ITERATIVE INSTRUMENTAL VARIABLE METHOD FOR ROBUST IDENTIFICATION OF MOTOR DRIVES

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***Abstract* - This paper highlights an adaptive weighted instrumental variable (WIV) algorithm for on-line system identification by applying numerically robust orthogonal Householder transformations to control design purpose. Thus, the speed DC servomotor parameters estimation on the basis of data obtained from closed loop experiments, without modeling the noise disturbing the system, by direct, indirect and joint input/output methods is presented. The approach for the selection and usage of instruments proposed shows that the algorithm obtains acceptable results : the choice of instruments has desirable statistical properties and the instruments produce asymptotically unbiased estimates.**

KEYWORDS
System identification, closed loop identification, instrumental variable, parameter estimation

1. INTRODUCTION

The problem of parametric identification of a linear system on the basis of data obtained from closed loop experiments has received considerable attention in the literature [9][12]. Often in the identification system area, a common concern and important problem is that the input and output measurements may be contaminated by noise. Another source of random noise in the measured data is that the system to be identified is also driven by disturbance at some point. It is a common problem in closed loop identification where many of the identification methods, that work well in open loop, fail when applied directly to measured input-output data[6][7]. The reason is the nonzero correlation between the input and the unmeasured output noise that is inevitable in adaptive control schemes. For low levels of noise by using least squares (LS) method, for example, may produce excellent estimates of the system parameters. However, with larger levels of noise may require some modifications in this method to overcoming the inconsistency problem induced by noise acting on the system. Many kinds of modified least square method have been developed such as the generalized least square (GLS) method, the extended least square (ELS) method and prediction error (PE) method, where the noise model needs to be estimated at same time as the system parameters are being estimated. Thus the results of these methods are inevitably dependent upon the accuracy of the noise model and some constraint conditions on it must be satisfied in these methods to obtain consistent parameter estimates[2][5]. In general, however, it is very difficult to model the noise accurately and it is also hard to know a priori whether the noise model satisfies these conditions. To overcoming the bias problem without modeling the noise, the instrumental variables (IV) method can be developed. It provides a promising way to obtain consistent estimates which have certain optimal properties by choosing proper instrument variables [1][2]. However, it seems at the initial stage of the development that there has not been any general and efficient technique to choose suitable instrument variables for the identification of linear systems.

Among the various efficiency issues, characterizing the performance of an algorithm, numerical and identification robustness are of great importance, this is, an algorithm with good numerical error properties, do not produce bias in estimates preserving good convergence properties when the input-output data are contaminated not only normal noise but also a small number of large errors that are often unavoidable and very difficult to pick them out before processing the data and tracking parameters that vary with time.

This paper presents a weighted instrumental variable (WIV) algorithm based on orthogonal transformation via QR factorization to obtain the properties below presented. Simulation and experiments results show the efficiency of algorithm with the instrument variables proposed compared with some non-robust and robust algorithms already developed [8] and its application to closed loop identification.

2. PROBLEM FORMULATION

Consider the ARX structure

$$y_t = b_1 u_{t-1} + \ldots + b_{nb} u_{t-nb} - c_1 y_{t-1} - \ldots - c_{nc} y_{t-na} + \xi_t \tag{1}$$

where u(t) and y(t) are the system input and output, respectively. ξ_t is an unknown noise disturbing the system. Denote :

$$\theta^T = (b_1, \ldots, b_{nb}, c_1, \ldots, c_{nc}) \tag{2}$$

$$\mathbf{a}_t^T = (u_{t-1}, \ldots, u_{t-nb}, -y_{t-1}, \ldots, -y_{t-na}) \tag{3}$$

Then the system eqn. 1 can be expressed by following vectorial form:

$$\mathbf{Y} = \mathbf{A}\theta + \Xi \tag{4}$$

where

$$\mathbf{Y}^T = [\mathbf{y}_1, \ldots, \mathbf{y}_n] \tag{5}$$

$$\mathbf{A}^T = [\mathbf{a}_1, \ldots, \mathbf{a}_p] \tag{6}$$

$$\Xi^T = [\xi_1, \ldots, \xi_n] \tag{7}$$

with p equal the dimension of the problem, i.e. $nb + nc$ and n is the number of samples.

