Analysis, Modeling and Control of Three-Phase Boost AC-DC Converters Using the Park Transformation

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Abstract - **This paper presents a technique for analysis, modeling and control of three phase AC-DC converters, applied to a bi-directional three-phase AC-DC converter without neutral point. The basic principle of operation of the converter will be presented then the proposed theoretical analysis and finally the results obtained by simulation.**

I. INTRODUCTION

The study of high power factor three-phase AC-DC converters is not diffused, so the applied methods are, most times, complexes or not efficient. This makes these methods not trustable, mainly in not studied converters.

So, it would be interesting to obtain a quick and simple methodology, but efficient and trustable for these converters. Using the Park transform, it's possible to obtain a method of analysis that presents the desirable characteristics.

It will be presented a methodology for analysis, modeling and control of three-phase AC-DC converters, applied to a particular case, to the three-phase, AC-DC converter, bi-directional, without neutral point, presented in Fig. 1, with three-phase sinusoidal and balanced input of 220V AC (line voltage) and output of 450V (DC), operating at a power of 12kW and at a switching frequency of 30KHz, so that the methodology can be extended to other converters, that's the objective.

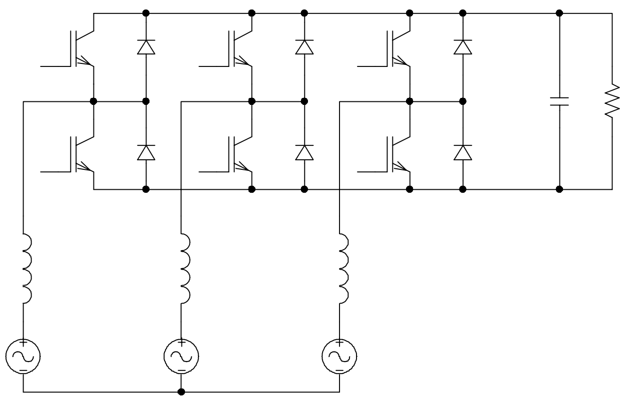

Fig. 1: Analyzed bi-directional three-phase AC-DC converter.

It can be observed that the circuit to be analyzed is traditionally used as an DC-AC converter, in fact, the methodology that will be presented can also be used in control of three-phase inverters or in active compensators of reactive power.

The presented study will focus techniques of analysis, modeling and control, so that components dimensioning and principles of functioning will not be presented.

II. OBTAINING THE CONVERTER MODEL FROM INPUT (AC)

The simplified circuit of Fig. 2 can represent the converter's circuit presented in Fig. 1, without loss of generality:

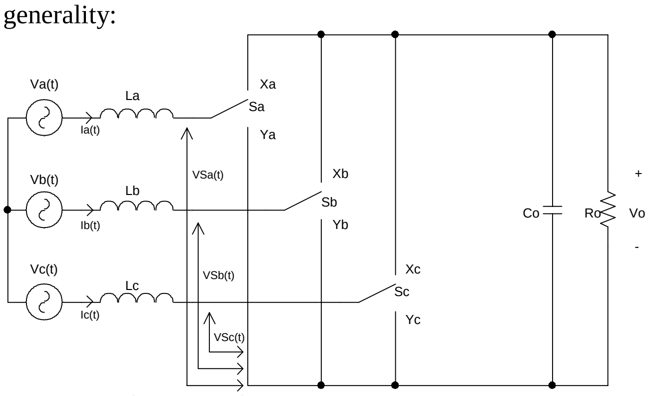

Fig. 2: Simplified circuit of bi-directional three-phase circuit presented in Fig. 1.

