A STUDY ON LOSSES IN CABLES AND TRANSFORMERS AND ANALYSIS OF POWER QUALITY PERFORMANCE

Fernando L. Tofoli, Aniel S. Morais, Carlos A. Gallo, Evandro A. S. Silva,
Fábio L. Albuquerque, Sérgio M. R. Sanhueza, Aloísio de Oliveira

Universidade Federal de Uberlândia – Faculdade de Engenharia Elétrica
Av. João Naves de Ávila, 2160, Campus Santa Mônica, Bloco "3N"
CEP 38400-902, Uberlândia, MG, Brasil, +55-34-32394166
E-mail: fernandolessa@pop.com.br

Abstract **– The use of electronic loads has increased significantly throughout the last decades. Such devices present an inherent non linearity characteristic due to the existence of switched power supplies in the input side, which convert the line ac voltage to the dc voltage level necessary to feed electronic circuits. This paper will focus on I^2R losses under non sinusoidal conditions, mainly in cables and transformers. For this purpose, a mathematical development that takes in account the presence of harmonic components in the I^2R losses calculation will be presented, so that the effects of the harmonic distortion in the power system components and consequently its impact on the power factor can be analyzed. Based on this study, a factor that evaluates the power quality performance in low voltage networks is presented. This factor is directly associated to the economical viability, once that it points when the losses minimization is interesting according to the cost/benefit ratio.**

KEYWORDS

I^2R losses, true power factor, harmonics, losses compensation, power quality factor.

I. INTRODUCTION

Electronic loads draw distorted currents even when fed by sinusoidal voltages. On the other hand the current distortion causes the voltage to be distorted as well, what can be seen in Figure I.

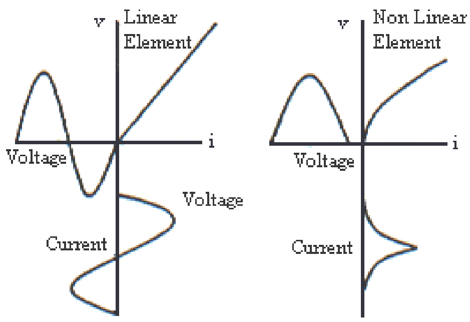

FIGURE I
Voltage and current waveform characteristics

One can notice in Figure I that a sinusoidal voltage applied to a linear load yields a sinusoidal current. However, distorted current waveform will result if a non linear element exists. Likewise, if a sinusoidal current is injected through a non linear impedance, the voltage across it will be distorted. Therefore, non linear loads indicate distorted voltage and/or distorted current waveforms, though the opposite is not true [6].

Such non linearity between the voltage and the current tends to cause several undesirable effects such as:

- Thermal overstress, what reduces equipments lifetime due to the overload in buses, feeders, cables and transformers;
- Isolation overstress;
- Unusable or undesirable operation of several devices;
- Additional losses;
- Power factor reduction.

Within this context, this paper presents a study on the effects of harmonic distortions in the power system components, mainly in cables and transformers.

II. THE INFLUENCE OF HARMONIC DISTORTIONS IN THE POWER SYSTEM COMPONENTS

A. Distorted currents in cables

According to Figure II, the pi model can be used in order to represent a conductor [4]. However, this model can be simplified, as it can be represented by a pure resistance, so that it becomes adequate for the purpose of this study, as shown in Figure III.

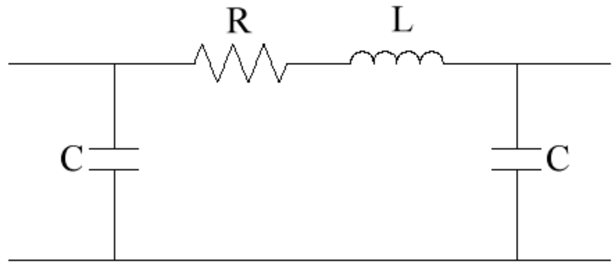

FIGURE II
Pi model representation of a cable

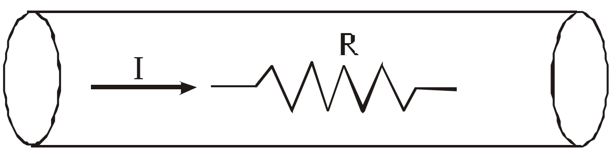

FIGURE III
Current flowing through a cable

Considering that the supply voltage is typically nearly sinusoidal i.e. THD_V<8%, while the rms current presents some significant harmonic content, as in (1),

$$I_{RMS} = \sqrt{\sum_{n=1} I_n^{\,2}} = \sqrt{I_1^2 + \sum_{n=2} I_n^{\,2}} \qquad (1)$$

the rms value of the current can be separated into a fundamental part and a harmonic part,

$$I_{RMS} = \sqrt{I_1^{\,2} + I_h^{\,2}} \qquad (2)$$

where I_1 is the fundamental current and I_h is the representation of the whole current harmonic spectrum, respectively.

