds to  $N_3 = (1.7320)N_1$ .  $N_2$  is obtained as a function of  $N_3$ . So, for tap  $N_2$  15.07%, corresponds to  $N_2 = 1.7320 \times 0.1507 N_1 = 0.26 N_1$ . The displacement 0° is achieved by using a star-star (Y/Y) transformer.

#### IV. COMPARISON BETWEEN ZIG-ZAG AND EXTENDED DELTA CONNECTIONS

Table 3 shows  $N_2$  and  $N_3$  as a function of  $N_1$  for Star Zig-Zag transformers.

Table 3:  $N_2$  and  $N_3$  as a function of  $N_1$  for Y-Z transformers

Displacement	$N_2$	$N_3$
0°(Y-Y)	1.0000	0.0000
7.5°	0.1506	0.9160
15°	0.2988	0.8164
22.5°	0.4419	0.7028
30°	0.5773	0.5773
37.5°	0.7028	0.4419
45°	0.8164	0.2988
52.5°	0.9160	0.1506

Table 4 shows  $N_2$  and  $N_3$  as a function of  $N_1$  for Star Extended-Delta transformers.

Table 4:  $N_2$  and  $N_3$  as a function of  $N_1$  for Star Extended-Delta transformers

Displacement	$N_2$	$N_3$
0°(Y-Y)	1.0000	0.0000
-22.5°	0.2610	1.3255
-15°	0.5175	0.8965
-7.5°	0.7653	0.4520
7.5°	0.7653	0.4520
15°	0.5175	0.8965
22.5°	0.2610	1.3255
30°	0.0000	1.7320

For obtaining 12-24-48-pulse converter, considering transformer voltage ratio 1:1 and using the results of tables 3 and 4, the total copper spent (including primary and secondary windings) of zig-zag and extended-delta transformers, in terms of  $N_1I_1$  will be obtained, where  $I_1$  is the primary RMS current and remembering that the RMS current inside the delta is  $I_1/1.7320$ . For other transformer voltage ratio the product  $N_1I_1$  is kept constant, because if in the secondary  $N$  increases,  $I$  decreases in the same proportion (or vice-versa).

## A. Copper Expent for 12-Pulse Operation

### A.1.) 12-Pulse Operation using star/zig-zag transformers

Using, for instance, the transformers:

- a) Y-Y (0°), Y-Z(30°) or b) Y-Z(7.5°), Y-Z(37.5°) or  
c) Y-Z(15°), Y-Z(45°) or d) Y-Z(22.5°), Y-Z(52.5°).

Results:

- a)  $\sum N_1 I_1 = 12.4638$   
b)  $\sum N_1 I_1 = 12.6339$   
c)  $\sum N_1 I_1 = 12.6912$   
d)  $\sum N_1 I_1 = 12.6339$

### A.2.) 12-Pulse Operation using star/extended-delta transformers (Y-ED)

Using, for instance, the transformers:

- e) Y-ED (-22.5°), Y-ED(7.5°) or  
f) Y-ED(-15°), Y-ED(15°) or  
g) Y-ED(-7.5°), Y-ED(22.5°) or  
h) Y-Y(0°), Y-D(30°). Results:

- e)  $\sum N_1 I_1 = 12.1576$   
f)  $\sum N_1 I_1 = 12.2105$   
g)  $\sum N_1 I_1 = 12.1576$   
h)  $\sum N_1 I_1 = 12.0000$

Considering, for instance, the cases f and c, that correspond to the use of Star Extended-Delta transformers ( $\pm 15^\circ$ ) (f), which uses identical units, needing only a phase sequence inversion for the unit (-15°) and so the more commonly applied and the use of the corresponding Star Zig-Zag transformers (c), results:

**Copper saving using extended-delta transformers: 3.94%**

## B. Copper Expent for 24-Pulse Operation

### B.1.) 24-Pulse Operation using star/zig-zag transformers

Using, for instance, the transformers:

- i) Y-Y (0°), Y-Z(30°); Y-Z(15°), Y-Z(45°) or  
j) Y-Z(7.5°), Y-Z(37.5°); Y-Z(22.5°), Y-Z(52.5°).

