Compensation Algorithms based on Instantaneous Powers Defined in the Phase Mode and in the $\alpha\beta 0$ Reference Frame

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***Abstract* – In this paper, compensation algorithms based on the instantaneous powers defined in the $\alpha\beta 0$ reference frame and in the phase mode (abc phases) are described. The main objective is to clarify some concepts, regarding to instantaneous powers defined directly in the phase mode, hereafter called as the *abc Theory*, and the instantaneous real and imaginary powers of the *pq Theory*. Compensation characteristics derived from each one of these set of power definitions, applied to a shunt active filter, are highlighted and simulation results are shown.**

KEYWORDS

Instantaneous real and imaginary theory, instantaneous active and non-active currents, Lagrange multipliers method, active filters.

I. Introduction

INSTANTANEOUS active and reactive power definitions, in a general sense, is of great interest in the field of Power Electronics. A consistent base of instantaneous power definitions is fundamental to design controllers for active power line conditioners.

The traditional active, reactive and apparent powers, defined in the frequency domain, and several other power quality indices that are derived from them, only serve for off-line calculation and analysis of power quality issues. In general, power definitions in the time domain offer a more robust basis for the use in controllers for power electronic devices, because they are also valid during transients.

Active filters have been developed since 1983, when one of the first prototypes based on instantaneous power theory has been reported [1][2]. Based on that theory, known as the *pq Theory*, other control strategies for active power line conditioners were derived [3][4][5][6], including its extension for the use in three-phase four-wire systems [7].

When compared with other active filter controllers, the controllers based on the *pq Theory* have being criticized mainly due to the following concerns.

1. Controllers based on the *pq Theory* need low-pass filters to separate control signals corresponding to the instantaneous real and imaginary powers into average and oscillating parts. These low-pass filters introduce attenuation and phase shift in the Bode diagram, which results in time delays that degenerate the dynamic performance of the active filter;
2. Controllers based on the *pq Theory* demand more calculations, since they need the use of Clark Transformation;
3. Under distorted and/or unbalanced system voltages, the shunt active filter does not compensate properly the load currents and injects harmonic currents into the network, which are not originally present in the load current.

The first argument above is really a problem, but not only for the *pq Theory*-based controllers. The synchronous-reference-frame-based controller also needs low-pass filters to separate the average portions of the direct (i_d) and quadrature (i_q) current components [8][9]. Under non-sinusoidal system voltages, the current minimization methods also need some kind of filtering to obtain an average load conductance to determine the instantaneous active portion of the load current [10][11][12][13].

The second argument above represents a cost that should be paid to gain flexibility to compensate independently the average or oscillating portion of the real (active) and/or imaginary (reactive) powers, and the instantaneous zero-sequence power as well. Without the use of Clarke Transformation, the complexity of other controllers that use directly the *abc* line currents and phase voltages increases quickly, when that flexibility is necessary [13]. For instance, the compensation of positive and negative-sequence components included in the real (active) and imaginary (reactive) power, separately from zero-sequence components, without the use of the *pq Theory*, becomes a drawback that needs to be overcome.

Finally, the third argument contains a little of misinterpretation of the original control algorithm as proposed by *Akagi et al.* [1]. This algorithm compensates the load current in order to guarantee constant instantaneous real power drained from the network. Therefore, under non-sinusoidal voltage conditions, the compensated current cannot become sinusoidal [14].

Some specialists have the opinion that "the best control strategy" is the one that guarantees compensated currents drained from the network, that are proportional (they have the same waveforms) to the system voltages. Under balanced harmonic-free system voltages, this strategy compensates the load current by forcing the compensated current to be sinusoidal and in phase with the system voltage. With other words, the compensated currents get the same waveform as the system voltages, suggesting that the network is "supplying a pure-resistive equivalent load".

Summarizing, under non-sinusoidal and/or unbalanced system voltages, it is impossible to implement a shunt active filter that satisfies simultaneously:

i) Constant real power drained from the network;
ii) Sinusoidal compensated current;
iii) Proportionality between the system voltage and the

compensated current.