According to [2], when the voltage is approximately sinusoidal, the fundamental current I_1 and the fundamental power factor "$cos\ f_1$" can be respectively given by (3) and (4).

$$I_1 = \frac{I_{RMS}}{\sqrt{1+THD_I^{\,2}}} \qquad (3)$$

$$\cos f_1 = TPF\sqrt{1+THD_I^{\,2}} \qquad (4)$$

where TPF is the true power factor considering the presence of harmonic components. Therefore the fundamental current can be divided in two components i.e. active and reactive current, according to (5).

$$\dot{I}_1 = I_1\underline{|f_1} = I_1\left(\cos f_1 + j\sin f_1\right) = I_{1p} + jI_{1q} \qquad (5)$$

where I_{1p} is the fundamental active current and I_{1q} is the fundamental reactive current, respectively.

On the other hand, the harmonic current is given by (6) [2].

$$I_h = \frac{I_{RMS}}{\sqrt{1+\dfrac{1}{THD_I^{\,2}}}} \qquad (6)$$

The losses due to distorted currents are basically proportional to the square of the rms current. However, one should also consider the resistance increase with the frequency due to the proximity and skin effects of conductors.

$$P_j = \sum_{n=1} R_n I_n^{\,2} = R_1 I_1^{\,2} + \sum_{n=2} R_n I_n^{\,2} = P_1 + P_h \qquad (7)$$

Once that the resistance increase with the frequency can be neglected, implying a slight underestimation in the losses calculation, the I^2R losses can be separated into the following parts:

i) I^2R losses caused by the fundamental active current (P_{1p}), which are inherent to the load operation and can not be compensated;

ii) I^2R losses caused by the fundamental reactive current (P_{1q}) and by the harmonic current (P_h), which are undesirable and can be adequately compensated.

Therefore, one can rewrite (7) as:

$$P_j = P_{1p} + P_{1q} + P_h = R_1\left(I_{1p}^{\,2} + I_{1q}^{\,2} + I_h^{\,2}\right) \qquad (8)$$

where R_1 is the resistance at the fundamental frequency.

Considering that the harmonic I^2R losses are essentially reactive, both reactive fundamental and harmonic I^2R losses can be added and defined as a single quantity called total reactive I^2R losses, represented by $P_{reactive}$.

$$P_j = P_{1p} + P_{reactive} \qquad (9)$$

B. Effects of harmonic components in transformers

The presence of harmonic components in transformers causes the copper losses (I^2R losses) and the iron losses (hysteresis and eddy current losses) to increase [1]. This paper will only consider the load-related losses, which are provided in (10).

$$P_{load} = P_{j(transf.)} + P_{EC} \qquad (10)$$

where P_{load} is the total load-related losses, $P_{j(transf.)}$ is the total I^2R losses, and P_{EC} is the total eddy current losses. Thus the following expressions are given.

$$P_{j(tranf.)} = P_{j1(tranf.)}\left(1+THD_I^{\,2}\right) \qquad (11)$$

$$P_{EC} = P_{EC1}\sum_{n=1}\left(\frac{I_n}{I_1}\right)^2 n^2 \qquad (12)$$

where $P_{j1(transf.)}$ represents the I^2R losses in the fundamental frequency, P_{EC1} is the eddy current losses in the fundamental frequency, n is the harmonic order, and I_n is the n-th order harmonic current.

III. BUDEANU'S MODEL

Even before the problems caused by harmonics became evident, Constantin I. Budeanu had developed in 1927 a mathematical model based on the frequency domain that separates the apparent power into three components i.e. active, reactive and distorted power [5], as the apparent power is given by:

$$S = V_{RMS}I_{RMS} = \sqrt{P^2 + Q^2 + D^2} \qquad (13)$$

where P, Q and D represent the active power, reactive power and distorted power, respectively. Thus, the apparent power can also be divided into a fundamental and a harmonic part, as follows.

$$S^2 = \left(V_1 I_1\right)^2 + \left(V_1 I_h\right)^2 + \left(V_h I_1\right)^2 + \left(V_h I_h\right)^2 \qquad (14)$$

$$S^2 = S_1^{\,2} + S_n^{\,2} \qquad (15)$$

where S_1 is the fundamental apparent power and S_n is the harmonic apparent power.

