# ROBUST STATE FEEDBACK CONTROL OF UNCERTAIN DISCRETE-TIME LINEAR SYSTEMS WITH ALMOST DEADBEAT RESPONSE

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***Abstract*— This paper addresses the problem of robust state feedback control of uncertain discrete-time systems in polytopic domains. The closed-loop poles are assigned into a circle with center at the origin of the complex plane and radius as small as possible, providing a transient response that tends to a deadbeat response. The proposed design is based on quadratic Lyapunov functions and is implemented through linear matrix inequalities. The resulting controller can stabilize and guarantee robust performance for this class of systems when conventional deadbeat control techniques fail, as shown through numerical examples.**

***Keywords* – Deadbeat control; Discrete-time linear systems; Linear matrix inequalities; Lyapunov functions; Polytopic uncertainties; Robust pole location; State feedback control.**

## I. INTRODUCTION

One important issue for discrete-time systems is the deadbeat control, that is, to drive any state to the zero in a finite number of steps through constant feedback control [1]. A classical approach to achieve deadbeat response for discrete-time linear systems is to assign all the poles of the closed-loop system at zero using conventional pole location techniques, as for instance the Ackermann's formula. In the case of multiinput systems, the state feedback gain is in general nonunique and some approaches give parameterized solutions for these gains [2], [3]. The problems of deadbeat regulation and tracking have been studied in several contexts, as in [4], where the deadbeat controller imposes prespecified bounds of overshoot and undershoot for the closed-loop system. In [5], necessary and sufficient conditions for deadbeat regulation and tracking in MIMO systems are given. Practical applications of deadbeat tracking schemes where the reference signal is a sinusoidal wave have been given for uninterruptible power supplies systems in [6], [7]. One common point among the prior mentioned studies and many others in the literature is that they do not take uncertainties in the system representation into account neither consider that the system can be affected by perturbations or can have unmodeled dynamics. Thus, a controller designed to provide a deadbeat response for the nominal system can lead to a poor closed-loop performance (even to unstable behaviors) when uncertainties are present.

The problem of synthesis of state feedback gains which provide robustness against perturbations for deadbeat regulators has been addressed in [8] by means of an unconstrained optimization problem. In [9] a design method for robust deadbeat controllers applied to systems with time-varying norm-bounded unstructured uncertainty is given in the frequency domain. A convex approach to determine a robust deadbeat controller that minimizes the norm of the closed-loop matrix to reduce the effect of unstructured uncertainties is presented in [10]. Although the above tests provide a certain degree of robustness for the system against perturbations and unstructured uncertainties, they do not deal with the presence of structured uncertainties in the system. This arises quite frequently in practice when the discrete-time linear system has parameters that are uncertain, which can be modeled through linear parameter-varying representations [11].

Lyapunov functions have been used to derive analysis and synthesis conditions for several classes of dynamic systems, including uncertain linear systems belonging to polytopic domains or that admit an affine representation (structured uncertainties). The use of a common Lyapunov matrix to assure the stability of the entire uncertain domain, called quadratic condition, has provided many important results in robust stability, control and filtering [12], [13], [14], [15]. This condition is appealing thanks to the low numerical complexity and because the tests derived are frequently expressed in terms of linear matrix inequalities (LMIs), providing to the problems polynomial time solutions (see [16], [17] for details on LMIs). In [18], it is shown that the feasibility of an LMI condition based on a quadratic Lyapunov function assures that all the eigenvalues of a precisely known matrix lie inside a circle on the complex plane with given center and radius. This condition has been recently extended for the case of uncertain systems in polytopic domains [19] where the problems of robust stability and control with pole location constraints have been studied through quadratic and parameter dependent Lyapunov functions. Notice that in the case of discrete-time uncertain systems, when the circle is centered at zero and the radius is as small as possible, the response of the system approaches to the deadbeat response.

The main objective of this paper is to present a design of a robust state feedback control that guarantees that all the poles of an uncertain discrete-time linear system lie inside a circle with center at zero and radius as small as possible, providing to the system an almost deadbeat response. The results here are derived for linear systems with polytopic uncertainties but can be directly applied to systems with an affine representation or for switched linear systems with arbitrary switching functions. The design is based on the quadratic Lyapunov function and expressed as an LMI test, easily implemented and solved through available softwares. Numerical examples show that the proposed condition can stabilize and guarantee robust performances for the class of systems under investigation in cases where conventional deadbeat techniques fail.

