COMPARISON BETWEEN PRONY'S METHOD AND FFT FOR ANALYZING POWER CONVERTERS SIGNALS

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Abstract **- This paper proposes a comparison between the Prony's method and the well-known Fast Fourier Transform (FFT), for analyzing signals of power converters. The two techniques have been applied to synthetic signals. The results show that Prony is a good alternative when one is dealing with short time windows of analyzed signals**

Keywords - Prony's method, FFT, spectral analysis

I. INTRODUCTION

In the last two decades, there has been a great development in power electronics. Costs are growing lower and the number of power electronics devices in industrial plants are increasing rapidly. It is well known that these devices distort the voltages and currents of utility's side. Standards like IEEE 519 [1] are getting more rigorous about the limits of voltage and current distortion. Thus harmonic and inter-harmonic estimation is an issue of increasing relevancy.

Traditionally, the Fast Fourier Transform has been applied to estimate the spectrum of currents and voltages of power systems [2]. Unfortunately, this mathematical tool has some limitations. There are basic premises in the application of DFT [3]. These premisses basically demands from the data system acquisition a great flexibility and mass memory. However, it is not an easy task to find out such acquisition systems.

Parametric modelling is an interesting alternative to achieve better spectral estimations. Higher resolution is one key of improvement area [4]. Prony's method is a parametric technique for fitting damped sinusoids to the data. It provides the amplitude, the initial phase, the damping factor and mainly the frequencies of these sinusoids. It is not strictly considered a spectral estimation technique, but has a close relationship to autoregressive (AR) spectral estimation. This paper propose a comparison between Prony's method and FFT to estimate the frequency content of responses of power converters.

II. FAST FOURIER TRANSFORM

The Discrete Fourier Transform (DFT) is a method to analyze signals in the frequency domain. It takes into account the samples $y[1], y[2], \cdots, y[N]$ of the data signal to generate N complex numbers $Y[k]$, through:

$$Y[k] = \sum_{i=1}^{N} y[i] e^{jk\frac{2\pi}{N}i} \qquad (1)$$

The Fast Fourier Transform is nothing more than an efficient way to compute the sum in (1). The direct computation of (1) leads to a number of operations proportional to N^2. The FFT requires a number proportional to $NlogN$ computations [5]. Although its high efficiency, misapplication of the FFT algorithm leads to incorrect results. The basic premisses to the correct application of FFT are:

1. The signal is stationary.
2. The sampling frequency is equal to the number of samples multiplied by the fundamental frequency assumed by the algorithm.
3. The sampling frequency is greater than twice the highest frequency in the signal to be analyzed.
4. Each frequency in the signal is an integer multiple of the fundamental frequency.

There are three major pitfalls as result of not following the premisses above [6]; namely aliasing, leakage and picket-fence effect. Aliasing can be mitigated by increasing the sampling frequency f_s. However, pseudoaliasing may still occur even if the highest frequency component is not higher than $f_s/2$. This may be caused by the presence of a fraction of a cycle of data or by white noise. The picket-fence is caused by the presence of a frequency that is not an multiple of $\frac{2\pi}{N\Delta T}$. The leakage is just a spreading of energy from one frequency into adjacent ones.

The problems mentioned above can be eliminated if the signal is truncated so that one considers an integer number of its cycles. Besides that, the Nyquist theorem must be obeyed. On the other hand the correct truncation of the signal demands a previous knowledge of the fundamental period of the signal. This means that the correct application of FFT can only be achieved if one knows the frequencies contained in the analyzed signal. In the next section we present the Prony's method. It can provide the frequencies in data signal.

