EFFECTS OF FACTS CONTROLLERS ON SMALL SIGNAL POWER SYSTEMS VOLTAGE STABILITY

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Abstract— **The purpose of this paper is to study the effects of three Flexible AC Transmission Systems (FACTS) controllers: Static Synchronous Compensator (STATCOM), Static Synchronous Series Compensator (SSSC) and Unified Power Flow Controller (UPFC) on small signal voltage stability. A system composed by a load fed by a generator via a loss-less transmission line is examined. The inclusion of these three FACTS controllers shows the improvement on voltage stability margin. This study is based on investigation of the eigenvalues of the linearized power system model in the framework of dynamic bifurcation theory. The results presented show that using this model in conjunction with the P-V curve, all the basic aspects of the small signal voltage stability can be easily addressed.**

***Keywords*—FACTS, Saddle-node bifurcation, SSSC, STATCOM, UPFC, Voltage stability**.

I. INTRODUCTION

The voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating conditions [1]. Since the rapid development of power electronics has made it possible to design power electronic equipment of high rating for high voltage systems, the problem resulting from transmission system may be, at least partly, improved by use of the equipment namely as Flexible AC Transmission Systems (FACTS) controllers.

Many analysis methodologies have been proposed to solve the problem of voltage collapse phenomena and are currently employed for such issue [2]. Most of these techniques are based on the identification of system equilibrium points that are typically referred to as points of voltage collapse. These points can be mathematically associated to saddle-node bifurcations points.

Local instability of an operation point is characterized by three kinds of local stability in algebraic-differential model of power system: Singularity induced bifurcation (characterized by infinite eigenvalues crossing the imaginary axis), saddle-node (characterized by a pure real eigenvalue crossing the imaginary axis) and Hopf bifurcation (characterized by a par of pure imaginary eigenvalues crossing the imaginary axis).

The bifurcation theory is used here to analyze the eigenvalues of system on the collapse (or bifurcation) point, characterizing the instability of the system [3], [4].

This study investigates the effects of three FACTS controllers, Static Synchronous Compensator (STATCOM), Static Synchronous Series Compensator (SSSC) and Unified Power Flow Controller (UPFC), on small signal power systems voltage stability of a system composed by a load fed by a generator via a loss-less transmission line. The system is modelled using the Power System Model (PSM) [5], [6].

II. FACTS CONTROLLERS

The FACTS is a concept based on power-electronic controllers, which enhance the value of transmission networks by increasing the use of their capacity. The potential benefits of FACTS equipment are now widely recognized by the power systems engineering and T&D communities.

The first group of FACTS controllers, such as Static Var Compensator (SVC) and Thyristor Controlled Series Capacitor (TCSC), used thyristor to control the reactor and capacitor banks, respectively. The second group of FACTS controllers, such as STATCOM, SSSC and UPFC are based on Voltage Source Converter (VSC) that employed GTOs to produce solid- state synchronous voltage sources at fundamental frequency [7].

III. POWER SYSTEM MODEL

The analysis of the STATCOM, SSSC and UPFC influence on the power system voltage stability can be accomplished for a single generator connected to a dynamic load where the FACTS controller is installed, as shown in Fig. 1. The study of impact of these FACTS is made separately.

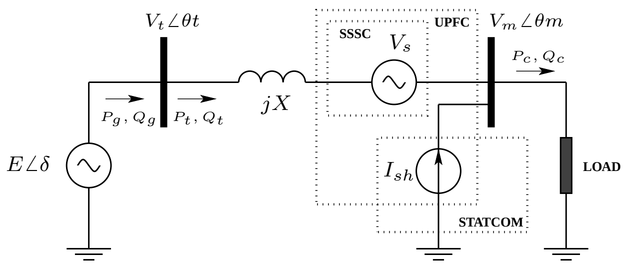

Fig. 1. Generator system - Transmission line - Dynamic Load including FACTS controllers.

