

# Fluorescent Lamp Model Employing Tangent Approximation

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**Abstract** – A circuit model simulating the electrical characteristics of a fluorescent lamp operating at high frequency is proposed. The model is based on a tangent approximation constructed by experimental results for several powers and frequencies. Simulations and experimental results are used to verify the analytical discussions, and, moreover, an electronic ballast example using the proposed model is presented to further demonstrate its applications.

## I. INTRODUCTION

Fluorescent lamps have been an important role in illumination system, owing to their higher efficiency and longer life compared to incandescent lamps. Fluorescent lamps have a negative dynamic resistance behavior, which makes necessary use a ballast to limit the current [1] [2] [3]. Fluorescent Lamps operated at high frequency (>20 kHz) have numerous benefits such as: higher lumen output, lower starting voltage and flicker absence [4] [5].

Improvements in the efficiency of illuminating systems are desirables. In the field of fluorescent lighting, topologies are developed to reduce the energy consumption [6] [7] [8] [9]. During the preliminary stage of high frequency electronic Ballast design, fluorescent lamp models are required to facilitate computer simulations. Thus it is necessary to develop a model which can predict the electrical characteristics of fluorescent lamps. It helps to simulate dimmable electronic ballast, where is necessary to know the fluorescent lamp behavior.

To verify the performance of dimmable electronic ballast or traditional electronic ballast, the fluorescent lamp model employed at PSPICE or others simulators becomes a useful tool that allows topologies analysis without implementation.

This paper presents a mathematical model of a fluorescent lamp based on regressive method employing a tangent function. This approach allows an easy way to determine the model parameters based on experimental data.

## II. THE TANGENT MODEL

Most of analyses of electronic ballast design reported so far represent the lamp as a resistor [14] [15] and do not include the characteristics of the fluorescent lamp in the ballast design consideration. However, it should be pointed out that the lamp characteristics could greatly affect the design of the ballast operation and choice of circuit components. The drawback in assuming the lamp as a resistor is that the lamp, as a low-pressure gas discharge

device, exhibits complex  $V \times I$  characteristics, which are different from those of resistor. When the lamp power is high, the resistance of the lamp discharge column is low, because the lamp gas is highly ionized (hot). When lamp power is low, the resistance of the discharge column is high, because the lamp gas is less ionized (cold). The lamp, when in thermal equilibrium, has a constant voltage drop. The main difference between the gas discharge lamp and the negative resistance is the thermal time constant; that of the negative resistance is on the order of milliseconds, that of the lamp is on the order of seconds [13]. The static and dynamic characteristics of fluorescent lamps represent the electric behavior in low frequency and high frequency respectively.

The fluorescent lamp model presented in this paper uses the dynamical characteristic of the lamp through an  $F$  function, which makes the relation between lamp voltage  $V_L$  with lamp current  $I_L$  and lamp power  $P_L$  (1).

$$V_L = F(I_L, P_L) \quad (1)$$

Experimental data were obtained through a Tektronix TDS 430A scope, by electronic ballast employing a Half-Bridge inverter and a LCC filter to supply a fluorescent lamp, according Fig. 1. The data were acquired to several power and frequency levels, as shown in Fig. 2. The power variation was achieved through the line voltage control, and the switching frequency variation was done through  $R_t$  variable resistor at the command circuit of the electronic ballast. The summarized prototype parameters are described in table I.

Fig. 2 shows different lamp voltage versus lamp current characteristics  $V_L \times I_L$  for several power and frequency levels that represent the behavior of a fluorescent lamp obtained by experimental results. Analyzing Fig. 2, we may conclude that the tangent function is a good  $V_L \times I_L$  characteristics representation. Two coefficients describe this function:  $A_P(P_L)$ , that represents the curve inclination, and  $B_P(P_L)$ , that represents the  $I_L$  tendency. In this case the function  $F$  in (1) has the following form:

$$V_L(I_L, P_L) = A_P(P_L) \cdot \tan\left(\frac{I_L}{B_P(P_L)}\right) \quad (2)$$