We seek to obtain a consistent parameter estimation for the parameter θ from the available observed data $\{y_t, u_t\}_1^n$ so that the error e_t between the measured output of the system y_t and the output of the associated model $\hat{y}_t$ is minimum in the least square sense, this is, the vector $\hat{\theta}$ that solves

$$\min \|\mathbf{A}\theta - \mathbf{b}\|_2^2 \tag{8}$$

3. DERIVATION OF THE ALGORITHM

Our algorithm uses orthogonal matrices to solve the least square problem in by QR factorization. The usage of orthogonal transformation for solving least squares problems is well established [10]. The usage of orthogonal transformation matrices is preferred because they are easy to invert, giving great accuracy and speed computationally, they are always perfectly conditioned and backward error analysis is simplified considerably when orthogonal transformations are used. The reason for this is that spectral and Euclidean norms, which are must commonly used in such analysis, are invariant under orthogonal transformations [11].

3.1 Instrumental Variable Method

The solution of the IV method is that a **Z** matrix is defined so that it is uncorrelated with the noise and correlated with the input and output. Therefore the following conditions must be satisfied

$$\underset{n\to\infty}{Lim}\left(\frac{1}{n}\right)\mathbf{Z}^T \Xi = 0 \tag{9}$$

$$\underset{n\to\infty}{Lim}\left(\frac{1}{n}\right)\mathbf{Z}^T\mathbf{X} = \mathbf{G} \tag{10}$$

where **G** is a nonsingular matrix. The eqn.(9)-(10) guarantee asymptotically unbiased parameter estimates.

In this paper, the set of instruments is chosen to be the delayed measurable inputs, i.e., the t-th row of the **Z** matrix is given by

$$\mathbf{z}_t = [\,u_{t-1} \quad u_{t-2} \quad \ldots \quad u_{t-p}\,] \tag{11}$$

where p is equal the dimension of the problem, i.e. $nb + nc$.

Proof of consistency

The standard proof of consistency for the IV estimator rely upon the uncorrelated of the sequence $\{\xi_t\}$. We now show that the estimates obtained by use of the proposed instruments in eqn.(11) is consistent when the sequence $\{\xi_t\}$ is correlated.

Consider the following stable model

$$y_t = b_1 u_{t-1} - c_1 y_{t-1} + \xi_t \tag{12}$$

where u_t, y_t and ξ_t are the system input, output and unknown correlated noise disturbing the system which is assumed statistically independent of u_t, respectively.

The **Z**, **A** and **Y** matrix are given by

$$\mathbf{Z} = \begin{bmatrix} u_0 & u_{-1} \\ u_1 & u_0 \\ \bullet & \bullet \\ u_{n-1} & u_{n-2} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} u_0 & -y_0 \\ u_1 & -y_1 \\ \bullet & \bullet \\ u_{n-1} & -y_{n-1} \end{bmatrix} \text{ and}$$

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ y_n \end{bmatrix}, \text{ respectively.}$$

The corresponding IV estimator is

$$\begin{bmatrix} \hat{b}_1 \\ \hat{c}_1 \end{bmatrix} = [\mathbf{Z}^T\mathbf{A}]^{-1}\mathbf{Z}^T\mathbf{Y} \tag{13}$$

and

$$\begin{bmatrix} \hat{b}_1 \\ \hat{c}_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{n}\sum_{t=1}^{n} u_{t-1}^2 & -\frac{1}{n}\sum_{t=1}^{n} u_{t-1}y_{t-1} \\ \frac{1}{n}\sum_{t=1}^{n} u_{t-1}u_{t-2} & -\frac{1}{n}\sum_{t=1}^{n} u_{t-2}y_{t-1} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{n}\sum_{t=1}^{n} u_{t-1}y_t \\ \frac{1}{n}\sum_{t=1}^{n} u_{t-2}y_t \end{bmatrix} \tag{14}$$

Stability of the system imply that the summations in eqn.(14) converge in probability to their expected values[6], i.e.,

$$\begin{bmatrix} \frac{1}{n}\sum_{t=1}^{n} u_{t-1}^2 & -\frac{1}{n}\sum_{t=1}^{n} u_{t-1} y_{t-1} \\ \frac{1}{n}\sum_{t=1}^{n} u_{t-1} u_{t-2} & -\frac{1}{n}\sum_{t=1}^{n} u_{t-2} y_{t-1} \end{bmatrix} \xrightarrow{prob} \begin{bmatrix} R_{uu}(0) & -R_{uy}(0) \\ R_{uu}(1) & -R_{uy}(-1) \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{n}\sum_{t=1}^{n} u_{t-1} y_t \\ \frac{1}{n}\sum_{t=1}^{n} u_{t-2} y_t \end{bmatrix} \xrightarrow{prob} \begin{bmatrix} R_{uy}(-1) \\ R_{uy}(-2) \end{bmatrix} \tag{15}$$

where $R_{\alpha\beta}(\tau) = E[\alpha_t \beta_{t-\tau}]$.