## II. PRELIMINARIES

Consider the system

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control vector and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ are the system matrices, supposed to be precisely known. Suppose that the state feedback control law

$$u(k) = Kx(x) \tag{2}$$

with $K \in \mathbb{R}^{m\times n}$, is applied to system (1). If the system is controllable, it is possible to determine $K$ such that the eigenvalues of the closed-loop system are equal to zero, that is,

$$\lambda_i(A + BK) = 0 \;\; , \quad i = 1, \ldots, n \tag{3}$$

thus providing a deadbeat response to the states system.

Consider now that the system is given by

$$x(k+1) = A(\alpha)x(k) + B(\alpha)u(k) \tag{4}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, and $A(\alpha) \in \mathbb{R}^{n\times n}$, $B(\alpha) \in \mathbb{R}^{n\times m}$ are not precisely known, but uncertain matrices, that belong to the polytopic domain

$$\mathcal{D} = \Big\{(A,B)(\alpha) \; : \; (A,B)(\alpha) = \sum_{j=1}^{N} \alpha_j (A,B)_j \; , \\ \sum_{j=1}^{N} \alpha_j = 1 \; , \; \alpha_j \geq 0, \quad j = 1, \ldots, N\Big\} \tag{5}$$

First of all, notice that it is very difficult (usually impossible) to determine a control gain $K$ such that $\lambda_i(A(\alpha) + B(\alpha)K) = 0$, $i = 1, \ldots, n$. The design of such gain for a particular pair $(A,B)(\alpha_n) \in \mathcal{D}$, where $\alpha_n$ can represent the nominal system, does not guarantee that all the pairs $(A,B)(\alpha) \in \mathcal{D}$ will have a deadbeat response. It is not uncommon that the uncertain system becomes unstable for a deadbeat gain designed for the nominal system.

It is worth to recall that the polytopic representation (5) describes the system subject to time-invariant or slow time-varying parameters [20], [21]. It can be also used to represent a system that originally is in the affine form, which is a very common representation of physical processes, and even can describe switched linear system when the subsystems are the vertices $(A,B)_j$ of the polytope $\mathcal{D}$ (see [22] for details).

The control problem stated here is: determine, if possible, a control gain $K$ for the state feedback law (2) such that the eigenvalues of the closed-loop system belong to a circle with center at zero and with radius as small as possible inside the unit circle, that is,

$$\lambda_i(A(\alpha) + B(\alpha)K) \in \mathcal{C}_r \;\; , \quad i = 1, \ldots, n \tag{6}$$

with $\mathcal{C}_r$ depicted in Figure 1.

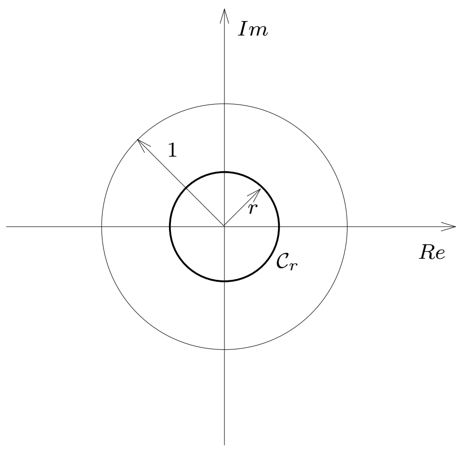


Fig. 1. Circle for pole location to provide an almost deadbeat response: center at zero and radius $r$ as small as possible, denoted by $\mathcal{C}_r$.

Conditions for pole location in different regions given in terms of LMIs can be found in the literature (see, for instance, [23], [18]). In the next section, a condition for pole location in the circle $\mathcal{C}_r$ will be given.

## III. DESIGN CONDITION

**Theorem 1** If there exists a symmetric positive definite matrix $W \in \mathbb{R}^{n\times n}$ and a matrix $Z \in \mathbb{R}^{m\times n}$ such that

$$\begin{bmatrix} rW & A_jW + B_jZ \\ WA_j' + Z'B_j' & rW \end{bmatrix} > 0 \; , \quad j = 1, \ldots, N \tag{7}$$

has a solution for a given $0 < r \leq 1$, then the state feedback control law (2) with gain

$$K = ZW^{-1} \tag{8}$$

stabilizes the system (4)-(5) and guarantees the pole location constraint (6).