Where  $A_P(P_L)$  and  $B_P(P_L)$  are power function that represent the steady state lamp behavior. The experimental data for several powers must be obtained to determine

these coefficients, where  $V_1$  and  $I_1$  are lamp voltage and current for low voltage, and  $V_2$  and  $I_2$  are to high voltage. The system solution is given by numerical methods, as shown in (3).

$$\begin{cases} V_1(I_L, P_L) = A_P(P_L) \cdot \tan\left(\frac{I_1}{B_P(P_L)}\right) \\ V_2(I_L, P_L) = A_P(P_L) \cdot \tan\left(\frac{I_2}{B_P(P_L)}\right) \end{cases} \quad (3)$$

Table II is constructed by the resolution of (3), for each experimental data, obtained by a OSRAM L40W/10S lamp.

Due to the  $A_P(P_L)$  value has a fewer variation (Fig. 3 (a)), an average value of  $A_P(P_L)$  was used. It is a way to simplify the model and avoid convergence error.  $A_P(P_L)$  is defined by (4):

$$A_P(P_L) = \frac{\sum_{i=1}^n A_P(P_L)}{n} \quad (4)$$

Where  $n$  is the number of experimental measures.

A second order polynomial regression (5) was used to obtain the curve that determines  $B_P(P_L)$ .

$$B_P(P_L) = ka \cdot P_L^2 + kb \cdot P_L + kc \quad (5)$$

Where  $ka$ ,  $kb$  and  $kc$  are  $B_P(P_L)$  polynomial coefficients.

Fig. 3 shows the tangent coefficients approximation curves obtained through Table II. Fig. 3(a) shows  $A_P(P_L)$  approximation curve, and Fig. 3(b) shows  $B_P(P_L)$  approximation curve indicated by solid line.

$A_P$  and  $B_P$  can be expressed by

$$A_P(P_L) = 81.323 \quad (6)$$

$$B_P(P_L) = 2.96 \cdot 10^{-4} \cdot P_L^2 + 2.34 \cdot 10^{-3} \cdot P_L + 0.025 \quad (7)$$

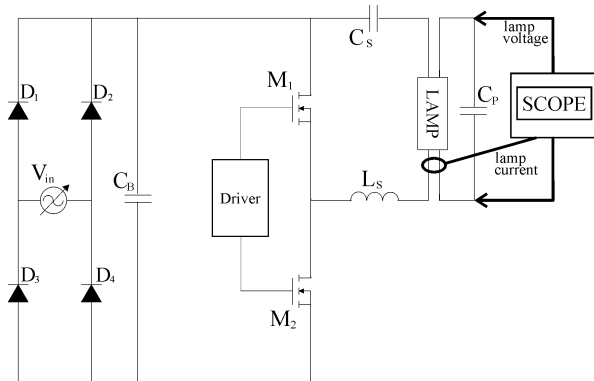


Fig. 1 – Electronic ballast prototype

TABLE I  
SUMMARIZED PROTOTYPE PARAMETERS

Resonant Filter Parameters	
$C_S$	Polypropylene capacitor, 147nF/250 Vac
$C_P$	Polypropylene capacitor, 8.2 nF/600 Vac
$L_S$	Inductor, 1.08 mH, 150 turns on core EE20 IP6-Thornton
Other Parameters	
$M_1$ - $M_2$	2xIRF840 (International Rectifier)
$D_1$ - $D_4$	4x1N4007
$C_B$	220 $\mu$ F, 350Vdc
$R_{Lamp}$	1xLamp, OSRAM L40W/10S

Fig. 4 shows the characteristics calculated from the model, which can be compared to experimental results shown in Fig. 2.

### III. SIMULATION AND EXPERIMENTAL RESULTS

Fig. 5 shows the model of the lamp used for simulation. The circuit is composed by resistors, capacitors, dependent voltage sources and dependent current sources.