Thus, a choice has to be made to design the appropriate controller for the active power line conditioner that guarantees one of the three options listed above. This paper clarifies some confusing points in the literature, regarding to instantaneous powers defined directly in the phase mode (abc instantaneous phase voltages and line currents), hereafter called as the *abc Theory*, and the real, imaginary and zero-sequence powers of the *pq Theory*. Moreover, the compensation characteristics derived from each set of power definitions will be highlighted.

II. Current Compensation Algorithms Based on the PQ Theory

The αβ0 transformation is an algebraic transformation of three-phase voltages and currents into a stationary reference frame, also known as *Clarke Transformation*. The αβ0 transformation of a three-phase voltage and its inverse are given by:

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{-1}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} \tag{2}$$

Similar equations hold on for the line currents i_a, i_b, i_c. One advantage of applying the αβ0 transformation is the automatic separation of the zero-sequence components into the 0-axis (v_0 and i_0 variables).

The instantaneous powers defined in the αβ0 reference frame are the **real** power p, the **imaginary** power q and the **zero-sequence** power p_0. They are given by:

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & v_\beta & -v_\alpha \end{bmatrix} \cdot \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \tag{3}$$

The instantaneous active three-phase power can be written in terms of αβ0 components as described in equation (4).

$$p_{3\phi} = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 = v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c = p + p_0 \tag{4}$$

This equation shows that the instantaneous active three-phase power $p_{3\phi}$ is always equal to the sum of the real power p and the zero-sequence power p_0.

On the other hand, if the α - β variables of the imaginary power q are replaced by their abc variables, the following equation can be written.

$$q = v_\beta i_\alpha - v_\alpha i_\beta = \frac{1}{\sqrt{3}}\left[v_{bc} \cdot i_a + v_{ca} \cdot i_b + v_{ab} \cdot i_c\right] \tag{5}$$

Based on equation (3) it is possible to achieve the real currents ($i_{\alpha p}$, $i_{\beta p}$). Since the zero-sequence power is extracted from the real power, these real currents will not produce imaginary and / or zero-sequence power, independently of harmonics and / or unbalances that the system might have. The real currents may be determined by using equation (6).

$$\begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \cdot \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \cdot \begin{bmatrix} p \\ 0 \end{bmatrix} \tag{6}$$

Transforming the real currents $i_{\alpha p}$ and $i_{\beta p}$ into abc variables, the equation (7) is achieved.

$$\begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = \frac{\left(v_1 . i_a + v_2 . i_b + v_3 . i_c\right)}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2} \begin{bmatrix} v_1 / 3 \\ v_2 / 3 \\ v_3 / 3 \end{bmatrix} \tag{7}$$

where,

$$\begin{cases} v_1 = v_{ab} - v_{ca} \\ v_2 = v_{bc} - v_{ab} \\ v_3 = v_{ca} - v_{bc} \end{cases} . \tag{8}$$

The same idea, based on equation (3), may be used to determine the imaginary currents ($i_{\alpha q}$, $i_{\beta q}$). The imaginary currents will produce imaginary power only, independently of harmonics and / or unbalances that the system might have. The imaginary currents may be achieved by using (9).

$$\begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} 0 \\ q \end{bmatrix} \tag{9}$$

Transforming these imaginary currents in terms of abc-variables, equation (10) is achieved.

$$\begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} = \frac{q \cdot \sqrt{3}}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2} \begin{bmatrix} v_{bc} \\ v_{ca} \\ v_{ab} \end{bmatrix} \tag{10}$$

Hereafter, the active and non-active currents, determined by means of minimization methods, are compared with the real and imaginary currents, respectively. It is possible to realize that the imaginary currents are the same as the non-active currents, independently of harmonics and/or unbalances that the system might have. However, differences appear between the real and the active currents under the presence of zero-sequence unbalances.