The dynamic load model proposed in [8], use a set of equations to represent the load in Fig. 1. This model consists of algebraic and differential equations, which can be expressed as follows:

$$\dot{x}_p = \frac{1}{T_p}[P_s(V) - x_p P_t(V)] \tag{1}$$

$$\dot{x}_q = \frac{1}{T_q}[Q_s(V) - x_q Q_t(V)] \tag{2}$$

where x_p and x_q are state variables, $\{P_s(V), Q_s(V)\}$ and $\{P_t(V), Q_t(V)\}$ are the the transient and steady-state load characteristics, respectively. T_p and T_q are the time constants that describe the load response.

A. STATCOM

The STATCOM is a shunt connected FACTS controller which integrates the SVC technique and the voltage source conversion. It resembles an ideal rotating synchronous condenser operating under no load conditions, generating a balanced three-phase voltage, with controlled amplitude and angle. This machine does not have inertia, and its response velocity is almost instantaneous, and does not affect the system impedance. Therefore, it can generate and absorb reactive power. Besides, it can exchange active power with the system if coupled to an appropriate energy source. [8], [9]. The most important function of the STATCOM is to control the bus voltage where it is connected. Its functional model is shown in Fig. 2. We will consider that the STATCOM will only exchange reactive power with the system, so the supply energy source can be eliminated [9].

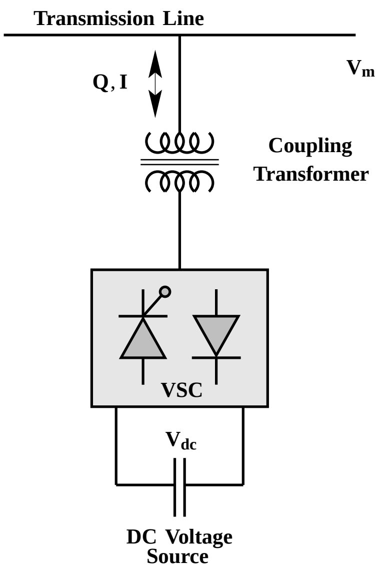

Fig. 2. Circuit for a Static Synchronous Compensator (STATCOM).

The STATCOM may be represented by the following set of equations [9]:

$$Q_{sh} = -I_s V_m \tag{3}$$

$$I_{sh} = \frac{K_u}{1 + sT_{sh}}(V_{m_{ref}} - V_m) \tag{4}$$

The power system can be represented by a set of differential and algebraic equations, which can be expressed as:

$$\dot{x} = f(x, y, \mu) \tag{5}$$

$$0 = f(x, y, \mu) \tag{6}$$

where x is a vector of dynamic state variables, y is a vector of algebraic variables and μ is a parameter, which can be varied slowly, such as nodal powers . For small signal stability analysis, we assume the system parameter variation is slow enough so that the model can be linearized around some equilibrium point as:

$$\Delta\dot{x} = J_1 \Delta x + J_2 \Delta y + \mathcal{B}\Delta u \tag{7}$$

$$0 = J_3 \Delta x + J_4 \Delta y \tag{8}$$

where J_1, J_2, J_3 and J_4 are Jacobian matrices of f and g related to dynamic state and algebraic variables, respectively, and $\mathcal{B}$ is the perturbation matrix. For the system shown in Fig. 1, considering only the presence of the STATCOM, the following state equations can be formulated according to nodal power balance methodology [5]:

$$\begin{bmatrix} \Delta\dot{E}'_q \\ \Delta\dot{E}'_d \\ \Delta\dot{E}_{FD} \\ \Delta\dot{x}_p \\ \Delta\dot{x}_q \\ \Delta\dot{I}_{sh} \end{bmatrix} = \begin{bmatrix} -\frac{x_d}{x'_d T'_{d0}} & 0 & \frac{1}{T'_{d0}} & 0 & 0 & 0 \\ 0 & -\frac{x_q}{x'_q T'_{q0}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_e} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{P_t}{T_p} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{Q_t}{T_q} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{sh}} \end{bmatrix} \cdot$$