The sources are defined in (8), (9), (10) and (11).

$$E_L = -V(1,2) + V(3) \cdot \tan\left(\frac{V(1,2)}{V(4)}\right) \quad (8)$$

$$G = V(1,2) \cdot V(1) \quad (9)$$

$$EA = 81.323 \quad (10)$$

$$EB = 2.96 \cdot 10^{-4} \cdot V(5)^2 + 2.34 \cdot 10^{-3} \cdot V(5) + 0.025 \quad (11)$$

where:

$E_L$ : lamp voltage behavior;

$G$ : lamp power variation;

$EA$ :  $A_P(P_L)$  coefficient approximation;

$EB$ :  $B_P(P_L)$  coefficient approximation.

$V(1,2)$ :  $I_L$  lamp current represented by  $R_S$  voltage drop, being  $R_S=1$ , then  $V(1,2)=I_L$ ;

$V(1)$ : lamp voltage;

$V(3)$ :  $R_a$  drop voltage  $V(3)=EA$ ;

$V(4)$ :  $R_b$  drop voltage  $V(4)=EB$ ;

$V(5)$ : Lamp Power variation.

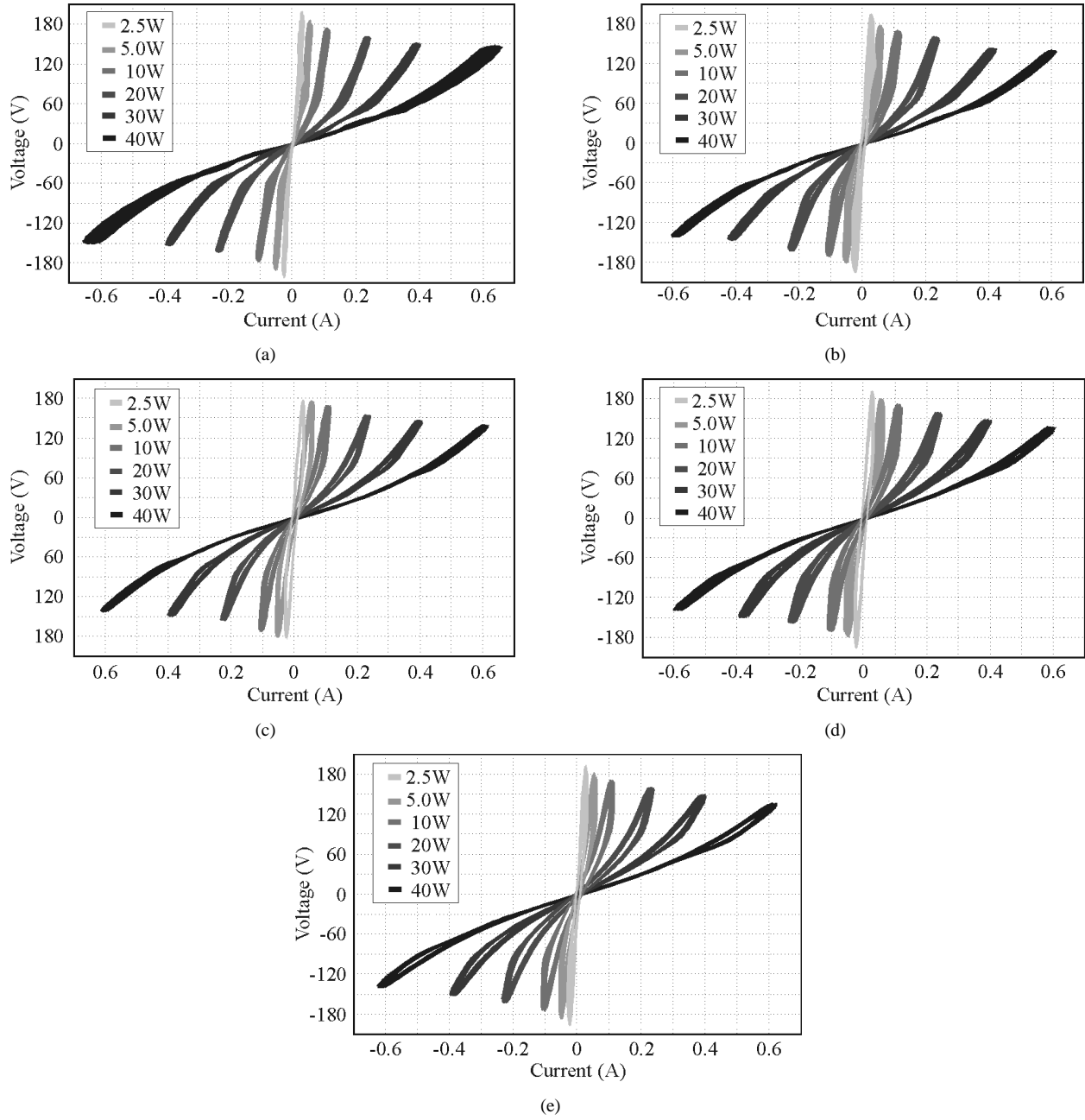


Fig. 2 – Experimental results ( $V_L \times I_L$ ) for several powers in: (a) 20kHz, (b) 30kHz, (c) 40kHz, (d) 50kHz and (e) 60kHz.

TABLE II  