III. Current Compensation Algorithms Based On Instantaneous Active And Non-Active Currents

A control algorithm for shunt current compensation is introduced, which is based on the set of power definitions presented by Fryze, in the 30's of the last century. The reactive (non-active) current of three-phase system is that component of the load current which does not produce any active power; but increases the current amplitude and the losses in the conductors. The non-active current can be determined through minimization methods. For the formulation, a hypothetical load current i_k, $k = (a, b, c)$, is assumed to consist of an *active* portion i_{pk} and a *non-active* portion i_{qk}, that is,

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} i_{pa} \\ i_{pb} \\ i_{pc} \end{bmatrix} + \begin{bmatrix} i_{qa} \\ i_{qb} \\ i_{qc} \end{bmatrix}. \tag{11}$$

The method involves minimizing the aggregate currents in the load, but under the constraint that the currents i_{qk} do not generate any active power. With this, the task consists of finding a minimum of:

$$L(i_{qa}, i_{qb}, i_{qc}) = (i_a - i_{qa})^2 + (i_b - i_{qb})^2 + (i_c - i_{qc})^2$$

Constrained by: (12)

$$g(i_{qa}, i_{qb}, i_{qc}) = v_a.i_{qa} + v_b.i_{qb} + v_c.i_{qc} = 0$$

The solution is well known for the calculation of the active currents [5][11][10], and is repeated here to compare with another method for finding directly non-active currents that are not found in the literature. This problem means that load currents should be minimized by extracting a maximum of non-active current and can be solved by applying the Lagrange Multipliers Method, which leads to the following system of equations.

$$\begin{bmatrix} 2 & 0 & 0 & v_a \\ 0 & 2 & 0 & v_b \\ 0 & 0 & 2 & v_c \\ v_a & v_b & v_c & 0 \end{bmatrix} \begin{bmatrix} i_{qa} \\ i_{qb} \\ i_{qc} \\ \lambda \end{bmatrix} = \begin{bmatrix} 2i_a \\ 2i_b \\ 2i_c \\ 0 \end{bmatrix} \tag{13}$$

Solving (13) for λ gives

$$\lambda = \frac{2(v_a.i_a + v_b.i_b + v_c.i_c)}{v_a^2 + v_b^2 + v_c^2} = \frac{2p_{3\phi}}{v_a^2 + v_b^2 + v_c^2} \tag{14}$$

By filling in (14) in (13), the instantaneous non-active currents are found to be:

$$\begin{bmatrix} i_{qa} \\ i_{qb} \\ i_{qc} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \frac{p_{3\phi}}{v_a^2 + v_b^2 + v_c^2} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{15}$$

Comparing (11) with (15), the active currents are given by:

$$\begin{bmatrix} i_{pa} \\ i_{pb} \\ i_{pc} \end{bmatrix} = \frac{p_{3\phi}}{v_a^2 + v_b^2 + v_c^2} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{16}$$

Therefore, the restriction imposed in the minimization method forces the active current calculated in (16) and the currents i_a, i_b, i_c of the generic load to produce the same instantaneous active three-phase power ($p_{3\phi}$), when multiplied by their respective phase voltages v_a, v_b, v_c, that is,

$$p_{3\phi} = v_a i_a + v_b i_b + v_c i_c = v_a i_{pa} + v_b i_{pb} + v_c i_{pc} \tag{17}$$

Hence, from the energy transportation point of view, they are fully equivalent. The difference is that the active currents i_{pa}, i_{pb}, i_{pc} (minimized currents) do not generate any instantaneous non-active (imaginary) power and have smaller rms values.

When the active currents achieved by equation (16) are compared with the real currents determined by equation (7), the only situation in which they will produce the same results is when the sum of the line currents of the load, as well as the sum of the phase voltages at the point of common coupling, is equal to zero. Otherwise, different results are achieved.

As explained, the real currents produce only real power. Contrarily, the active currents in (16) can produce real power, as well as zero-sequence power. Another interesting aspect is that, depending on the unbalances in the system voltages, the compensated currents drained from the network may become unbalanced and contain zero-sequence components that are not originally present in the load currents, which constitutes in a serious drawback.

A new methodology is proposed here to achieve the non-active currents. It is based on the concepts of the imaginary power, extracted from the *pq Theory* [1], together with the Lagrange Multipliers method. Therefore the task consists of finding a minimum of:

$$L(i_{qa}, i_{qb}, i_{qc}) = (i_a - i_{pa})^2 + (i_b - i_{pb})^2 + (i_c - i_{pc})^2$$

Constrained by: (18)

$$g(i_{qa}, i_{qb}, i_{qc}) = v_{bc}.i_{pa} + v_{ca}.i_{pb} + v_{bc}.i_{pc} = 0$$

By the same way, this problem can be solved applying the Lagrange Multipliers Method, which leads to the following system of equations.