**Proof** Multiplying (7) by $\alpha_j$ and applying the sum operator $\sum_{j=1}^{N}$ to the resulting expression, one has

$$\begin{bmatrix} rW & A(\alpha)W + B(\alpha)Z \\ WA(\alpha)' + Z'B(\alpha)' & rW \end{bmatrix} > 0 \quad (9)$$

Using the variables transformation $Z = KW$ from (8), it is possible to write

$$\begin{bmatrix} rW & (A(\alpha) + B(\alpha)K)W \\ W(A(\alpha) + B(\alpha)K)' & rW \end{bmatrix} > 0 \quad (10)$$

and applying Schur complement to (10), one can find

$$\frac{(A(\alpha) + B(\alpha)K)}{r} W \frac{(A(\alpha) + B(\alpha)K)'}{r} - W < 0 \quad (11)$$

The feasibility of (7) is a necessary and sufficient condition to guarantee the feasibility of (11), thanks to the convexity of the problem. Denoting the unit circle in Figure 1 as $\mathcal{C}_1$, the existence of a solution to (11) is sufficient to assure that all the eigenvalues $\lambda_i\Big(\frac{A(\alpha)+B(\alpha)K}{r}\Big) \in \mathcal{C}_1$ which is equivalent to say that $\lambda_i\Big(A(\alpha)+B(\alpha)K\Big) \in \mathcal{C}_r$, $i = 1, \dots, n$. □

Theorem 1 deserves some remarks:

R1) Condition (7) is a test with $N$ LMIs written at the vertices of polytope $\mathcal{D}$, whose feasibility allows to determine a robust control gain that solves problem (6). Notice that the problem data are the vertices $A_j$, $B_j$, $j = 1, \dots, N$, that describe the uncertain system model. The design parameter is the scalar $r$, chosen in the interval $0 < r \leq 1$ and the problem variables are the matrices $W$ and $Z$. These matrices can be automatically determined (in polynomial time) using standard LMI solvers, as the LMI Control Toolbox of Matlab [17].

R2) The existence of a fixed fixed gain $K$, from Theorem 1, guarantees that all the matrices $A(\alpha)+B(\alpha)K$ will have the eigenvalues inside the circle $\mathcal{C}_r$, which assures a robust performance for the closed-loop uncertain system, measured in terms of overshoot and settling times that can be viewed as a function of the radius $r$.

R3) As Theorem 1 relies on the quadratic condition, it can be directly applied to the class of the discrete-time switched linear systems, that change arbitrarily fast from one vertex $(A, B)_j$ to another vertex $(A, B)_k$, $j, k = 1, \dots, N$, $j \neq k$ following arbitrary switching rules. In this case, Theorem 1 assures that all the eigenvalues of each one ofthe subsystems $(A, B)_j$, $j = 1, \dots, N$ of the switched system lie inside the circle $\mathcal{C}_r$ and that the system is stable for any arbitrary switching rule, since the feasibility of the pole location condition (7) implies in the feasibility of the quadratic stability condition $(A(\alpha) + B(\alpha)K)W(A(\alpha) + B(\alpha)K)' - W < 0$, since, for $0 < r \leq 1$,

$$\Big(A(\alpha) + B(\alpha)K\Big)W\Big(A(\alpha) + B(\alpha)K\Big)' - W <$$
$$\Big(A(\alpha) + B(\alpha)K\Big)W\Big(A(\alpha) + B(\alpha)K\Big)' - r^2W < 0 \quad (12)$$

R4) Theorem 1 can be used to determine a stabilizing gain for uncertain systems in the form (4)-(5) that have arbitrarily time-varying parameters. In this case, the concept of eigenvalue is not applicable to the system anymore, and one must evaluate the feasibility of Theorem 1 for $r = 1$ to determine the stabilizing gain.

R5) As the condition (7) is expressed in terms of LMIs, it is easy to incorporate constraints in the matrices $W$ and $Z$ to deal with structural constrained control or static output feedback control (see [19]) for details.

R6) Recently, conditions of pole location in circular regions based on quadratic or on parameter dependent Lyapunov functions or switched Lyapunov functions have been given [19], [24], [25], [26]. It was shown that parameter dependent Lyapunov functions or switched Lyapunov functions can produce more stringent design specifications than quadratic Lyapunov functions at the price of using gain-scheduled or switching control techniques.