 $A_P$  AND  $B_P$  EXPERIMENTAL RESULTS

Freq.	20 kHz		30 kHz		40 kHz		50 kHz		60 kHz		Mean	
Power	$A_p$	$B_p$	$A_p$	$B_p$	$A_p$	$B_p$	$A_p$	$B_p$	$A_p$	$B_p$	$A_p$ mean	$B_p$ mean
2,5	110,75	0,0273	135,94	0,0315	222,38	0,0471	78,63	0,0181	74,82	0,0198	124,50	0,028767
5	59,60	0,0387	64,02	0,0440	75,19	0,0462	69,54	0,0450	98,61	0,0552	73,39	0,045848
10	52,92	0,0769	53,26	0,0821	62,66	0,0842	54,67	0,0829	73,82	0,0892	59,47	0,083057
20	63,95	0,1894	65,47	0,1877	71,45	0,2005	69,91	0,1976	69,61	0,1968	68,08	0,194402
30	73,73	0,3412	81,29	0,3863	69,19	0,3406	72,25	0,3391	78,10	0,3553	74,91	0,352502
40	87,89	0,6032	90,94	0,5971	82,76	0,5803	83,92	0,5745	92,42	0,6257	87,59	0,596150
Mean	74,81		81,82		97,27		71,49		81,23		81,32	

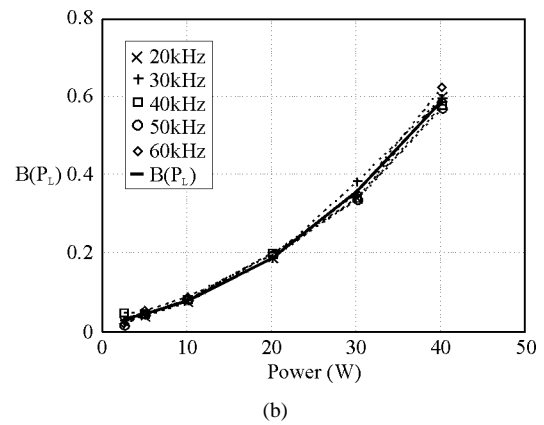
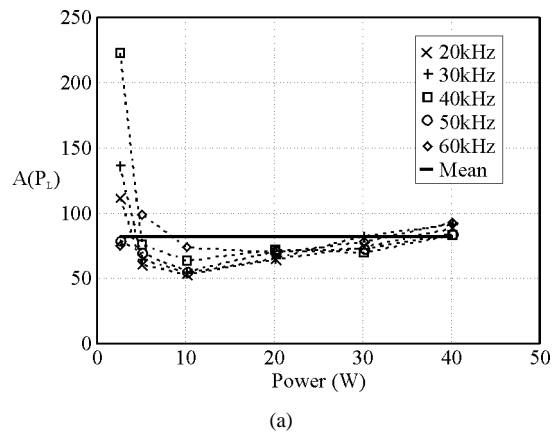


Fig. 3 – Tangent function coefficients as function of power: (a)  $A_p \times P_L$  and (b)  $B_p \times P_L$ .