III. PRONY'S METHOD

Prony's method was developed by the French mathematician Baron Gaspard de Prony in 1795. The objective was to fit exactly p exponential curves to a data set of $2p$ points. The modern version was presented by Hildebrand in 1956 [7]. Let $y[1], y[2], \cdots, y[N]$ be samples of the signal measured at equally-spaced time instants $\Delta T, 2\Delta T, \cdots, N\Delta T$. It is required to fit a model of the form:

$$\begin{aligned}\widehat{y}[k] &= \sum_{i=1}^{p}\left[A_i\cos(k\omega_i\Delta t)+B_i\sin(k\omega_i\Delta T)\right]\\ &= \sum_{i=1}^{p}\left(c_i e^{j\omega_i\Delta Tk}+c_i^{*}e^{-j\omega_i\Delta Tk}\right),\\ &\qquad\qquad k=1,2,\cdots,N,\end{aligned} \tag{2}$$

where the coefficients c_i and c_i^* are complex conjugates. Putting $z_i = e^{j\omega_i\Delta t}$, equation (2) may be written as:

$$\widehat{y}[k] = \sum_{i=1}^{p}\left(c_i z_i^k + c_i^* z_i^{-k}\right), \qquad k=1,2,\cdots,N. \tag{3}$$

The z_i are the nonlinear parameters of the model and c_i are the linear parameters. The objective is to find c_i and z_i that give $y[k]=\widehat{y}[k]$ for all k. Substituting the samples of the signal in equation (3) and putting it in matrix form, we obtain:

$$\begin{bmatrix} z_1 & z_2 & \cdots & z_p & z_1^{-1} & z_2^{-1} & \cdots & z_p^{-1}\\ z_1^2 & z_2^2 & \cdots & z_p^2 & z_1^{-2} & z_2^{-2} & \cdots & z_p^{-2}\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\ z_1^N & z_2^N & \cdots & z_p^N & z_1^{-N} & z_2^{-N} & \cdots & z_p^{-N}\end{bmatrix}\cdot\begin{bmatrix} c_1\\ c_2\\ \vdots\\ c_p\\ c_1^*\\ c_2^*\\ \vdots\\ c_p^*\end{bmatrix} = \begin{bmatrix} y[k]\\ y[2]\\ \vdots\\ \vdots\\ y[N]\end{bmatrix}, \tag{4}$$

where $N \geq 2p$. Equation (4) can be compactly written as $\mathbf{Z\,C} = \mathbf{Y}$. Note that each z_i is complex and of unit magnitude. They are the roots of a polynomial of degree $2p$ and therefore, satisfy:

$$\begin{aligned}z^{2p} + a_1 z^{2p-1} + \cdots + a_{p-1}z^{p+1} + a_p z^p +\\ +a_{p-1}z^{p-1} + \cdots + a_1 z + 1 = 0.\end{aligned} \tag{5}$$

The roots of the polynomial occur in complex conjugate pairs. As a consequence, the coefficients of the polynomial in equation (5) are symmetric.

Now, let us construct the $(1 \times N)$ row vector:

$$\mathbf{A} = [1\ a_1\ a_2\ \cdots\ a_{p-1}\ a_p\ a_{p-1}\ \cdots\ a_1\ 1\ 0\ 0\ \cdots\ 0]. \tag{6}$$

Clearly $\mathbf{AZC} = \mathbf{AY} = \mathbf{0}$. Successive row vectors may be constructed by right-shifting the elements of $\mathbf{A}$ and moving its last element to the first position. Pre-multiplying $\mathbf{Y}$ by each of these row vectors, we obtain:

$$\begin{aligned}&y[k] + a_1 y[k+1] + a_2 y[k+2] + \cdots + a_p y[k+p]\\ &+a_{p-1}y[k+p+1] + \cdots + a_2 y[k+2p-2]\\ &+a_1 y[k+2p-1] + y[k+2p] = 0\\ &k = 1,2,\cdots,N-2p,\end{aligned} \tag{7}$$

Solution of these equations provides the coefficients of the $(2p)^{th}$ degree polynomial and the roots of this polynomial are the z_i. The corresponding frequencies are calculated from $z_i = e^{j\omega_i\Delta T}$. Finally, the solution of equations (4) in the least-square sense yields the linear parameters c_i.

In summary, Prony's method for finding the frequency spectrum of the signal is:

(i) Assume a suitable value for p and compute the coefficients a_i of a polynomial of degree $2p$. This step involves the least-square solution of the system of equations in (7).