In next section, some numerical examples are presented to illustrate how the proposed condition can improve the performance of this class of dynamic systems.

## IV. NUMERICAL EXAMPLES

The first example shows how a state feedback deadbeat controller designed for a nominal system can lead to an unstable behavior in the presence of uncertainties. Consider the system

$$x(k + 1) = (A_0 + \theta A_\theta)x(k) + B_0u(k) \quad (13)$$

where the matrices of the nominal system (randomly generated) are

$$A_0 = \begin{bmatrix} 0.33 & 0.19 & 0.56 & 0.30 \\ 0.14 & 0.66 & 0.93 & 0.50 \\ 0.64 & 0.45 & 0.98 & 0.40 \\ 0.78 & 0.75 & 0.17 & 0.67 \end{bmatrix}; \; B_0 = \begin{bmatrix} 0.49 & 0.87 \\ 0.07 & 0.66 \\ 0.46 & 0.96 \\ 0.32 & 0.15 \end{bmatrix} \quad (14)$$

and the matrix related with the uncertain parameter $\theta$ is

$$A_\theta = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

For instance, using the function PLACE from the Control System Toolbox of Matlab, it is possible to determine a gain $K_{db}$ to place the poles of the nominal system at 0.001, 0.0001, 0.0002, 0.0003, that is, $\max \mid \lambda_i(A_0 + B_0K_{db}) \mid \leq 0.001$, $i = 1, \dots, 4$. Observe that this pole location assures a response which is very close to the ideal deadbeat response for the nominal system. The control gain is given by

$$K_{db} = \begin{bmatrix} 41.5958 & 87.7447 & 32.4128 & 66.9894 \\ -20.1237 & -41.8722 & -16.2024 & -32.0757 \end{bmatrix} \quad (16)$$

However, the system performance can be deteriorated if a small perturbation around the nominal operation point occurs. Consider that the uncertain parameter $\theta$ is such that

$$0 \leq \theta \leq 0.1 \quad (17)$$

The eigenvalues of the uncertain closed-loop system $x(k+1) = ((A_0 + \theta A_\theta) + B_0 K_{db})x(k)$ are shown in Figure 2. Observe that the uncertain closed-loop system is unstable,

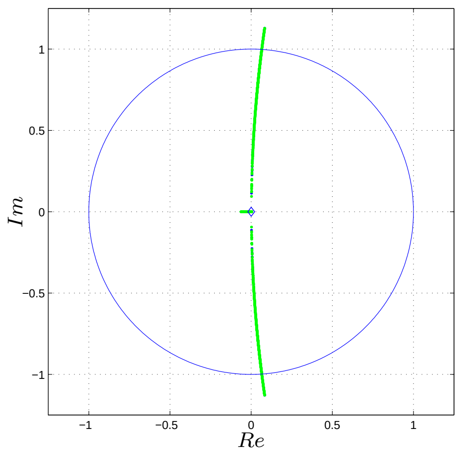


Fig. 2. Eigenvalues of the uncertain closed-loop system with the deadbeat controller designed for the nominal system, that is, $\lambda_i((A_0 + \theta A_\theta) + B_0 K_{db})$, $i = 1, \ldots, 4$, with (14)-(17).

since not all the eigenvalues lie inside the unit circle.

To illustrate the performance degradation of the closed-loop uncertain system, Figure 3 shows a time simulation of the closed-loop system subject to $\delta(k)$, an impulse input, described by

$$\begin{aligned} x(k+1) &= ((A_0 + \theta A_\theta) + B_0 K_{db})x(k) + B_\delta \delta(k) \\ y(k) &= Cx(k) \end{aligned} \quad (18)$$

with $B'_\delta = [0\ 0\ 0\ 1]$ and $C = [1\ 0\ 0\ 0]$.

Notice that the system presents a deadbeat response for the nominal condition, represented by the curve with $\theta = 0$ in Figure 3, as expected, but the performance is deteriorated as the uncertain parameter $\theta$ increases, leading to the instability for $\theta = 0.1$.