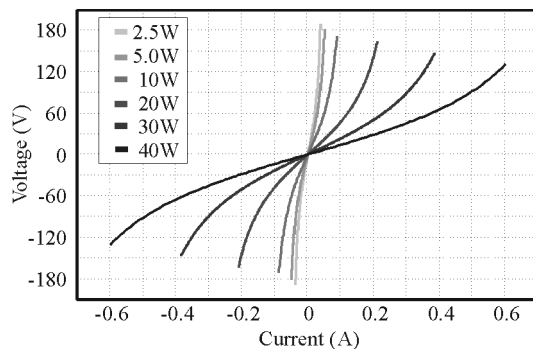


Fig. 4 –  $V \times I$  Lamp model characteristics

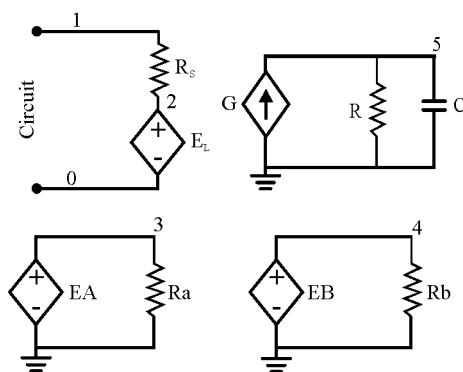


Fig. 5 – Circuit model

PSPICE simulation circuit is shown in Fig. 6. Table III shows summarized simulated parameters, and model components.

Fig. 7 shows the lamp current and voltage waveforms in steady-state to several lamp power levels to switching frequency of 50 kHz. Fig. 7(a) shows lamp current and voltage to 40 W, obtained by experimental data. Fig. 7(b) shows the lamp current

and voltage simulated to 40 W. To verify the performance of the employed model Fig. 7(c) and (d) shows the lamp current and voltage to 20 W experimental and simulated results respectively. Fig. 7(e) and (f) shows the same waveforms to lamp power of 10W.

Analyzing the results and by simulation in PSPICE, we may conclude that the efficacious of the presented model is according of experimental results; therefore the model becomes an useful tool to use to simulate the behavior of a fluorescent lamp.

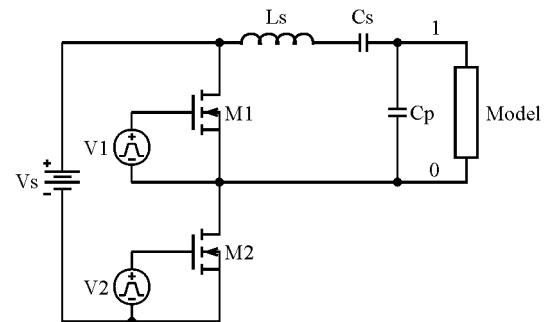


Fig. 6– Simulated Circuit

TABLE III  
SUMMARIZED MODEL PARAMETERS

Model components	
C	C=2 mF
R	R=1 $\Omega$
R <sub>a</sub>	R <sub>a</sub> =1 $\Omega$
R <sub>b</sub>	R <sub>b</sub> =1 $\Omega$
R <sub>s</sub>	R <sub>s</sub> =1 $\Omega$
Other Parameters	
C <sub>s</sub>	147 nF
C <sub>p</sub>	8.2 nF
L <sub>s</sub>	1.08 mH
M1,M2	IRF840
V <sub>s</sub>	VDC
V1, V2	VPULSE