From Frechet's theorem[6]

$$\begin{bmatrix} \hat{b}_1 \\ \hat{a}_1 \end{bmatrix} \xrightarrow{prob} \begin{bmatrix} R_{uu}(0) & -R_{uy}(0) \\ R_{uu}(1) & -R_{uy}(-1) \end{bmatrix}^{-1} \begin{bmatrix} R_{uy}(-1) \\ R_{uy}(-2) \end{bmatrix} \tag{16}$$

Developing the inverse in eqn.(16), we have

$$\begin{bmatrix} R_{uu}(0) & -R_{uy}(0) \\ R_{uu}(1) & -R_{uy}(-1) \end{bmatrix}^{-1} = \frac{1}{\Delta} \begin{bmatrix} -R_{uy}(-1) & R_{uy}(0) \\ -R_{uu}(1) & R_{uu}(0) \end{bmatrix} \tag{17}$$

where $\Delta = R_{uu}(1) R_{uy}(0) - R_{uu}(0) R_{uy}(-1)$.

Thus,

$$\begin{bmatrix} \hat{b}_1 \\ \hat{a}_1 \end{bmatrix} \xrightarrow{prob} \frac{1}{\Delta} \begin{bmatrix} -R_{uy}(-1) & R_{uy}(0) \\ -R_{uu}(1) & R_{uu}(0) \end{bmatrix} \begin{bmatrix} R_{uy}(-1) \\ R_{uy}(-2) \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} \hat{b}_1 \\ \hat{a}_1 \end{bmatrix} \xrightarrow{prob} \frac{1}{\Delta} \begin{bmatrix} R_{uy}(0) R_{uy}(-2) - R_{uy}^2(-1) \\ R_{uu}(0) R_{uy}(-2) - R_{uu}(1) R_{uy}(-1) \end{bmatrix} \tag{19}$$

From eqn.(12), multiplying for u_{t-1} e u_{t-2}, result

$$u_{t-1} y_t = b_1 u_{t-1} u_{t-1} - a_1 u_{t-1} y_{t-1} + u_{t-1} \xi_t \tag{20}$$

$$u_{t-2} y_t = b_1 u_{t-2} u_{t-1} - a_1 u_{t-2} y_{t-1} + u_{t-2} \xi_t \tag{21}$$

and

$$R_{uy}(-1) = b_1 R_{uu}(0) - a_1 R_{uy}(0) \tag{22}$$

$$R_{uy}(-2) = b_1 R_{uu}(1) - a_1 R_{uy}(-1) \tag{23}$$

Substituting eqn.(22)-(23) into eqn.(19), we have

$$\begin{bmatrix} \hat{b}_1 \\ \hat{a}_1 \end{bmatrix} \xrightarrow{prob} \begin{bmatrix} b_1 \\ a_1 \end{bmatrix} \tag{24}$$

Thus, with the proposed instruments, produce asymptotically unbiased estimates, this is, $\hat{b}_1$ and $\hat{c}_1$ converge in probability to the true parameters b_1 and c_1 and the estimator obtained is consistent. The proof was with two parameters for simplicity of analysis but can be extended to any one with similar result.

3.2 On-line Identification Algorithm (WIV)

In many applications, the structure of the model may be known, but its parameters may be known and changing with time because of change in operation conditions, aging of equipment, etc., rendering off-line parameter estimation techniques ineffective. Thus, this work was motivated by developing of an algorithm that provide frequent estimates of the parameters by properly processing the I/O data on-line and to adapt itself to possible variation of the parameters with time.