It is considered sinusoidal and balanced input, so that the imput voltages are given by:

$$\begin{cases} V_a(t) = V_P \cdot sen(w \cdot t) \\ V_b(t) = V_P \cdot sen(w \cdot t - 120°) \\ V_c(t) = V_P \cdot sen(w \cdot t + 120°) \end{cases} \quad (1)$$

Then:

$$\begin{cases} V_{ab}(t) = V_a(t) - V_b(t) = \sqrt{3} \cdot V_P \cdot sen(w \cdot t + 30°) \\ V_{bc}(t) = V_b(t) - V_c(t) = \sqrt{3} \cdot V_P \cdot sen(w \cdot t - 90°) \\ V_{ca}(t) = V_c(t) - V_a(t) = \sqrt{3} \cdot V_P \cdot sen(w \cdot t + 150°) \end{cases} \quad (2)$$

Then, from circuit presented in Fig. 2, it can be observed that when S_a is in X_a position, $V_{Sa}(t) = V_o$, and when S_a is in Y_a position, $V_{Sa}(t) = 0$, from this observation, and following the same ratiocination for S_b and S_c, it can be written:

$$\begin{cases} V_{Sa}(t)=[1-D_a(t)]\cdot Vo \\ V_{Sb}(t)=[1-D_b(t)]\cdot Vo \\ V_{Sc}(t)=[1-D_c(t)]\cdot Vo \end{cases} \Rightarrow \begin{cases} Da(t)=\begin{cases}0\,, & Sa\to Xa\\ 1\,, & Sa\to Ya\end{cases} \\ Db(t)=\begin{cases}0\,, & Sb\to Xb\\ 1\,, & Sb\to Yb\end{cases} \\ Dc(t)=\begin{cases}0\,, & Sc\to Xc\\ 1\,, & Sc\to Yc\end{cases} \end{cases} \tag{3}$$

Where V_o corresponds to output voltage, which by now will be considered constant. So, the equivalent circuit shown in Fig. 3 can represent the converter presented in Fig 2.

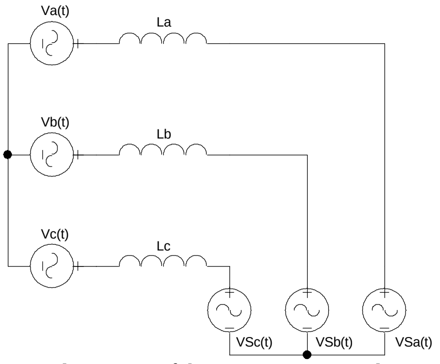

Fig. 3. Equivalent circuit of the converter presented in Fig. 2 (and consequently equivalent to the presented in Fig. 1).

So, from circuit presented in Fig. 3:

$$\begin{cases} [V_a(t)-V_b(t)]=[V_{La}(t)-V_{Lb}(t)]+[V_{Sa}(t)-V_{Sb}(t)] \\ [V_b(t)-V_c(t)]=[V_{Lb}(t)-V_{Lc}(t)]+[V_{Sb}(t)-V_{Sc}(t)] \\ [V_c(t)-V_a(t)]=[V_{Lc}(t)-V_{La}(t)]+[V_{Sc}(t)-V_{Sa}(t)] \end{cases} \tag{4}$$

Defining:

$$\begin{cases} D_{ab}(t)=Da(t)-Db(t) \\ D_{bc}(t)=Db(t)-Dc(t) \\ D_{ca}(t)=Dc(t)-Da(t) \end{cases} \quad \text{and} \quad \begin{cases} i_{ab}(t)=i_a(t)-i_b(t) \\ i_{bc}(t)=i_b(t)-i_c(t) \\ i_{ca}(t)=i_c(t)-i_a(t) \end{cases} \tag{5}$$

So, (4) reduces to:

$$\begin{cases} V_{ab}(t)=L\cdot\dfrac{di_{ab}(t)}{dt}-Vo\cdot D_{ab}(t) \\ V_{bc}(t)=L\cdot\dfrac{di_{bc}(t)}{dt}-Vo\cdot D_{bc}(t) \\ V_{ca}(t)=L\cdot\dfrac{di_{ca}(t)}{dt}-Vo\cdot D_{ca}(t) \end{cases} \tag{6}$$

Or, in vectorial form:

$$\overrightarrow{V_{abc}}=L\cdot\frac{d}{dt}\overrightarrow{I_{abc}}+Vo\cdot\overrightarrow{D_{abc}} \tag{7}$$