(ii) Compute the roots z_i of the polynomial and extract the frequencies from the roots.

(iii) The linear parameters c_i are obtained by solving equations (4) in the least-square sense.

IV. SIMULATIONS AND RESULTS

The simulations in this section have been accomplished in MATLAB. The first case presents a simulation of a voltage source converter (VSC) depicted in Fig. 1. The windowed current signal of phase a and its FFT are shown in Fig. 2(a) and Fig. 2(b) respectively. It can be noted that a little bit more than three cycles of the signal were windowed. It also can be noted that the FFT in Fig. 2(b) presents a great number of spectral rays up to 300 Hz (fifth harmonic) and the biggest one does not happen at the expected 60 Hz fundamental. The proposed Prony technique was applied to analyze the same windowed signal. Its first five estimated frequencies along with their respective amplitudes and initial phases are shown in table 1. From table 1, one can see that there is only two significative amplitude values related to 60 and 301 Hz, in other words, the fundamental and fifth harmonics. This result fits well with the theoretical spectrum of the data signal, shown in Fig. 3 up to 300 Hz. One should observe that the amplitude shown in Fig. 3 are half of the amplitudes related to the sinusoids that constitute the signal. This is due to symmetric property of Fourier transform of real signals.

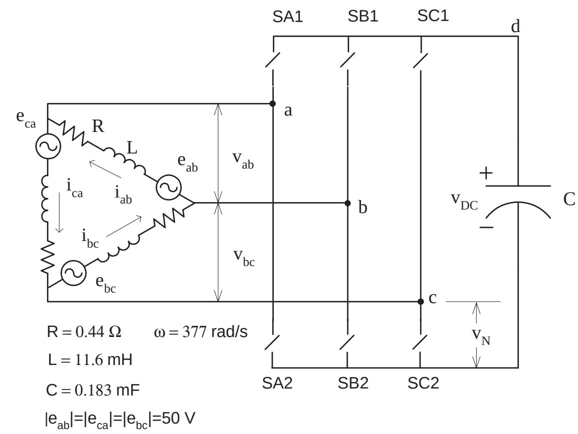


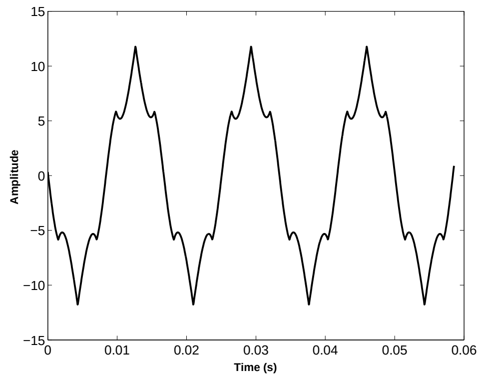
Figure 1 Voltage Source Converter

This simple example shows the eficiency of Prony's method and that FFT must be used very carefully.

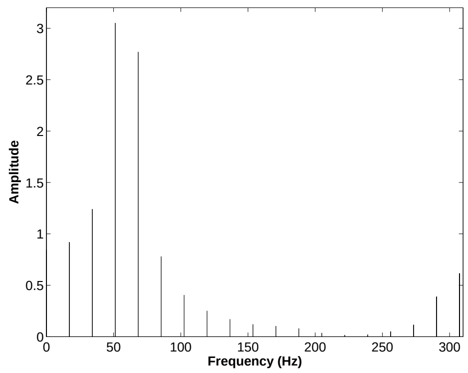
For the second simulation, consider the synthetic waveform generated by:

$$\begin{aligned}y_u(t) &= 200\cos(\omega_1 t) + 50\cos(5\omega_1 t) + 70\cos(7\omega_1 t)\\ &+50\cos(19\omega_1 t) + 30\cos(25\omega_1 t) + 30\cos(45\omega_1 t),\end{aligned} \tag{8}$$

where $\omega_1 = 2\pi 40$ rads/sec. The Prony method was ap-



(a) Windowed current signal



(b) FFT of the windowed signal

Figure 2 Time domain and Fourier representation of phase a current.