$$\begin{bmatrix} 2 & 0 & 0 & v_{bc}/\sqrt{3} \\ 0 & 2 & 0 & v_{ca}/\sqrt{3} \\ 0 & 0 & 2 & v_{ab}/\sqrt{3} \\ v_{bc} & v_{ca} & v_{ab} & 0 \end{bmatrix} \begin{bmatrix} i_{pa} \\ i_{pb} \\ i_{pc} \\ \lambda \end{bmatrix} = \begin{bmatrix} 2i_a \\ 2i_b \\ 2i_c \\ 0 \end{bmatrix} \tag{19}$$

Solving (19) for λ gives

$$\lambda = \frac{2\sqrt{3}(v_{bc}.i_a + v_{ca}.i_b + v_{ab}.i_c)}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2} = \frac{6q}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2} \tag{20}$$

By filling in (20) in (19), the instantaneous active currents are found to be:

$$\begin{bmatrix} i_{pa} \\ i_{pb} \\ i_{pc} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \frac{(v_{bc}.i_a + v_{ca}.i_b + v_{ab}.i_c)}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2} \begin{bmatrix} v_{bc} \\ v_{ca} \\ v_{ab} \end{bmatrix} \tag{21}$$

Comparing (11) with (21) one sees that the non-active currents are given by:

$$\begin{bmatrix} i_{qa} \\ i_{qb} \\ i_{qc} \end{bmatrix} = \frac{\sqrt{3} \cdot q}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2} \begin{bmatrix} v_{bc} \\ v_{ca} \\ v_{ab} \end{bmatrix} \quad , \qquad (22)$$

which is the same as the imaginary currents defined in (10), but can present different results from that defined in (15), in the presence of zero-sequence components. Thus, although the same symbol is used, the non-active currents achieved in (15) cannot be confused with that from (22). The restriction imposed in the minimization method forces the non-active currents calculated in (22) and the currents i_a, i_b, i_c of the generic load to produce the same imaginary power, as defined in the *pq Theory*, that is,

$$q = \left(v_{bc} i_a + v_{ca} i_b + v_{ab} i_c\right)/\sqrt{3} = \left(v_{bc} i_{qa} + v_{ca} i_{qb} + v_{ab} i_{qc}\right)/\sqrt{3} \qquad (23)$$

Note that the non-active currents i_{qa}, i_{qb}, i_{qc} do not generate any instantaneous active power.

IV. Simulation Results

Three cases have been simulated in the MATLAB simulator in order to verify that, in the presence of zero-sequence components, the compensation method derived from (15) presents some drawbacks.

Fig. 1 shows the basic principle of shunt current compensation. Two control strategies can be derived from the previous discussions about current decompositions. In the first case, the control algorithm is based in the *pq Theory* and the shunt compensator provides the imaginary currents of the load, calculated as in (10). Therefore, the compensated currents drained from the network are given by

$$\begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \frac{\left(v_{bc} . i_a + v_{ca} . i_b + v_{ab} . i_c\right)}{v_{ab}^2 + v_{bc}^2 + v_{ca}^2} \begin{bmatrix} v_{bc} \\ v_{ca} \\ v_{ab} \end{bmatrix} \quad . \qquad (24)$$

In the second case, the control algorithm is based on the decomposition method into active and non-active currents. In this case, the active currents are calculated as given in (16) and the shunt compensator provides the non-active currents, calculated as given in (15). Therefore, the compensated currents drained from the network becomes

$$\begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} = \frac{p_{3\phi}}{v_a^2 + v_b^2 + v_c^2} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad . \qquad (25)$$

The first case describes the performance of equations (24) and (25) in the presence of a zero-sequence component in the load current only. Next, a case comprising a zero-sequence component in the system voltage only is shown. Finally, a test case with zero-sequence components simultaneously in the system voltage and in the load current is presented. The total simulation time for each case is equal to 40ms.