The interest problem may be couched as

$$\mathbf{Z}^T \mathbf{A} \theta = \mathbf{Z}^T \mathbf{Y} \tag{25}$$

where $\mathbf{Z}_{n\times p}$, $\mathbf{A}_{n\times p}$, $\theta_{p\times 1}$ and $\mathbf{Y}_{n\times 1}$ are the instrumental variable matrix, data matrix, parameters vector and output vector, respectively.

The eqn.(25) can be rewritten as

$$\mathbf{Z}^T \mathbf{W} \mathbf{A} \theta = \mathbf{Z}^T \mathbf{W} \mathbf{Y} \tag{26}$$

where $\mathbf{W}_{n\times n}$ and $\mathbf{W}_n = diag\left(\lambda^{n-1}, \lambda^{n-2}, ..., 1\right)$, with $0 < \lambda < 1$. The scalar λ is known as the *forgetting factor* and it is used to place less weight on past data.

Developing both sides in eqn.(26), as **Z**, **W**, **A** and **Y** are known, result

$$\mathbf{S} \theta = \mathbf{b} \tag{27}$$

where $\mathbf{S}_{p\times p} = \mathbf{Z}^T \mathbf{W} \mathbf{A}$ and $\mathbf{b}_{p\times 1} = \mathbf{Z}^T \mathbf{W} \mathbf{Y}$. It is worth emphasizing that the resulting order of the **S** matrix and of the **b** vector are lower than order of the **A** matrix and of the **Y** vector, because p is equal to the number of parameters that will be estimated, implying less computational effort and, consequently, greater speed to solution of θ.

Generically, the **Z**, **W**, **A** matrices and the **Y** vector are given by

$$\mathbf{Z}^T = \begin{bmatrix} u_0 & u_1 & \dots & u_{n-1} \\ u_{-1} & u_0 & \dots & u_{n-2} \\ \bullet & \bullet & \dots & \bullet \\ u_{-p+1} & u_{-p+2} & \dots & u_{n-p} \end{bmatrix};$$

$$\mathbf{W} = \begin{bmatrix} \lambda^{n-1} & 0 & . & . & 0 \\ 0 & \lambda^{n-2} & 0 & . & . \\ . & 0 & . & 0 & . \\ . & . & 0 & \lambda & 0 \\ 0 & . & . & 0 & 1 \end{bmatrix};$$

$$\mathbf{A} = \begin{bmatrix} u_0 & u_{-1} & \dots & u_{1-nb} & -y_0 & -y_{-1} & \dots & -y_{1-nc} \\ u_1 & u_0 & \dots & u_{2-nb} & -y_1 & -y_0 & \dots & -y_{2-nc} \\ \bullet & \bullet & \dots & \bullet & \bullet & \bullet & \dots & \bullet \\ u_{n-1} & u_{n-2} & \dots & u_{n-nb} & -y_{n-1} & -y_{n-2} & \dots & -y_{n-na} \end{bmatrix}$$

and

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ y_n \end{bmatrix}$$

Hence, $\mathbf{S}_{p\times p} = \mathbf{Z}^T \mathbf{WA}$ result

$$\mathbf{S} = \begin{bmatrix} \sum_{t=1}^{n} u_{t-1}^2 \lambda^{n-t} & \dots & \sum_{t=1}^{n} u_{t-1} u_{t-nb} \lambda^{n-t} & -\sum_{t=1}^{n} u_{t-1} y_{t-1} \lambda^{n-t} & \dots & -\sum_{t=1}^{n} u_{t-1} y_{t-nc} \lambda^{n-t} \\ \sum_{t=1}^{n} u_{t-2} u_{t-1} \lambda^{n-t} & \dots & \sum_{t=1}^{n} u_{t-2} u_{t-nb} \lambda^{n-t} & -\sum_{t=1}^{n} u_{t-2} y_{t-1} \lambda^{n-t} & \dots & -\sum_{t=1}^{n} u_{t-2} y_{t-nc} \lambda^{n-t} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \sum_{t=1}^{n} u_{t-p} u_{t-1} \lambda^{n-t} & \dots & \sum_{t=1}^{n} u_{t-p} u_{t-nb} \lambda^{n-t} & -\sum_{t=1}^{n} u_{t-p} y_{t-1} \lambda^{n-t} & \dots & -\sum_{t=1}^{n} u_{t-p} y_{t-nc} \lambda^{n-t} \end{bmatrix} \tag{28}$$

and $\mathbf{b}_{p\times 1} = \mathbf{Z}^T \mathbf{WY}$ is

$$\mathbf{b} = \begin{bmatrix} \sum_{t=1}^{n} u_{t-1} y_t \lambda^{n-t} \\ \sum_{t=1}^{n} u_{t-2} y_t \lambda^{n-t} \\ \bullet \\ \sum_{t=1}^{n} u_{t-p} y_t \lambda^{n-t} \end{bmatrix} \tag{29}$$