Table 1 Prony parameters

Frequency (Hz)	Amplitude	Initial phase (degree)
60	9.0999	87.3
301	1.5859	64.0
487	0.0212	85.2
697	0.0042	188.7
1010	0.0494	143.7

plied to 1000 samples of the signal with $\Delta T = 0.1ms$. The window of the signal is shown in Figure 4. Table 2 shows the frequency estimation along with their associated magnitudes. We see that the results were quite good. The FFT of this signal is shown in Figure 5. The result is also good. But if the frequency of the second component was 216 Hz instead 200 Hz, the FFT would result in an spread spectral around the true frequency. This signal is shown in Figure 6 The FFT is shown in Figure 7. On the other hand, the Prony estimation keeps a good estimation as shown in Table 3. The estimation can be even better if the order of the model is augmented to 10. The result is shown in 4.

The result shown on Table 4 shows that augmenting the

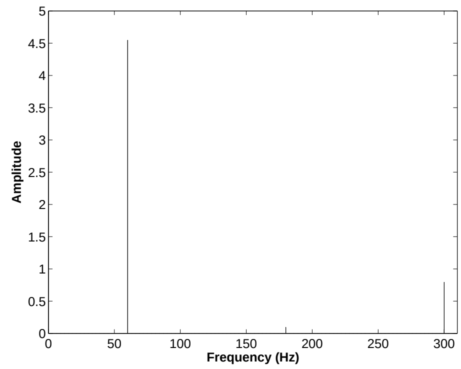


Figure 3 Theoretical spectrum of phase a current.

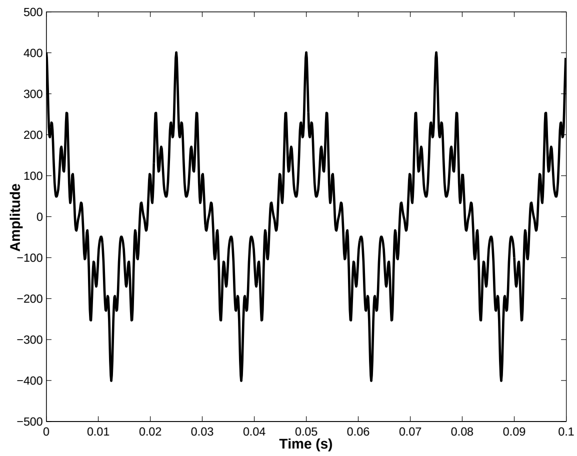


Figure 4 Signal with six components.

Table 2 Prony estimation for frequencies of the signal with six components-without noise.

Frequency (Hz)	38.7	200.6	279.8	760	1000	1800
Amplitude	196.83	49.51	69.97	49.96	29,98	0,994

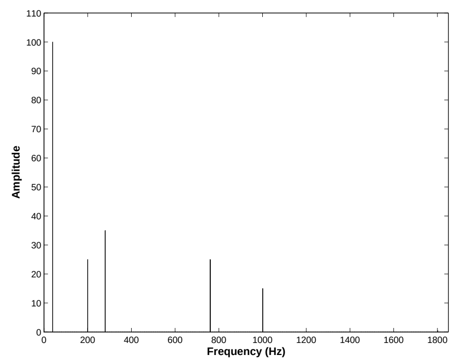


Figure 5 FFT of the signal with six components.

order of the model, we obtain better results. Augmenting the order of the model beyond the number of components that really exists in the analyzed signal, means that Prony's method will estimate spurious frequencies that in

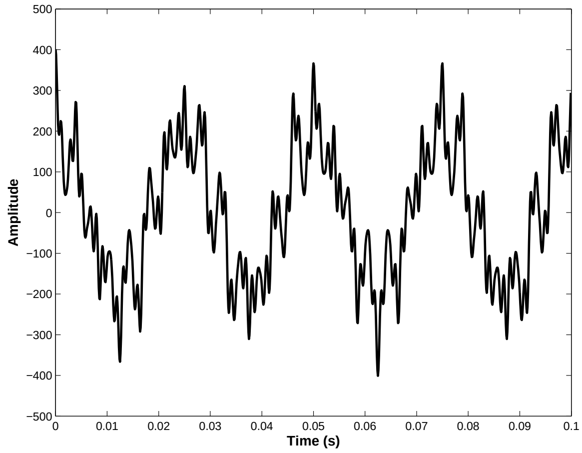


Figure 6 Signal with six components-one interharmonic.