Fig. 2 shows the considered system voltages, which are balanced and composed only from the fundamental positive-sequence component. The load currents contain a fundamental positive-sequence component plus a zero-

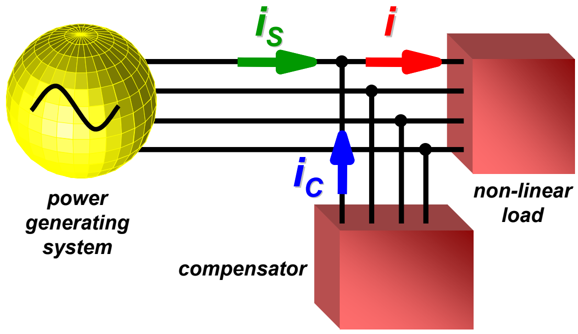

Fig. 1. Principle of shunt current compensation

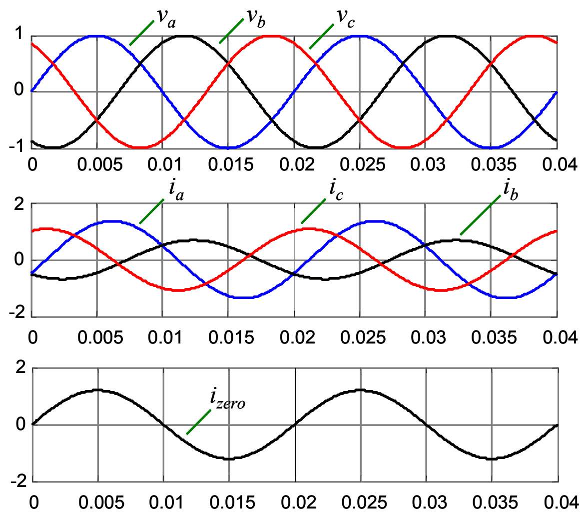

Fig. 2. System voltages and load currents with zero-sequence unbalance

sequence component, also at the fundamental frequency. It may be observed, that the positive-sequence component of the load current is lagged by 60 degrees from the system voltage, and the zero-sequence component is in phase with the voltage.

Fig. 3 shows the compensated currents drained from the network when the shunt compensator is providing the imaginary current of the load. In other words, the compensated currents are given as in (24). It may be verified that the compensated currents present the same unbalance as that of the load currents. This means that the zero-sequence currents are not compensated, as it was expected. Since the imaginary power of the load is being provided by the shunt compensator, the compensated current contains a smaller positive-sequence component that is in phase with that of the system voltage.

Fig. 4 shows the results when the active and non-active current decomposition method is applied. The compensated currents become balanced, since the presence of a zero-sequence component in the current only does not produce any zero-sequence power and do not contribute to the active currents given by (25). In other words, when there is an unbalance due to zero-sequence components in the load current only, in the second compensation method the neutral current of the load is treated as non-active current and is compensated by the shunt compensator.

The system voltages and load currents considered in the second simulation case is shown in Fig. 5, where the load currents are balanced and the system voltage contains a zero-sequence component at the fundamental frequency. As expected, the zero-sequence components in the voltage do not affect the calculation of the imaginary current in the first control strategy, and the compensated currents, given by (24), continue being balanced, but now in phase with the voltages and present smaller rms values, if compared with those of the load current, as it may be observed in Fig. 6.

Contrarily, the second compensation method is influenced by zero-sequence voltages and inserts an undesirable zero-sequence component in the compensated current, as can be seen in Fig. 7, not present in the original load current.

Finally, the third simulation case considers the presence of zero-sequence components, simultaneously in the system voltage and load current. Fig. 8 shows the system voltage with zero-sequence unbalance at the fundamental frequency, whereas the load current is unbalanced by a 2nd harmonic.

The results for both control strategies are shown in Fig. 9 and Fig. 10. Again, the first method presents better performance, since it does not alter the zero-sequence components in the load current, whereas the second method achieves compensated currents (active currents) that are modified by the presence of zero-sequence components in the system voltage.

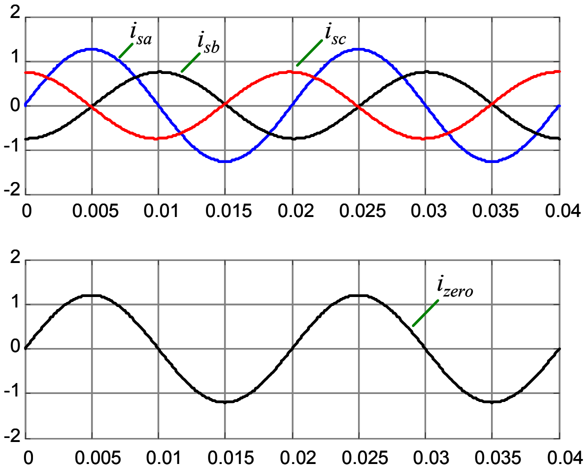

Fig. 3. Compensated currents when the imaginary current of the load is being compensated (case #1)