The ratio between S_n and S_1 can be described by the following expression [3]:

$$\left(\frac{S_n}{S_1}\right)^2 = THD_I^{\,2} + THD_V^{\,2} + \left(THD_I THD_V\right)^2 \qquad (16)$$

Assuming that THD_I is typically much greater than THD_V i.e. THD_V<8% and THD_I>20%, the S_n/S_1 ratio can be expressed as follows, where the result provides an error lower than 1% if compared to that given by (16).

$$\frac{S_n}{S_1} = THD_I \tag{17}$$

If THD_V<5%, this ratio can be represented by (18), where the result provides an error equal to about 0.15% if compared to that given by (16).

$$\frac{S_n}{S_1} = \sqrt{THD_I^{\,2} + THD_V^{\,2}} \tag{18}$$

IV. THE POWER QUALITY FACTOR

Once that part of the I^2R losses can be compensated, it is desirable that such compensation is performed according to the economical viability. The cost of losses is proportional to the square of the rms current, and it can be represented in Figure IV [7].

It can be seen that when the fundamental reactive current is equal to 40% of the active current, the cost of losses is equal to approximately 2% of the active energy cost, while the cost of the compensation is equal to 0.25%.

Based on the simplified S_n/S_I expression given in (17), a factor that points the need of the compensation according to the cost/benefit ratio is presented and proposed as follows:

$$\begin{aligned} PQF_{\%} &= \left(2 - \frac{P_j}{P_{1p}}\right) \times 100 \\ &= \left(2 - \frac{P_{1p} + P_{1q} + P_h}{P_{1p}}\right) \times 100 \end{aligned} \tag{19}$$

Substituting all the necessary parameters in (19), it provides:

$$PQF_{\%} = \frac{2 \times (TPF)^2 - 1}{(TPF)^2} \times 100 \tag{20}$$

This expression depends on the true power factor, that on the other hand depends on the current harmonic distortion, according to (4). When the power factor is unity and the harmonic distortion is null, the factor $PQF_{\%}$ is maximum and equal to 100%, as the power quality profile is considered optimum. When the fundamental reactive and harmonic losses are significant, this empirical expression provides values lesser than 100%. In critical situations, this expression may even provide negative values, and in this case the value will not be considered negative but null, implying the worst situation for the power quality.

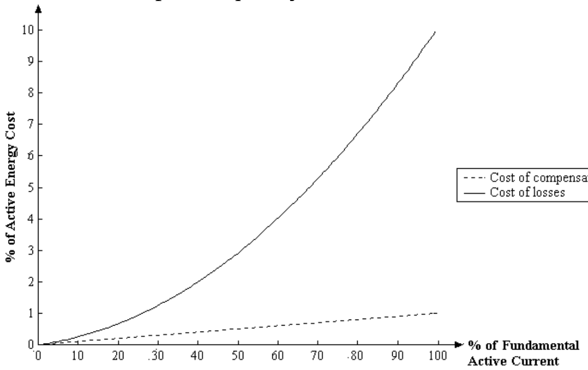

FIGURE IV
Cost of losses and compensation

Therefore this factor provides values varying from 0% to 100%, that can be classified as follows:

(i) 0% $\le PQF_{\%}$ <20% (critical): In this case, the power quality is deteriorated, and there is the need of imminent investigation and possible compensation of the reactive losses;

(ii) 20% $\le PQF_{\%} \le$ 50% (acceptable): The power quality profile is considered satisfactory, although a more detailed analysis is needed in order to determine whether the reactive losses compensation is economically viable;

(iii) 50%<$PQF_{\%} \le$ 100% (optimum): The power quality profile is in accordance with the allowed patterns, and there is no need of compensation.

V. CASE STUDY

An airline reservation center with 240 workstations in an office building located in the United States, whose electrical wiring schematic is presented in Figure V, showed signs of harmonic overloading near the workstations. The total load of the reservation center is about 300kW, distributed among three three-phase 480V feeders and a single-phase 277V feeder, and the transformer T_2 load is 60kW, correspondent to the computer workstations.