Results:

- i)  $\sum N_1 I_1 = 25.1550$  (a+c)  
j)  $\sum N_1 I_1 = 25.2678$  (b+d)

### B.2.) 24-Pulse Operation using star/extended-delta transformers (Y-ED)

Using, for instance, the transformers:

- k) Y-ED(-22.5°)Y-ED(7.5°); Y-ED(-7.5°)Y-ED(22.5°) or  
l) Y-ED(-15°), Y-ED(15°); Y-Y(0°), Y-D(30°).

Results:

- k)  $\sum N_1 I_1 = 24.3152$  (e+g)  
l)  $\sum N_1 I_1 = 24.2105$  (f+h)

Considering, for instance, the cases i and l, results:

**Copper saving using extended-delta transformers: 3.90%**

## C. Copper Expent for 48-Pulse Operation

### C.1.) 48-Pulse Operation using star/zig-zag transformers

Using, for instance, the transformers:

- m) Y-Y (0°), Y-Z(30°), Y-Z(15°), Y-Z(45°);  
Y-Z(7.5°), Y-Z(37.5°), Y-Z(22.5°), Y-Z(52.5°).

Results:

- m)  $\sum N_1 I_1 = 50.4228$  (i+j)

### C.2.) 48-Pulse Operation using star/extended-delta transformers (Y-ED)

Using, for instance the transformers:

- n) Y-ED(-22.5°)Y-ED(7.5°)Y-ED(-7.5°)Y-ED(22.5°)  
Y-ED(-15°), Y-ED(15°), Y-Y(0°), Y-D(30°). Results:

- n)  $\sum N_1 I_1 = 48.5257$  (k+l)

Considering, for instance, the cases m and n, results:

**Copper saving using extended-delta transformers: 3.91%**

## D. Winding Insulation for 12-Pulse Operation

In all the following cases the insulation aspect will be considered in terms of  $\sum (N_2 + N_3)$  per leg, in the secondary (Tables 3 and 4).

### D.1.) 12-Pulse Operation using star/zig-zag transformers

Using, for instance, the transformers:

- o) Y-Y (0°), Y-Z(30°) or p) Y-Z(7.5°), Y-Z(37.5°) or  
q) Y-Z(15°), Y-Z(45°) or r) Y-Z(22.5°), Y-Z(52.5°).

Results:

- o)  $\sum (N_2 + N_3) = 2.1546$   
p)  $\sum (N_2 + N_3) = 2.2113$   
q)  $\sum (N_2 + N_3) = 2.2304$   
r)  $\sum (N_2 + N_3) = 2.2113$

### D.2.) 12-Pulse Operation using star/extended-delta transformers (Y-ED)

Using, for instance, the transformers:

- s) Y-ED (-22.5°), Y-ED(7.5°) or  
t) Y-ED(-15°), Y-ED(15°) or  
u) Y-ED(-7.5°), Y-ED(22.5°) or  
v) Y-Y(0°), Y-D(30°). Results:

- s)  $\sum (N_2 + N_3) = 2.8038$   
t)  $\sum (N_2 + N_3) = 2.8280$   
u)  $\sum (N_2 + N_3) = 2.8038$   
v)  $\sum (N_2 + N_3) = 2.7320$

Considering, for instance, the cases t and q, that correspond to the use of Star Extended-Delta transformers ( $\pm 15^\circ$ ) (t), which uses identical units and

the use of the corresponding Star Zig-Zag transformers (q), results:

**Global winding insulation reduction using Star-Zig-Zag transformers : 26.79%**

#### E. Winding Insulation for 24-Pulse Operation

E.1.) 24-Pulse Operation using star/zig-zag transformers

Using, for instance, the transformers:

w) Y-Y (0°), Y-Z(30°); Y-Z(15°), Y-Z(45°) or  
x) Y-Z(7.5°), Y-Z(37.5°); Y-Z(22.5°), Y-Z(52.5°).