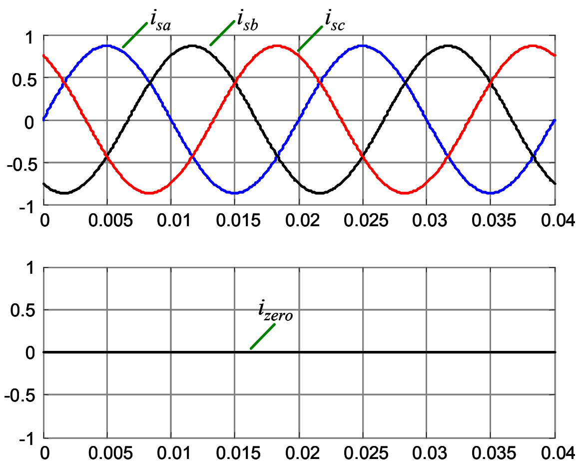

Fig. 4. Compensated currents comprising only the active current of the load (case #1)

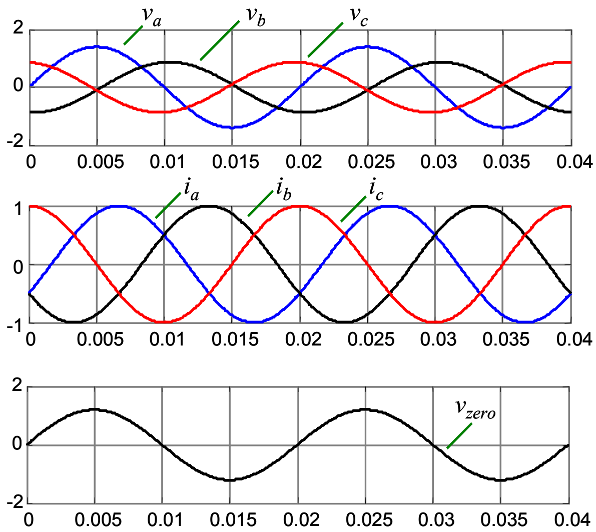

Fig. 5. System voltages with zero-sequence unbalance and load currents

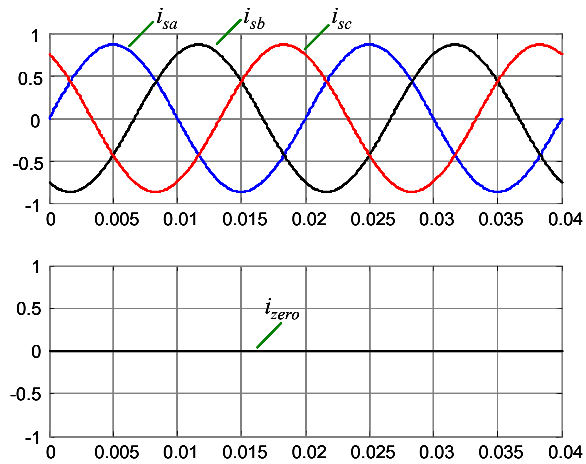

Fig. 6. Compensated currents when the imaginary current of the load is being compensated (case #2)

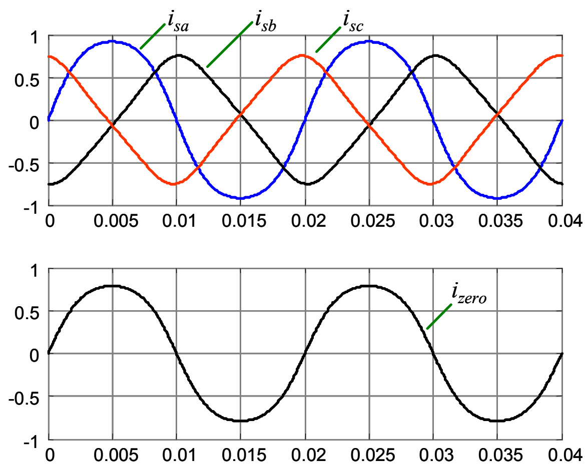

Fig. 7. Compensated currents comprising only the active current of the load (case #2)