The measured results are shown in Table I. The necessary parameters for the calculation of the power losses neglecting the proximity and skin effects are determined then, and the results are presented in Table II.

In order to establish a comparison with the results obtained in Table II, the power losses considering the proximity and skin effects are also calculated, where the resistance increase with the frequency is shown in Figure VI and the values for cables 2, 3 and 4 are presented in Table III, Table IV and Table V, respectively. No results are presented for the current in cable 1 because its harmonic content was not measured and is considered unknown.

The obtained values are summarized in Table VI. It can be seen that the error is low, except for the cable 3. It means that when the harmonic current rms value in amperes is significant, the error tends to be greater, and the skin and proximity effects must be considered in the calculation. Although the measured current in the cable 4 is low, the losses are significant, due to the high resistance value. This can be explained once that this parameter is inversely proportional to the conductor size and when multiplied by the square of the rms current, causes higher losses.

The load-related losses in transformer T_2 are evaluated and the results are shown in Table VII. The I^2R losses in the fundamental frequency are assumed to be equal to 2.5% of the transformer load, and the eddy current losses in the fundamental frequency are assumed as 5% of the I^2R losses in the fundamental frequency.

One can see in Table VII that the I^2R losses in non sinusoidal conditions are about twice those in the fundamental frequency, while the eddy current losses are about fifteen times those in the fundamental frequency.

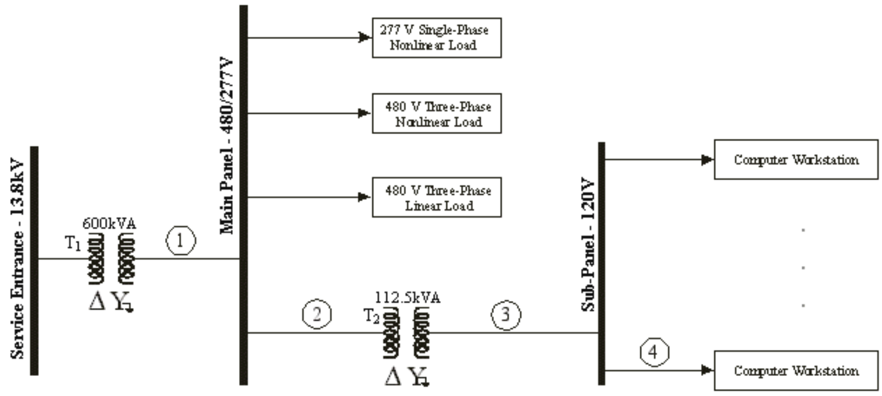

FIGURE V
Airline reservation center schematic

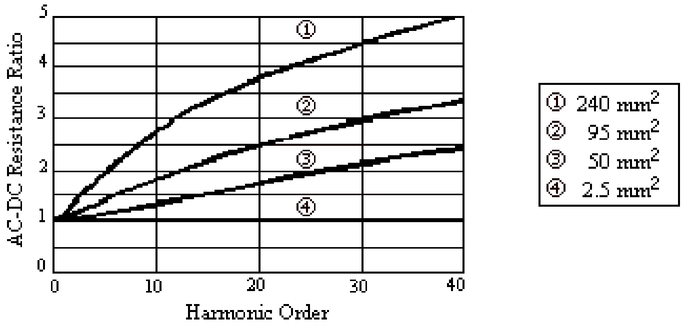

FIGURE VI
Resistance increase as a function of the frequency

TABLE I
Measured results

Cable Segment	Conductor Size (mm^2)	Resistance (ohms)	Rated Current (A)	Measured Voltage (V)	THD_V (%)	Measured Current (A)	THD_I (%)	*TPF*
Cable 1	240	0.001186	688	479	2.8	380	16	0.90
Cable 2	50	0.034633	150	475	4.0	86	30	0.89
Cable 3	185	0.004935	280	206	5.5	237	99	0.69
Cable 4	2.5	69.04266	21	116	7.8	6.2	100	0.64

TABLE II
Power losses neglecting the proximity and skin effects

Cable Segment	I_1 (A)	cos f_1	I_{1p} (A)	I_{1q} (A)	I_h (A)	P_{1p} (W)	P_{1q} (W)	P_h (W)	$P_{reactive}$ (W)	P_j (W)
Cable 1	375.23	0.91	342	154.37	60.04	138.73	28.27	4.28	32.55	171.28
Cable 2	82.37	0.93	76.54	30.45	24.71	202.90	32.10	21.15	53.25	256.15
Cable 3	168.42	0.97	163.53	40.31	166.74	131.98	8.02	137.21	145.23	277.21
Cable 4	4.38	0.91	3.97	1.86	4.38	1087.08	239.92	1327.00	1566.92	2654.00