Theorem 1 can cope with the uncertainty, allowing to improve the performance of this system. Observe that this system with one uncertain parameter can be represented by an $N = 2$ vertices polytope: $A_1 = A_0$, $B_1 = B_0$ (vertex 1) and $A_2 = A_0 + 0.1A_\theta$, $B_2 = B_0$ (vertex 2). Choosing $r = 0.1$, Theorem 1 gives as solution the robust

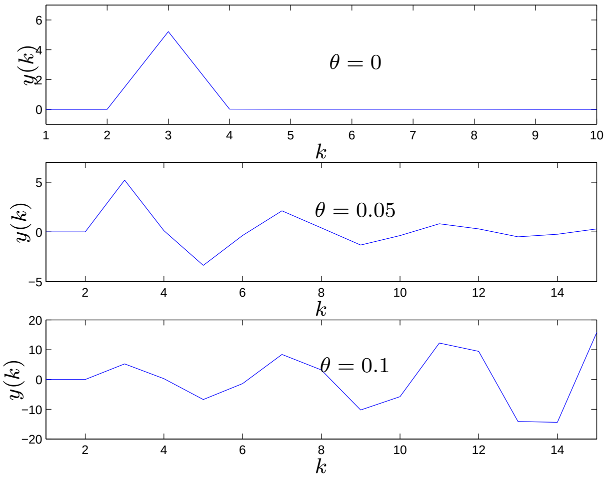


Fig. 3. Impulse response of the uncertain closed-loop system with the deadbeat controller designed for the nominal system, that is, $\lambda_i((A_0 + \theta A_\theta) + B_0 K_{db})$, $i = 1, \ldots, 4$, with (14)-(17).

state feedback gain

$$K_{L1} = \begin{bmatrix} -0.9563 & 1.2449 & 1.2380 & 0.4676 \\ -0.2777 & -1.5611 & -1.6628 & -1.0624 \end{bmatrix} \quad (19)$$

The uncertain closed-loop system $x(k+1) = ((A_0 + \theta A_\theta) + B_0 K_{L1})x(k)$ has the eigenvalues shown in Figure 4.

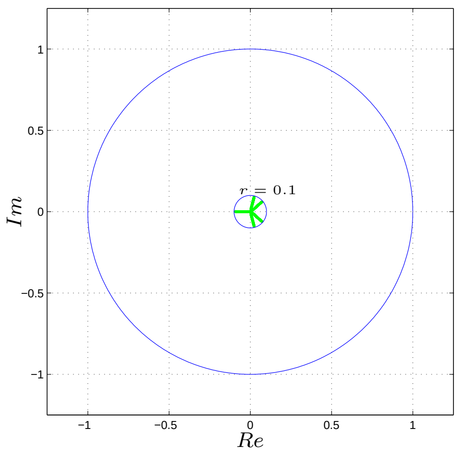


Fig. 4. Eigenvalues of the uncertain closed-loop system with the robust controller designed through Theorem 1, $\lambda_i((A_0 + \theta A_\theta) + B_0 K_{L1})$, $i = 1, \ldots, 4$, with (14), (15), (17), (19).

Observe that this system is stable. Moreover, all the eigenvalues are located inside the circle with center at zero and radius $r = 0.1$, guaranteed by the robust gain determined through Theorem 1. There is an apparent difference in the root locus of Figure 4 (robust almost deadbeat design) and the one of Figure 2 (conventional deadbeat design based on the nominal model). This difference can be viewed also in the time simulation of the system subject to the impulse input, as in (18), but with the robust state feedback gain (19), shown in Figure 5.

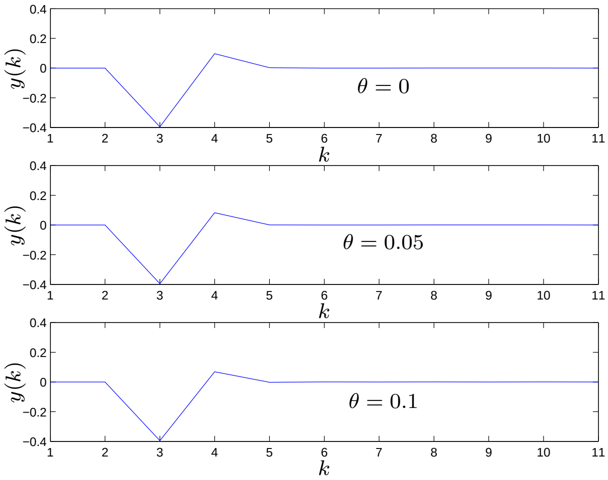


Fig. 5. Impulse response of the uncertain closed-loop system with the robust controller designed through Theorem 1, that is, $\lambda_i((A_0+\theta A_\theta)+B_0 K_{L1})$, $i = 1,\ldots,4$, with (14), (15), (17), (19).