Results:

w)  $\sum (N_2+N_3)=4.3850$  (o+q)

x)  $\sum (N_2+N_3)=4.4226$  (p+r)

E.2.) 24-Pulse Operation using star/extended-delta transformers (Y-ED)

Using, for instance, the transformers:

y) Y-ED(-22.5°)Y-ED(7.5°); Y-ED(-7.5°)Y-ED(22.5°)  
or z) Y-ED(-15°), Y-ED(15°); Y-Y(0°), Y-D(30°).

Results:

y)  $\sum (N_2+N_3)=5.6076$  (s+u)

z)  $\sum (N_2+N_3)=5.5600$  (t+v)

Considering, for instance, the cases w and z, results:

**Global winding insulation reduction using Star-Zig-Zag transformers : 26.79%**

#### F. Winding Insulation for 48-Pulse Operation

F.1.) 48-Pulse Operation using star/zig-zag transformers

Using, for instance, the transformers:

α) Y-Y (0°), Y-Z(30°), Y-Z(15°), Y-Z(45°);  
Y-Z(7.5°), Y-Z(37.5°), Y-Z(22.5°), Y-Z(52.5°).

Results:

α)  $\sum (N_2+N_3)=8.8076$  (w+x)

F.2.) 48-Pulse Operation using star/extended-delta transformers (Y-ED)

Using, for instance, the transformers:

β) Y-ED(-22.5°)Y-ED(7.5°)Y-ED(-7.5°)Y-ED(22.5°)  
Y-ED(-15°), Y-ED(15°), Y-Y(0°), Y-D(30°). Results:

β)  $\sum (N_2+N_3)=11.1676$  (y+z)

Considering, for instance, the cases α and β, results:

**Global winding insulation reduction using Star-Zig-Zag transformers : 26.79%**

#### G. Winding Power

A comparison between the zig-zag and extended-delta winding power will be made taking into account the product NI only in the **secondary**. For this purpose, it is necessary to subtract the primary product NI (6, 12, 24 for 12-24-48 pulse operations,

respectively). Considering the same cases above, results:

G.1.) 12-Pulse Operation (cases c and f)

**Winding power reduction using extended-delta transformers: 7.74%**

The obtained result agree with [3].

G.2.) 24-Pulse Operation (cases i and l)

**Winding power reduction using extended-delta transformers: 7.74%**

G.3.) 48-Pulse Operation (cases m and n)

**Winding power reduction using extended-delta transformers: 7.74%**

#### H. Harmonics and Simulations

The characteristic harmonic contents of the the a.c side currents of multiconverter systems are given by:

$$N'=Pq'\pm 1 \quad (17)$$

Where: N' = Harmonic order

P = Number of Pulses

q' = 0,1,2,3,4,.....

So, for 12-24-48 pulse operations, results:

H.1.) 12-pulse operation

N'=1,11,13,23,25,35,37,47,49, ....

H.2.) 24-pulse operation

N'= 1,23,25,47,49,.....

H.3.) 48-pulse operation

N'= 1, 47,49,....

Where N'= 1 is the fundamental component.

An analysis, of course, shows that using both zig-zag or extended-delta transformers, results in the same characteristic harmonics, at balanced steady state conditions.

Prototypes of such zig-zag and extended delta transformers, 2 [KVA], have been built and tested at our laboratories and simulated voltage and current waveshapes of multiconverter systems, at balanced steady state conditions, match with experimental ones [1],[2],[4],[5],[6],[9],[10],[11],[12],[15].

#### V. CONCLUSION

Static Converters are responsible by current harmonic injection in the electric system. There are two solutions to mitigate harmonics. The first one is the installation of filters (passives and more recently actives). The second one is increase the converter's pulse number, by using the suitable zig-zag or extended-delta transformers, as presented in this paper.

A detailed procedure for obtaining the taps of zig-zag and extended delta transformers has been presented in this paper, using phasor diagrams and applying also the sine law.

For obtaining a 48-pulse converter and using the results of tables 3 and 4, the total copper spent of zig-zag transformers in terms of  $N_1 I_1$  is  $50.42 N_1 I_1$  (including primary and secondary windings). The total copper spent of extended-delta transformers for the same conditions is  $48.52 N_1 I_1$ . So the extended delta connection expends less copper for the purpose of obtaining a 48-pulse converter (total copper saving of 3.91%). The analysis of winding power, shows that the reduction was about 7.74% for the extended-delta connection. The harmonic contents of the a.c side currents of both transformers are identical (characteristic harmonics). The zig-zag connection is better only with respect to the lower secondary winding required insulation. For 48-pulse global analysis, the reduction was about 26.79%.