TABLE III
Power losses in cable 2 considering the proximity and skin effects

n	I_n/I_1	I_n (A)	R_n/R_1	R_n (ohms)	P_{jn} (W)
1	1.0000	82.373	1.000	0.034633	235.00
3	0.0140	1.153	1.068	0.036988	0.05
5	0.2500	20.593	1.136	0.393438	16.68
7	0.1500	12.356	1.182	0.040936	6.25
9	0.0100	0.824	1.273	0.044088	0.03
11	0.0600	4.942	1.364	0.047240	1.15
13	0.0400	3.295	1.455	0.050391	0.55
15	0.0030	0.247	1.545	0.053508	0.003
17	0.0200	1.647	1.636	0.056660	0.15
19	0.0150	1.236	1.727	0.059812	0.09
21	0.0010	0.082	1.818	0.062963	0.0004
23	0.0120	0.988	1.909	0.066115	0.06
25	0.0110	0.906	2.000	0.069267	0.05
				P_j (W)	260.09

TABLE IV
Power losses in cable 3 considering the proximity and skin effects

n	I_n/I_1	I_n (A)	R_n/R_1	R_n (ohms)	P_{jn} (W)
1	1.0000	168.424	1.000	0.004935	140.00
3	0.7700	129.687	1.400	0.006909	116.21
5	0.4600	77.475	1.700	0.008390	50.36
7	0.2700	45.475	2.000	0.009871	20.41
9	0.2000	33.685	2.200	0.010858	12.32
11	0.1820	30.653	2.400	0.011845	11.13
13	0.1510	25.432	2.700	0.013325	8.62
15	0.1140	19.200	2.800	0.013819	5.09
17	0.0850	14.316	3.000	0.014806	3.03
19	0.0600	10.105	3.100	0.015300	1.56
21	0.0420	7.074	3.300	0.016287	0.81
23	0.0510	8.590	3.400	0.016780	1.24
25	0.0320	5.390	3.500	0.017274	0.50
				P_j (W)	371.30

TABLE V
Power losses in cable 4 considering the proximity and skin effects

n	I_n/I_1	I_n (A)	R_n/R_1	R_n (ohms)	P_{jn} (W)
1	1.0000	4.384	1.000	69.0427	1327
3	0.8135	3.566	1.000	69.0427	878.19
5	0.5162	2.263	1.000	69.0427	353.60
7	0.2198	0.964	1.000	69.0427	64.11
9	0.0566	0.248	1.000	69.0427	4.25
11	0.0949	0.416	1.000	69.0427	11.95
13	0.0741	0.325	1.000	69.0427	7.29
15	0.0241	0.106	1.000	69.0427	0.77
17	0.0350	0.153	1.000	69.0427	1.63
				P_j (W)	2648.78

TABLE VI
Power losses in the airline reservation center facility

Cable Segment	P_{1p} (W)	P_{1q} (W)	P_h (W)	$P_{reactive}$ (W)	P_j (W)	Error (%)
Cable 2	202.90	32.10	25.09	57.19	260.09	1.51
Cable 3	131.98	8.02	231.30	239.3	371.30	25.34
Cable 4	1087.08	239.92	1321.78	1561.70	2648.78	0.19

TABLE VII
Load-related losses in transformer T_2

Transformer	$P_{j1(transf.)}$ (W)	P_{EC1} (W)	$P_{j(transf.)}$ (W)	P_{EC} (W)	P_{load} (W)
T_2	1500	75	2970.15	1139.87	4110.02

VI. DIAGNOSTIC PROVIDED BY THE POWER QUALITY FACTOR

The power quality factor can be applied to the studied case, and the diagnostic is provided in Table VII. For this calculation, the values obtained in Table II are employed, where the influence of the proximity and skin effects is not considered.