Where:

$$\overrightarrow{V_{abc}}=\frac{1}{\sqrt{3}}\cdot\begin{bmatrix}V_{ab}(t)\\ V_{bc}(t)\\ V_{ca}(t)\end{bmatrix}=\frac{1}{\sqrt{3}}\cdot\begin{bmatrix}V_a(t)-V_b(t)\\ V_b(t)-V_c(t)\\ V_c(t)-V_a(t)\end{bmatrix} \tag{8}$$

$$\overrightarrow{I_{abc}}=\frac{1}{\sqrt{3}}\cdot\begin{bmatrix}i_{ab}(t)\\ i_{bc}(t)\\ i_{ca}(t)\end{bmatrix}=\frac{1}{\sqrt{3}}\cdot\begin{bmatrix}i_a(t)-i_b(t)\\ i_b(t)-i_c(t)\\ i_c(t)-i_a(t)\end{bmatrix} \tag{9}$$

$$\overrightarrow{D_{abc}}=\frac{1}{\sqrt{3}}\cdot\begin{bmatrix}D_{ab}(t)\\ D_{bc}(t)\\ D_{ca}(t)\end{bmatrix}=\frac{1}{\sqrt{3}}\cdot\begin{bmatrix}D_a(t)-D_b(t)\\ D_b(t)-D_c(t)\\ D_c(t)-D_a(t)\end{bmatrix} \tag{10}$$

It is known that the Park transform, applied to any vector $\overline{X_{abc}}$ is given by:

$$\overline{X_{dq0}}=\overline{B}^{-1}\cdot\overline{X_{abc}} \tag{11}$$

Where:

$$\overline{B}^{-1}=\sqrt{\frac{2}{3}}\cdot\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \cos(w\cdot t) & \cos(w\cdot t-120°) & \cos(w\cdot t+120°)\\ -\sin(w\cdot t) & -\sin(w\cdot t-120°) & -\sin(w\cdot t+120°)\end{bmatrix} \tag{12}$$

And, obviously the inverse transform is given by:

$$\overline{X_{abc}}=\overline{B}\cdot\overline{X_{dq0}} \tag{13}$$

It must be recorded that the Park transform guarantees that the power does not vary, that is, the transformation is orthogonal, so:

$\overline{B}^{-1}=\overline{B}^{t}$ or $\overline{B}=\overline{B}^{-1^{t}}$.

Then, applying (13) in (7):

$$\overline{B}\cdot\overrightarrow{V_{dq0}}=L\cdot\frac{d}{dt}\left[\overline{B}\cdot\overrightarrow{I_{dq0}}\right]-Vo\cdot\overline{B}\cdot\overrightarrow{D_{dq0}} \tag{14}$$

Multiplying (14) by $\overline{B}^{-1}$:

$$\left[\overline{B}^{-1}\cdot\overline{B}\right]\cdot\overrightarrow{V_{dq0}}=L\cdot\overline{B}^{-1}\cdot\left\{\frac{d}{dt}\left[\overline{B}\cdot\overrightarrow{I_{dq0}}\right]\right\}-Vo\cdot\left[\overline{B}^{-1}\cdot\overline{B}\right]\cdot\overrightarrow{D_{dq0}} \tag{15}$$

Then, finally it's obtained:

$$\begin{bmatrix}V_0(t)\\ V_d(t)\\ V_q(t)\end{bmatrix}=L\cdot\begin{bmatrix}\dfrac{dI_0(t)}{dt}\\ \dfrac{dI_d(t)}{dt}\\ \dfrac{dI_q(t)}{dt}\end{bmatrix}-Vo\cdot\begin{bmatrix}D_0(t)\\ D_d(t)\\ D_q(t)\end{bmatrix}+L\cdot w\cdot\begin{bmatrix}0\\ -I_q(t)\\ I_d(t)\end{bmatrix} \tag{16}$$

Applying the Park transform to the line voltages:

$$\begin{cases} V_0(t)=0 \\ V_d(t)=\sqrt{\dfrac{3}{2}}\cdot Vp \\ V_q(t)=0 \end{cases} \tag{17}$$