This example shows another interesting feature of Theorem 1. Consider the uncertain parameter $0 \leq \theta \leq \bar{\theta}$. The relationship between the minimum radius $r_{min}$ for which Theorem 1 remains as a function of $\bar{\theta}$ is shown in Figure 6. Notice that as $\bar{\theta}$ increases, $r_{min}$ increases

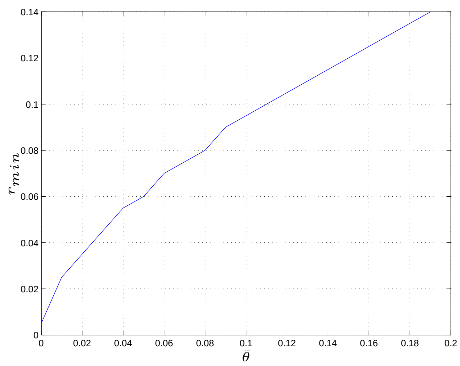


Fig. 6. Relationship between the minimum radius $r_{min}$ for which Theorem 1 is feasible and the upper bound on the uncertain parameter $\bar{\theta}$, for the system (13)-(15).

indicating that for this example, it is necessary to make the pole location constraints less stringent in Theorem 1 to cope with larger interval uncertainty.

The second example aims on the structural constrained control problem. Suppose the system (4)-(5) with two randomly generated vertices given by

$$A_1 = \begin{bmatrix} 0.0860 & 0.5029 & 0.3034 \\ 0.9012 & 0.7865 & 0.7636 \\ 0.8092 & 0.8762 & 0.7448 \end{bmatrix};\ B_1 = \begin{bmatrix} 0.4630 \\ 0.9490 \\ 0.8430 \end{bmatrix} \tag{20}$$

$$A_2 = \begin{bmatrix} 0.3596 & 0.9895 & 0.3580 \\ 0.0167 & 0.9317 & 0.1752 \\ 0.2674 & 0.3962 & 0.3943 \end{bmatrix};\ B_2 = \begin{bmatrix} 0.8190 \\ 0.7350 \\ 0.7050 \end{bmatrix} \tag{21}$$

The objective here is to determine the minimum radius for which Theorem 1 has a solution when only the states $x_2(k)$ and $x_3(k)$ are available for feedback, which leads to a structured constrained gain given by $K_C = [0\ k_2\ k_3]$. Applying constraints on the structure of the matrices $Z$ and $W$, one has that Theorem 1 is feasible for the minimum radius $r = 0.41$, with the solution

$$K_C = \begin{bmatrix} 0 & -1.1382 & -0.5127 \end{bmatrix} \tag{22}$$

$$Z_C = \begin{bmatrix} 0 & -10.7755 & -13.7783 \end{bmatrix}$$
$$W_C = \begin{bmatrix} 0.6199 & 0 & 0 \\ 0 & 6.3584 & 6.9004 \\ 0 & 6.9004 & 11.5536 \end{bmatrix} \tag{23}$$

If the full state information was available for the controller, the minimum radius could be reduced to $r = 0.29$, with the control gain given by

$$K = \begin{bmatrix} -0.3903 & -1.1291 & -0.5564 \end{bmatrix} \tag{24}$$

The third example deals with a more numerically complex design problem: the discrete-time state feedback control of an electrical circuit, the $RLC$ circuit shown in Figure 7.

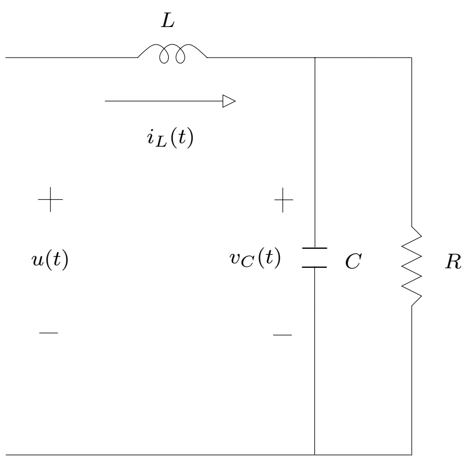


Fig. 7. Circuit with uncertain $R$, $L$ and $C$ parameters subject to a state feedback control law.