TABLE VIII
Application of the power quality factor

Cable Segment	P_{1p} (W)	P_{1q} (W)	P_h (W)	$PQF_{\%}$ (%)	Power Quality Status
Cable 1	138.73	28.27	4.28	76.54	Optimum
Cable 2	202.90	32.10	21.15	73.76	Optimum
Cable 3	131.98	8.02	137.21	0	Critical
Cable 4	1087	239.92	1327	0	Critical

If the power quality factor provides negative values e.g. for cables 3 and 4, its value is considered null. According to Table VIII, the power quality factor points cables 3 and 4 as critical cases where the compensation must be performed, once that both conductors present high harmonic content and low power factor, as it is shown in Figure VII.

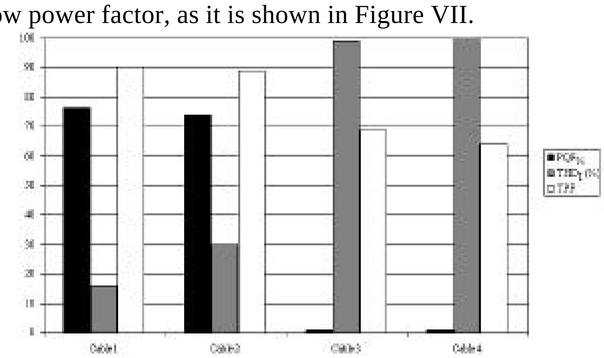

FIGURE VII
Profile provided by the power quality factor

TABLE IX
Overall cost of losses

Cable Segment	$P_{reactive}$ (W)	Annual Cost Of Losses (US$)
Cable 1	32.55	14.25
Cable 2	53.25	23.32
Cable 3	145.23	63.61
Cable 4	1566.92	686.31
Total	1797.95	787.49

To estimate the cost of the energy, the workstations are assumed to operate 12 hours per day and 365 days per year with electric energy at US$ 0.10 per kilowatt-hour, as the results are given in Table IX. One can notice that the losses must be compensated in cables 3 and 4, once that they are responsible for about 95% of the overall losses in the cables.

VII. MINIMIZATION OF I^2R LOSSES

In order to minimize I^2R losses, the following practical alternatives are recommended and should be considered according to the cost/benefit ratio:

i) Increase the conductors size or run a parallel conductor in cases where the losses are significant e.g. in cables 3 and 4;

ii) Increase the true power factor where its value is less than 0.7 e.g. in cables 3 and 4, by placing capacitor banks observing the resonance phenomenon risk, or even installing capacitors in series with inductors to avoid it. This alternative should also consider the cost/benefit ratio according to the study presented above;

iii) Consider the current harmonic content in the plant design and in cases where new loads are being added to the plant.

VIII. CONCLUSION

A study on the effects of the harmonic components in cables and transformers was carried out. A method for the calculation of I^2R power losses in cables regarding low voltage networks was also presented. It was shown that such losses are significant and must be compensated according to the economical viability. A factor that evaluates the power quality performance is proposed, and it works as a good reference to recommend the need of the reactive I^2R losses compensation.

ACKNOWLEDGMENT

The authors acknowledge CAPES, Fapemig and CNPq for the financial support to this work.

REFERENCES

[1]. A. C. Delaiba, “Comportamento de Transformadores com Cargas Não Lineares: Uma Abordagem Analítica, Experimental e Numérica pelo Método dos Elementos Finitos”. São Paulo, 1997. Tese de Doutorado – EPUSP.

[2]. R. C. Dugan, M. F. Mcgranaghan, H. W. Beaty, “Electrical Power Systems Quality”. Mc Graw-Hill, 1995.

[3]. A. E. Emanuel, “The Need for a Simple and Practical Resolution of the Apparent Power”. European Transactions on Electrical Power Engineering. Vol. 3, 1991, n. 1, p. 103.

[4]. R. D. Fuchs “Transmissão de Energia Elétrica - Linhas Aéreas”. Livros Técnicos e Científicos S.A, vol. 1, 1977.

[5]. A. M. Lopes, A. Oliveira, J. C. Oliveira, L. F. Pagotti, “A Influência das Componentes Harmônicas na Compensação da Potência Reativa”. SBQEE ’96, Uberlândia, MG, Brazil, June, 1996, p. 190-199.

[6]. A. A. Moustafa, A. M. Moussa, M. A. El-Gammal, “Separation of Customer and Supply Harmonics in Electrical Power Distribution Systems”. ICHQP 2000, Orlando, FL, USA, October, 2000, pp. 1035-1040.

[7]. S. Svensson, “Preferred Methods for Power-Related Measurements”. ICHQP ’98, Athens, Greece, October, 1998, pp. 238-243.