The substituting in (16), it is obtained:

$$\begin{cases} \sqrt{\dfrac{3}{2}}\cdot Vp=L\cdot\dfrac{dI_d(t)}{dt}-Vo\cdot D_d(t)-w\cdot L\cdot I_q(t) \\ 0=L\cdot\dfrac{dI_q(t)}{dt}-Vo\cdot D_q(t)+w\cdot L\cdot I_d(t) \end{cases} \tag{18}$$

This way, from (18), the equivalent circuits of sequence **d** and **q** are presented in Fig. 4:

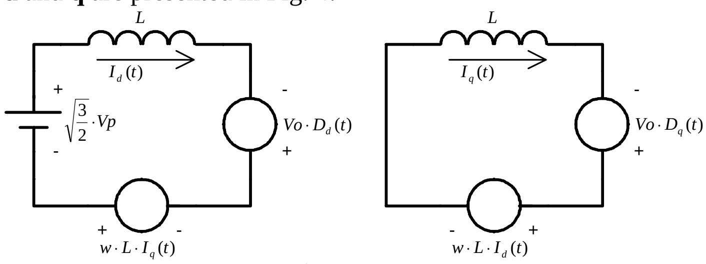

Fig. 4: Equivalent circuits of sequence d and q, respectively.

Applying low disturbations to variables in (18) and using Laplace transform:

$$I_d(s) = \frac{Vo}{L}\cdot\left[D_d(s)\cdot\frac{s}{s^2+w^2}+D_q(s)\cdot\frac{w}{s^2+w^2}\right]$$
$$I_q(s) = \frac{Vo}{L}\cdot\left[-D_d(s)\cdot\frac{w}{s^2+w^2}+D_q(s)\cdot\frac{s}{s^2+w^2}\right] \quad (19)$$

III. Obtaining the Converter Model from the Output (DC)

From the equivalent circuit presented in Fig. 2, it can be observed that:

$$Io(t) = I_a(t)\cdot[1-D_a(t)]+I_b(t)\cdot[1-D_b(t)]+I_c(t)\cdot[1-D_c(t)] \quad (20)$$

Knowing that the sum of three currents in phase must be equal to zero, as the input has no neutral point, it can be written, in vectorial form:

$$Io(t) = -\overrightarrow{I_{abc}}^{\,t}\cdot\overrightarrow{D_{abc}} \quad (21)$$

Applying, then, the Park transform in (21):

$$Io(t) = -I_d(t)\cdot D_d(t) - I_q(t)\cdot D_q(t) \quad (22)$$

Then, the equivalent circuit, seen by the output, is presented in Fig. 5:

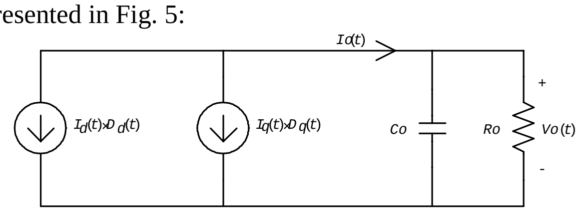

Fig. 5: Equivalent circuit, seen by the output (DC).

From the circuit of Fig. 5, it can be easily obtained:

$$\frac{Vo(s)}{Io(s)} = \frac{Ro}{1+s\cdot Ro\cdot Co} \quad (23)$$

More, adding low disturbations to the variables in (16), considering the perturbations are sufficiently low that the product of the two disturbations can be neglected, and using the Laplace transform:

$$Io(s) = -\left[I_d\cdot D_d(s)+D_d\cdot I_d(s)+I_q\cdot D_q(s)+D_q\cdot I_q(s)\right] \quad (24)$$

It can be seen that the circuit behaving depends on the point of operation, this way, I_d, I_q, D_d and D_q are considered invariant and calculated for a determined point of operation. Then, it is known that:

$$\begin{cases} P_{IN} = V_d\cdot I_d + V_q\cdot I_q \\ Q_{IN} = V_d\cdot I_q - V_q\cdot I_d \end{cases} \quad (25)$$