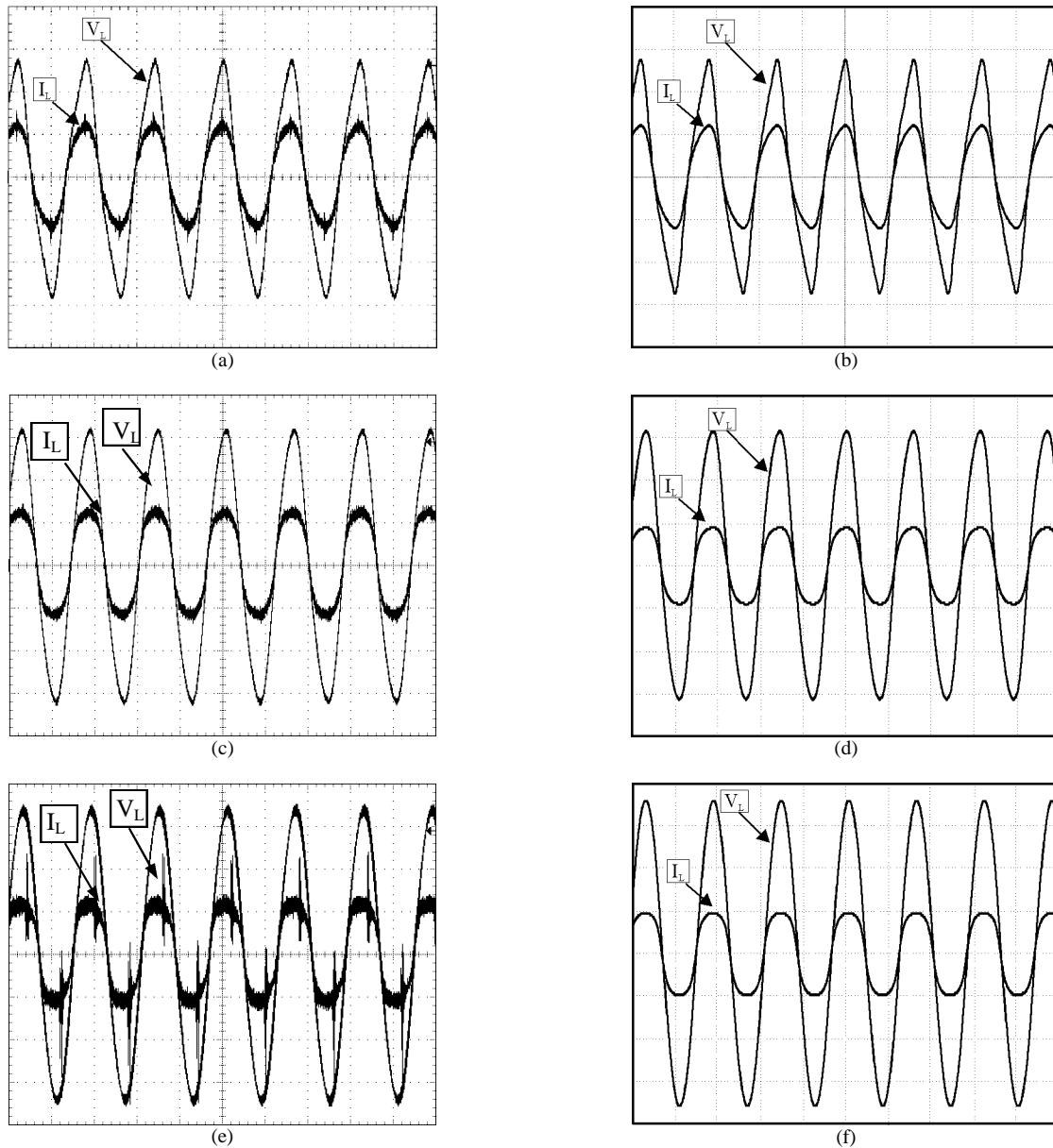


Fig. 7– Current and voltage waveform (12.5 $\mu$ s/div): (a) experimental (b) simulated (40W 50V/div; 500mA/div); (c) experimental (d) simulated (20W 50V/div; 200mA/div);(e) experimental (f) simulated (10W 50V/div; 100mA/div);

#### IV. CONCLUSION

Fluorescent lamp model presented has proven to be a useful tool in the design of electronic ballast. The experimental results show that the high frequency lamp characteristics variation can be negligible. This approach allows an easy way to determine the model parameters by experimental data.

The approximations done in some parameters do not affect the performance of the model, reducing convergence errors, and becoming the simulation easier and faster without compromise the fluorescent lamp model performance.

Simulation shown in this paper validates the model when compared with experimental results. This model presents a reduced convergence error when compared to the cubic model [13], what was verified when two models were used to simulate the same electronic ballast.

The proposed model is useful to other simulators, even his programs contain sources that could be described by functions.

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