From eqn.(28)-(29), we can observe that the elements of the **S** matrix and of the **b** vector are summations that depend of the actual and immediately former values, based on the dimension of the problem, of the input and output measures. This imply in generating, directly, i.e., in each sample, **S** and **b**, without need of a priori batch matritial operations, as in eqn.(26), with advantage that the order problem is lower to application of the QR factorization.

Thus, the problem may be couched as that of finding the solution of

$$\underset{\hat{\theta}}{\text{minimize}} \| \mathbf{S}\theta - \mathbf{b} \|_2^2 \tag{30}$$

Applying QR factorization via House-holder orthogonal transformations, we have

$$\underset{\hat{\theta}}{minimize} \| \mathbf{Q}^T\mathbf{S}\theta - \mathbf{Q}^T\mathbf{b} \|_2^2 \tag{31}$$

and

$$\underset{\hat{\theta}}{minimize} \| \mathbf{R}\theta - \mathbf{d} \|_2^2 \tag{32}$$

where $\mathbf{Q}_{p\times p}$ is an orthogonal matrix, $\mathbf{R}_{p\times p}$ is an upper triangular matrix and $\mathbf{d}_{p\times 1}$ is a resulting vector. Hence, the minimum point of eqn.(30) may be found by solving $\mathbf{R}\hat{\theta} = \mathbf{d}$ by back substitution. The algorithm receive an initial batch data to initial estimation and the updating is obtained for simple acquisition of input and output data and insert it into summations of the matrix **S** and of the vector **b**, this is, in the k-th sample, we have

$$\mathbf{S}_{new} = \mathbf{S} + \lambda \begin{bmatrix} u_k^2 & \cdots & u_k u_{k-nb+1} & -u_k y_k & \cdots & -u_k y_{k-nc+1} \\ u_{k-1}u_k & \cdots & u_{k-1}u_{k-nb+1} & -u_{k-1} y_k & \cdots & -u_{k-1} y_{k-nc+1} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ u_{k-p+1}u_k & \cdots & u_{k-p+1}u_{k-nb+1} & -u_{k-p+1} y_k & \cdots & -u_{k-p+1} y_{k-nc+1} \end{bmatrix} \tag{33}$$

and

$$\mathbf{b}_{new} = \mathbf{b} + \lambda \begin{bmatrix} u_k y_k \\ u_{k-1} y_k \\ \bullet \\ u_{k-p+1} y_k \end{bmatrix} \tag{34}$$

4. APPROACHES TO CLOSED LOOP IDENTIFICATION

In this paper, the proposed instrumental variable algorithm WIV will be applied to identification by direct approach, indirect approach and two stages method belong to joint input-output approach [6, 7, 12,14,15,16]. We will illustrate the applicability of the algorithm to identify sufficiently accuracy models, which represent the dynamic behavior of the plant in closed loop, important in identification for control, considering it as an alternative adaptive algorithm to closed loop identification.

4. RESULTS

In this section, we present an experimental application to show the applicability of the algorithm to closed loop identification as a basis to identification for control.

4.1 Experimental results

Our experiment is to identify speed DC servomechanism of the our control and automation laboratory. This identification process is divided in three steps: 1) Open loop identification as shown on Fig. 1, 2) Closed loop identification utilizing the direct method, shown on Fig. 2, 3) Closed loop identification utilizing the indirect method, shown on Fig. 3.

The both 2 and 3 steps we use the proportional controller of K_p = 1, to illustrate the application of the algorithm to closed loop identification and we are concerned only with the identified model analysis.