As V_q is equal to zero:

$$I_d = \frac{P_{IN}}{V_d} \quad \text{and} \quad I_q = \frac{Q_{IN}}{V_d} \quad (26)$$

More, from the equivalent circuits presented in Fig.5, knowing that the average voltages on the inductors are zero:

$$D_d = -\frac{V_d}{Vo} - \frac{w\cdot L\cdot I_q}{Vo} \quad \text{and} \quad D_q = \frac{w\cdot L\cdot I_d}{Vo} \quad (27)$$

Then, substituting (24), (26) and (27) in (23):

$$Vo(s) = I_d(s)\cdot\sqrt{\frac{3}{2}}\cdot\frac{Ro\cdot Vp}{Vo}\cdot\frac{\left[1-s\cdot\frac{2\cdot Po\cdot L}{3\cdot Vp^2}\right]}{1+s\cdot Ro\cdot Co}+$$
$$-I_q(s)\cdot\sqrt{\frac{2}{3}}\cdot\frac{Ro\cdot Q_{IN}\cdot L}{Vp\cdot Vo}\cdot\frac{s}{1+s\cdot Ro\cdot Co} \quad (28)$$

It can be observed that, if the reactive power is zero, the second term in (22) is zero, obtaining a transfer function very simple. Two current loops and one voltage loop can be used, with conventional controllers, where the reference for the current loop for **q** sequence is null and the reference for the current loop of sequence **d** is given by the voltage loop. The architecture of the proposed control system is presented in Fig.6:

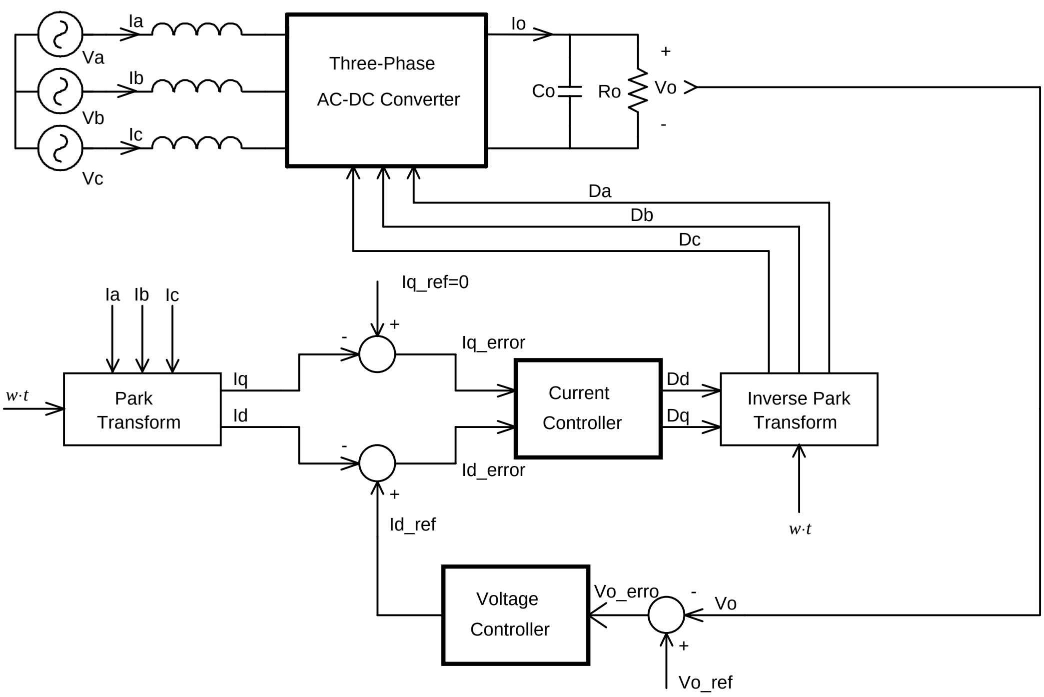

Fig. 6:. Proposed architecture for the control system.