This circuit is largely used as the final stage of static power converters (DC-DC or DC-AC). It can be modeled by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{25}$$

with $x(t)' = [v_C(t)\ \ i_L(t)]$, where $v_C(t)$ is the capacitor voltage and $i_L(t)$ is the inductor current, and

$$A = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix} \ ;\ B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix} \tag{26}$$

Suppose that the parameters $R$, $L$ and $C$ are uncertain, lying in the intervals

$$\begin{array}{l} 0.1R_n \leq R \leq 1.9Rn \quad , \\ 0.9L_n \leq L \leq 1.1Ln \quad , \quad 0.9C_n \leq C \leq 1.1Cn \end{array} \tag{27}$$

with the nominal values $R_n = 12\Omega$, $L = 1.3mH$, $C_n = 25\mu F$ are borrowed from [27]. Observe that the uncertainty in $L$ and $C$ represents deviations of $10\%$ from their respective nominal values, while for $R$, the uncertainty represents $90\%$ of deviation.

A discrete-time model for this system can be obtained using the zero-order hold method, with a sampling period $T_s = 1/10800$, as in [27]. A grid in the space of the three uncertain parameters $R$, $L$ and $C$, leads to a $N = 2^6 = 64$ vertices polytope, since the system matrices in the discrete-time model

$$A_d = \begin{bmatrix} a_{d11} & a_{d12} \\ a_{d21} & a_{d22} \end{bmatrix} \quad , \quad B_d = \begin{bmatrix} b_{d11} \\ b_{d21} \end{bmatrix} \tag{28}$$

have six uncertain parameters.

This example is numerically more complex than the prior ones, but Theorem 1 can still determine a robust controller for the system, with a control gain

$$K = \begin{bmatrix} -0.2865 & -11.2201 \end{bmatrix} \tag{29}$$

that guarantees the pole location inside a circle with radius $r = 0.84$, as shows Figure 8.

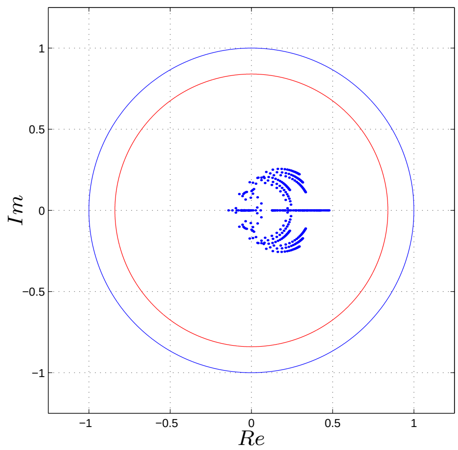


Fig. 8. Eigenvalues of the uncertain closed-loop $RLC$ circuit system with the robust controller designed through Theorem 1 (29). All the eigenvalues, marked by dots, lie inside the circle with $r = 0.84$.

It is worth to mention that for this application, the load parameter $R$ can be time-varying even exhibiting fast time variations, as in the case of switched loads. Recalling remark R4, Theorem 1 can be used to determine a stabilizing control for system (26) subject to any arbitrary time-variation of the uncertain parameters in (27). By choosing $r = 1$, one has that a stabilizing gain is given by

$$K = \begin{bmatrix} -0.5949 & -12.2544 \end{bmatrix} \tag{30}$$

## V. CONCLUSION

This paper presents a sufficient LMI condition to determine a robust state feedback controller with pole location constraints, suitable to design a controller that can provide to the closed-loop uncertain discrete-time system an almost deadbeat response. The approach can also be applied to determine stabilizing controllers in the case when the system parameters can vary arbitrarily. Numerical examples illustrate how the proposed condition can improve the results for the class of systems under investigation, providing robust performance when conventional deadbeat techniques can lead the system to instability. Also, structural constraints are shown to be easily incorporated to the given condition, that can be applied in the control of more complex systems indicating its potential as a design tool for control systems engineering with applicability on power electronics. As perspectives of future work, one can cite the use of the technique proposed in the paper to design control gains for practical applications.

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