However, taking into account all aspects, the extended delta transformer is less expensive to manufacture and therefore is preferred over the zig-zag transformer [3].

## VI. REFERENCES

- [1] A.J.J.Rezek, "Modelling and implementation of multiconverter systems", (In Portuguese), DSc Thesis, FEE/UNICAMP, Campinas., Brasil, may, 1991.
- [2] A.J.J.Rezek, "Permanent and transient condition analysis of an AC/DC electric conversion system" (In Portuguese), M.Sc Thesis, EFEI, Itajubá, Brasil, june, 1986.
- [3] L.Carlsson. "Extended-delta converter transformer for 12 - pulse operation in HVDC projects", International HVDC Seminar, Rio de Janeiro, Brasil, 1986.
- [4] J.P.G.Abreu; A.J.J.Rezek; A.Candido, "Modelling and implementation of 48-pulse multiconverter", Proceedings IEEE-ICHPS, Bologna, Italy, september, 1994, pp. 50-54.
- [5] J.C.G.Siqueira; G.H.F.Floriano, "Harmonic spectrum of a periodic function by using the Fast Fourier Transform" (In portuguese), EFEI/CPq/DET, Itajubá, Brasil, 1987.
- [6] A.J.J.Rezek; J.P.G.Abreu; V.F.Silva; M.S.Miskulin, "New alternatives for HVDC transmission" (In Portuguese), Cigré, VI ERLAC, Foz do Iguaçu, PR, Brasil, may, 1995.
- [7] E.Ulmann, "Power transmission by direct current", Springer-Verlag, Berlin, Heidelberg, New York, 1975.
- [8] E.W.Kimbark, "Direct current transmission", Vol I, Wiley Intercience, New York, 1971.
- [9] A.J.J.Rezek; J.P.G.Abreu; V.F.Silva; M.S.Miskulin, "Increasing the converter pulse number: Harmonic contents and power factor improvement" (In Portuguese), Cigré, VII ERLAC, Puerto Iguazú, Argentina, may, 1997.
- [10] J.P.G.Abreu; A.J.J.Rezek; V.F.Silva; L.E.B.Silva; M.S.Miskulin, "Harmonics elimination in multiconverter systems by using a special autotransformer (ADZ)" Proceedings AMSE Modelling, Simulation & Control, AMSE Press, vol. 38, n° 4, pp. 45-53, France, 1992.
- [11] A.J.J.Rezek; M.S.Miskulin; J.P.G.Abreu, "Computer aided design applied in power electronics" (In Portuguese), Proceedings II Power Electronics Seminar, pp. 102-109, UFSC, Florianópolis, SC, Brasil, 1989.
- [12] A.J.J.Rezek; J.P.G.Abreu; V.F.Silva; M.S.Miskulin, "Converters for application in HVDC systems", International Conference on Contribution of Cognition to Modeling, AMSE, july 6-8, Lyon-Villeurbanne, France, 1998.
- [13] Katsuhiko Matsuno; Taizou Hasegawa; Yasushi Oue, "Development of Static Var Generation for Power Systems and its Application Effects", System Engineering Department, The Kansai Electric Power Co., Inc., Japan, 2000.
- [14] E. Cengelci; P. Enjeti; W. Gray "A new modular motor-modular inverter (MM-MI) concept for medium voltage adjustable speed drive systems", Proceedings PESC 99, USA., pp. 1972-1979.
- [15] A.J.J. Rezek; J.P.G. Abreu; J.M.E. Vicente; J.A.Cortez; V.F.Silva, M.S.Miskulin "Power factor improvement of line-commutated graetz converters by increasing their number of pulses", Proceedings V Brazilian Power Electronics Conference, V COBEP, Fóz do Iguaçu, PR, Brasil, 1999, Vol. 2, pp. 551-556.