The architecture for the control system presented in Fig.6 can be represented by the block diagram shown in Fig.7:

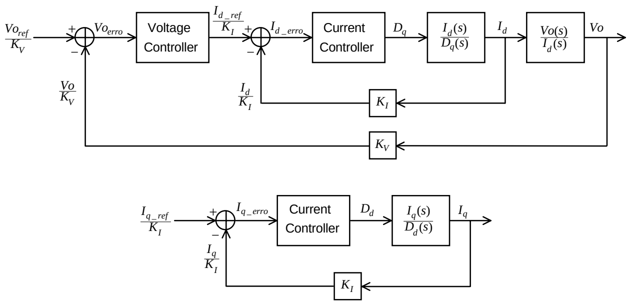

Fig.7: Block diagram representation for the control system presented in Fig.6.

It should be noted then, when applied the anti-transformation to the duty cycles, it's obtained "line duty cycles", so it's needed to use a logical to determine the "phase duty cycles" that effectively will be modulated to command the switches, then the proposed logical is:

Table 1: Logical used to determine the real duty cycles.

D_{ab}	D_{bc}	D_{ca}	D_a	D_b	D_c
> 0	> 0	> 0	No possible		
> 0	> 0	< 0	$-D_{ca}$	D_{bc}	0
> 0	< 0	> 0	D_{ab}	0	$-D_{bc}$
> 0	< 0	< 0	D_{ab}	0	$-D_{bc}$
< 0	> 0	> 0	0	$-D_{ab}$	D_{ca}
< 0	> 0	< 0	$-D_{ca}$	D_{bc}	0
< 0	< 0	> 0	0	$-D_{ab}$	D_{ca}
< 0	< 0	< 0	No possible		

IV. Obtained Results by Simulation.

Simulations were done using the software Pspice, based on the theoretical analysis presented. The project were done supposing the converter operating as a rectifier, with P_{IN}=12KW and Q_{IN}=0. The obtained results are presented below:

A. *Obtained results for the converter operating in nominal condition, with P_{IN}=12KW and Q_{IN}=0:*

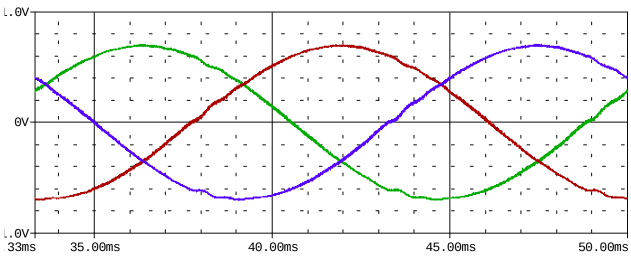

Fig. 8: "Line" duty cycles.

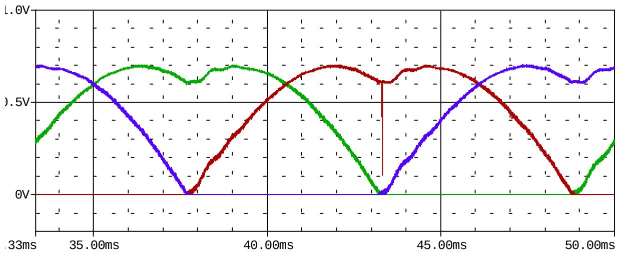

Fig. 9: "Phase" duty cycles.

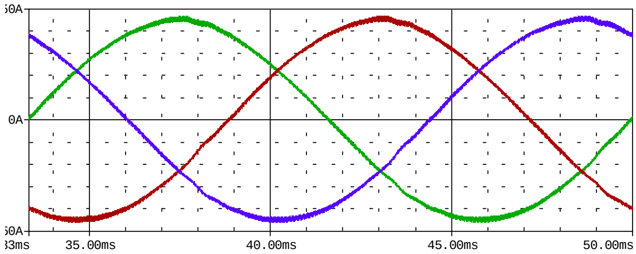

Fig. 9: Phase currents.