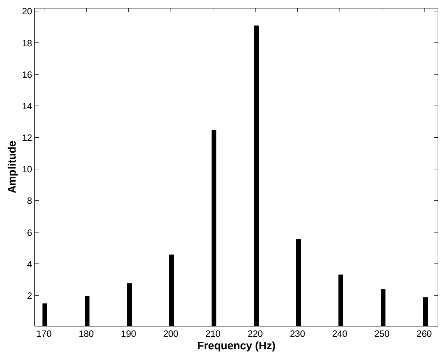


Figure 7 FFT of the signal with six components - one interharmonic.

Table 3 Prony estimation for frequencies of the signal with six components-without noise and with interharmonic.

Frequency (Hz)	40.80	215.60	280.20	760	1000	1800
Amplitude	196.04	49.83	70.11	50.03	30.01	1.00

Table 4 Prony estimation for frequencies of the signal with six components-without noise and with interharmonic (order 10).

Frequency (Hz)	40	216	280	760	1000	1800
Amplitude	200	50	70	50	30	1

general are associated with very small amplitudes. These spurious components can be eliminated from the estimation by a thresholding proceeding.

Prony´s method is known to be very sensitivity to noise. The next simulation intended to show the effectiveness of augmenting the order of the model if there is noise in the signal. Consider the signal described in (8). If we added to it a white noise (zero mean, unit variance), the Prony estimation would degenerate. The Prony estimation for the six more dominant components is shown in Table 5. They are all wrong. When the order of the model is augmented to 30, the results are exact.

Table 5 Prony estimation for frequencies of the signal with six components-with noise.

Frequency (Hz)	89.2	719.6	1005.9	2426.4	3560.5	4524.6
Amplitude	4.22	0.09	18.06	1.04	0.098	0.87

V. THEORETICAL CONSIDERATIONS ABOUT PRONY'S METHOD AND FFT

It is worth emphasizing that the Fast Fourier Transform and the Prony's method have two distinct goals. The FFT seeks amplitudes and phases associated to a set of discrete frequencies previously known. The result shown by FFT is a graphic and to known the dominant frequencies in the signal, we are supposed to look for the peaks of this graphic. Otherwise, the Prony´s method determines directly the dominant frequencies in the signal.

The two methods are computationally efficient, but the FFT is faster than Prony's method because the latter relies on the solution of a polynomial equation. On the other hand, FFT provides unprecise results if there is no previous knowledge of the fundamental period of the analyzed signal.

The results in the previous section allow us to make an analogy between the FFT and the Prony's method. The former one needs a previous knowledged of the frequencies in the signal, while the latter needs the order. Besides, we can say that the bigger the size of the window, the better is the result of FFT. While the bigger is the order of model, the better is the estimation of Prony.

The problem of the determination of the order of model is a difficult task. The most usual approach is to use the the Akaike's criteria [8]:

$$AIC[p] = Nln(\sigma^2) + 2p, \qquad (9)$$

where N is the number of samplings and σ^2 is the variance of model error when the order is set in p. As p increase, the fitness is better, so the variance is reduced. But, at a certain point, the increase of order does not provoke any variation in σ^2 and the improper increment is penalized by the second parcel. We must choose p so the criteria $AIC[p]$ is minimum.

VI. CONCLUSIONS

A comparative study between FFT and Prony's method was presented in this paper. The simulations shows that the Prony method is an efficient tool to estimate the frequencies in the signal. They also shows that FFT must be used very carefully. Prony's method is capable to deal with difficult spectrums with the presence of interharmonics. Nevertheless, the order of the polynomial must be known a priori.

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