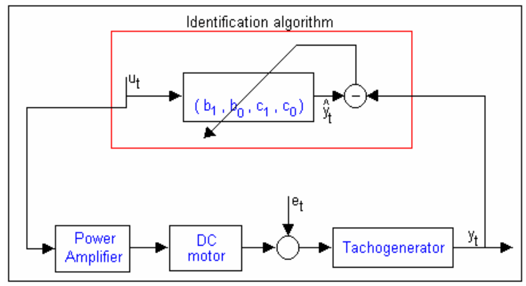

Figure 1. Step 1 : Open loop identification

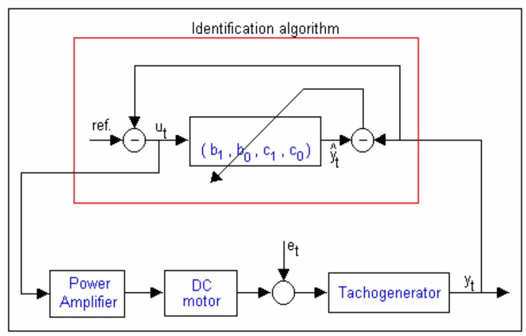

Figure 2. Closed loop identification by direct method.

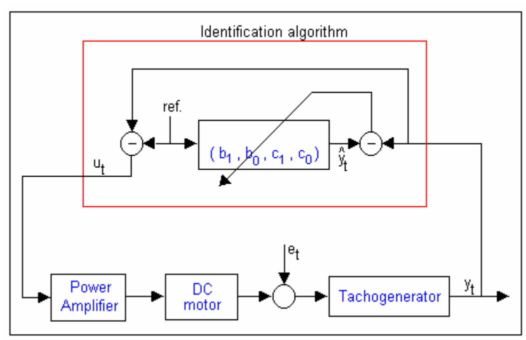

Figure 3. Closed loop identification by indirect method.

To this experiment λ is taken as 0.95, 25 pairs of input-output data were utilized to initial estimation, the sample period T is taken 10ms and the total of points is 600. The input and reference signals are taken as the voltage of 4.0V. The Tab. 1 shows the obtained comparative results of the estimates of parameters in open loop and by closed loop identification methods presented.

Parameters	Open loop	Direct method	Indirect method
a_1	-0.370435	-0.295373	-0.365008
a_0	-0.570980	-0.579290	-0.548080
b_1	0.082360	0.094971	0.082336
b_0	-0.030469	-0.010481	-0.023247

Table 1. Identified parameters using the proposed algorithm to open loop and closed loop (direct and indirect methods) identification.

Considering the open loop results as true parameters, we can note that the indirect method was better. This because the input data to algorithm consist the reference signal which is statistically independent of the noise disturbing the system. However, it is important emphasizing the effect of the nonlinearity from power amplifier circuit and the noise signals from the acquisition data system.

The Fig. 4 shows the estimated and real curves of the three cases.

The results clearly show the accuracy of the estimation by direct and indirect methods, this is, the real and estimated curves are practically equals and, thus, it shows the applicability of the algorithm to closed loop identification in the sense that the identified model represent the dynamic behavior of the plant in closed loop which is important in identification for control.

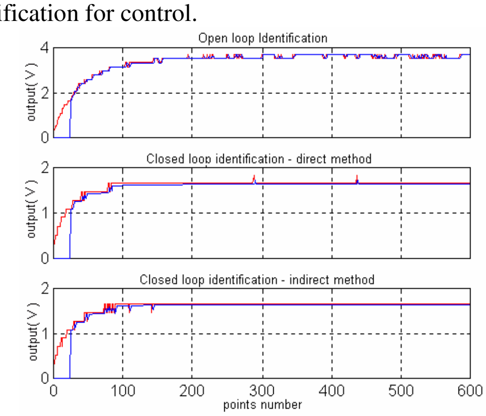

Figure 4. Output estimation to the three cases above mentioned with the real curve in red and the estimated curve in blue.

5. FINAL REMARKS

An adaptive instrumental variable algorithm has been developed for system identification in this paper. A choice of instruments was proposed and the QR factorization by Householder orthogonal transformation was implemented. The proof of consistency of the proposed Weighted Instrumental Variable (WIV) method has been established and simulation results show a robust characteristic as accurate unbiased parameter estimation, fast convergence speed and its efficiency in dealing with outliers contained in input and output observations. The algorithm was applied to open loop and closed loop (direct and indirect methods) identification to extend its applicability to control schemes where the controller design is done iteratively based on identified model, this is, it is also applied to identification for control.

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