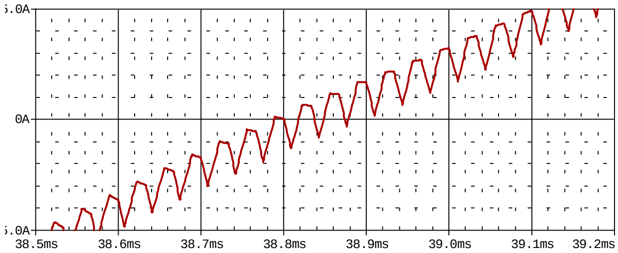

Fig. 10: Zero crossing of phase current $I_a(t)$.

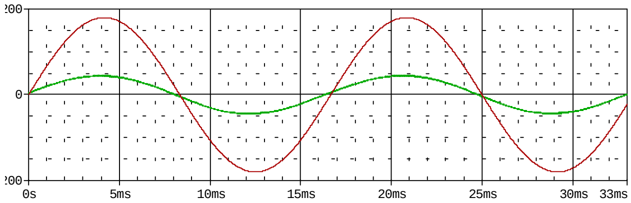

Fig. 11: Voltage and current in phase A.

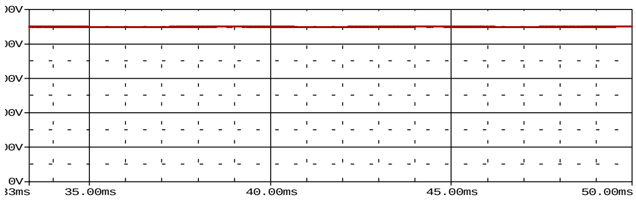

Fig. 12:. Output voltage.

B. *Obtained results for the converter operating in other interesting points, like inverter or reactive power compensator, where only references for P_{IN} and Q_{IN} were modified:*

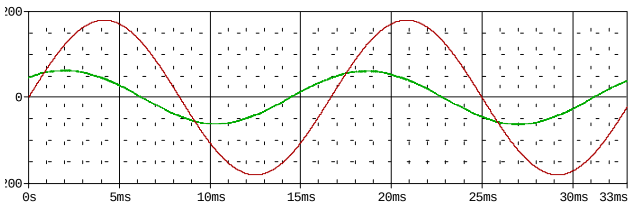

Fig. 13: Current and voltage in phase A, for P_{IN} = 12KW and Q_{IN} = -12KVAr.

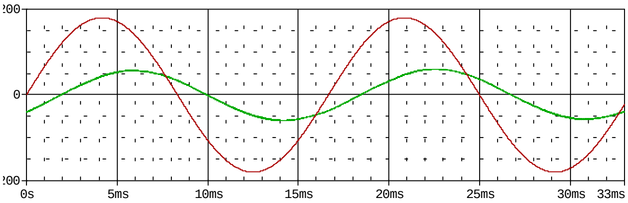

Fig. 14: Current and voltage in phase A, for P_{IN} = 12KW and Q_{IN} = 12KVAr.

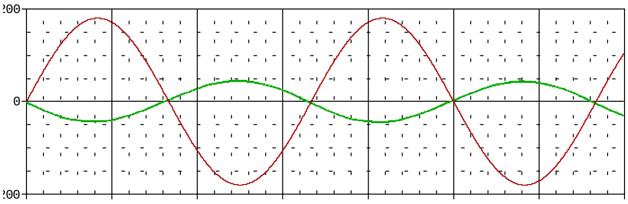

Fig. 15: Current and voltage in phase A, for P_{IN} = -12KW and Q_{IN} = 0.

V. CONCLUSIONS

Firstly, it must be emphasized that the use of Park transform permits to view the converter in a different way from that we used to see it, even that it is not used such tool directly in its control.

In other hand, using the Park transform to control the converter, it is seen that the currents will follow a sinusoidal form naturally, so that the controllers (conventional) will determine only the magnitude and phase of these currents.

In the case of the analyzed converter, it is observed the facility with which it can vary the active power, being able, inclusive, to do it operate as an inverter, and the reactive power, being able to have capacitive or inductive behaving.

So, in general form, it can be concluded that the obtained results are better than expected, so that the extension of the proposed methodology, for other converters is very flattering.

VIII. REFERENCES

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