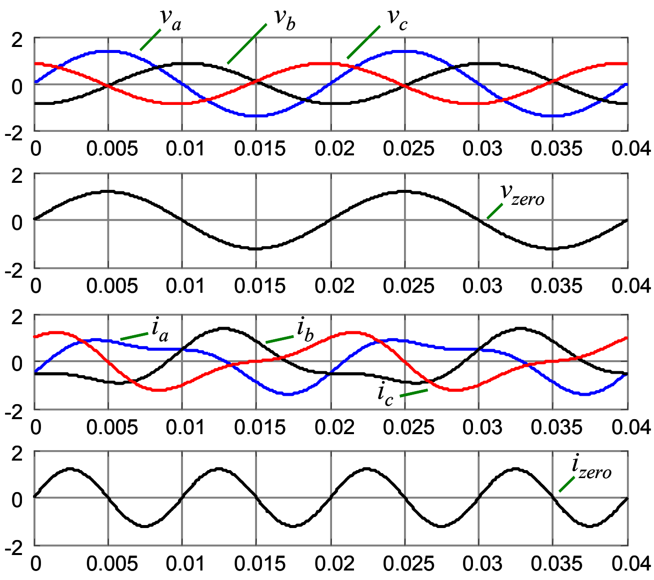

Fig. 8. System voltages with zero-sequence unbalance and load currents with zero-sequence unbalance at second harmonic

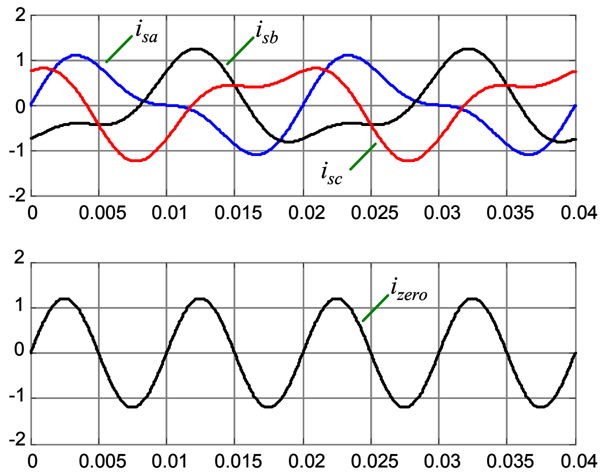

Fig. 9. Compensated currents when the imaginary current of the load is compensated (case #3)

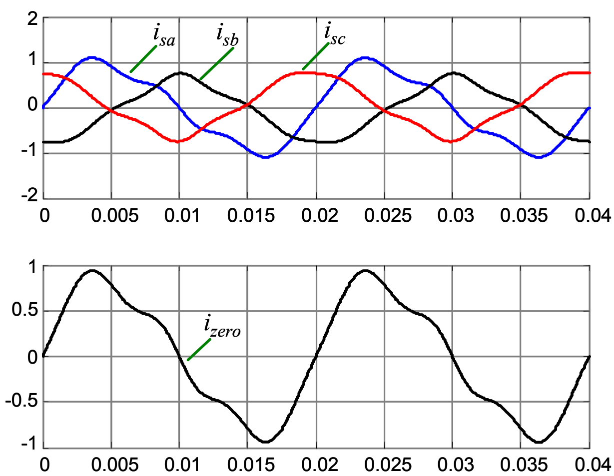

Fig. 10. Compensated currents comprising only the active current of the load (case #3)

V. CONCLUSIONS

Compensation algorithms based on the *pq Theory* and *abc Theory* were presented, under the presence of zero-sequence components. The controller based on the active currents does not treat coherently the zero-sequence components in the load current, since it is compensated (treated as non-active currents) or not (treated as active currents), depending on the absence or presence of zero-sequence components in the system voltages.

To overcome such problem, the use of the real and imaginary currents is imperative as determined in the approach based on the *pq Theory,* and additionally to take the decision if the zero-sequence currents of the load should or should not be compensated separately and completely, as already proposed in previous works, listed in the reference, dealing with shunt active filters for three-phase four-wire systems.

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