$$\begin{bmatrix} \Delta E'_q \\ \Delta E'_d \\ \Delta E_{FD} \\ \Delta x_p \\ \Delta x_q \\ \Delta I_{sh} \end{bmatrix} + \begin{bmatrix} \frac{K_a}{T'_{d0}} & \frac{K_v}{x'_d T'_{d0}} & 0 & 0 \\ -\frac{K'_a}{T'_{q0}} & \frac{K'_v}{T'_{q0}} & 0 & 0 \\ 0 & -\frac{K_e}{T_e} & 0 & 0 \\ 0 & 0 & 0 & K_{pd} \\ 0 & 0 & 0 & K_{qd} \\ 0 & 0 & 0 & -\frac{K_u}{T_{sh}} \end{bmatrix} \cdot \begin{bmatrix} \Delta\Theta_t \\ \Delta V_t \\ \Delta\Theta_m \\ \Delta V_m \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{2_g} & A_{4_g} & 0 & 0 & 0 & 0 \\ R_{2_g} & R_{4_g} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -A_{cd} & 0 & 0 \\ 0 & 0 & 0 & 0 & -R_{cd} & V_m \end{bmatrix} \cdot$$

$$\begin{bmatrix} \Delta E'_q \\ \Delta E'_d \\ \Delta E_{FD} \\ \Delta x_p \\ \Delta x_q \\ \Delta I_{sh} \end{bmatrix} + \begin{bmatrix} -A_{1_g} - A_{1_t} & A_{3_g} - A_{2_t} & \cdots \\ -R_{1_g} - R_{1_t} & R_{3_g} - R_{2_t} & \cdots \\ A_{1_m} & -A_{3_m} & \cdots \\ R_{1_m} & -R_{3_m} & \cdots \end{bmatrix}$$

$$\begin{matrix} A_{1_t} & -A_{3_t} \\ R_{1_t} & -R_{3_t} \\ -A_{1_m} & -A_{2_m} - A_{1_c} \\ -R_{1_m} & -R_{2_m} - R_{1_c} + I_{sh} \end{matrix} \Bigg] \cdot \begin{bmatrix} \Delta\Theta_t \\ \Delta V_t \\ \Delta\Theta_m \\ \Delta V_m \end{bmatrix} \tag{10}$$

The coefficients A and R represent local sensitivity functions of active and reactive powers, respectively. They are related to the state and algebraic variables. As can be seen in (9) and (10), the rotor angle deviation and the rotor speed deviation are not include in the modelling as state variables since the system studied represent a problem strictly of voltage stability.

An important characteristic of this model used is the maintenance of the structure of the system. The power flow Jacobian of the system can be derivative of J_4, as follow below:

$$J_{FC} = \begin{bmatrix} -A_{1_m} & -A_{2_m} - A_{1_c} \\ -R_{1_m} & -R_{2_m} - R_{1_c} + I_{sh} \end{bmatrix} \tag{11}$$

If the Jacobian J_4 is not singular, so the vector of algebraic variables can be eliminated. Then the representation of state-space system can be obtained as:

$$\Delta\dot{x} = \mathcal{A}\Delta x + \mathcal{B}\Delta u \tag{12}$$

where:

$$\mathcal{A} = J_1 - J_2 J_4^{-1} J_3 \tag{13}$$

is the system state matrix.

B. SSSC

The SSSC is a series FACTS controller that is based on voltage source conversion. It consists of a VSC that converts a dc voltage into a three-phase ac voltage at fundamental frequency [7]. The SSSC has the capability to improve the stability margin of the system making the series line compensation where it is installed. The output ac voltage of SSSC can be kept in quadrature with the line current, making the SSSC exchange only reactive power with the power system. The behavior of SSSC can be similar to a controllable series capacitor and a controllable series reactor where the degree of compensation can be constant and its output voltage magnitude will depend only of the loading factor that the SSSC will be submitted [10]. Fig. 3 depicts the basic circuit of a series connected SSSC with the transmission line.

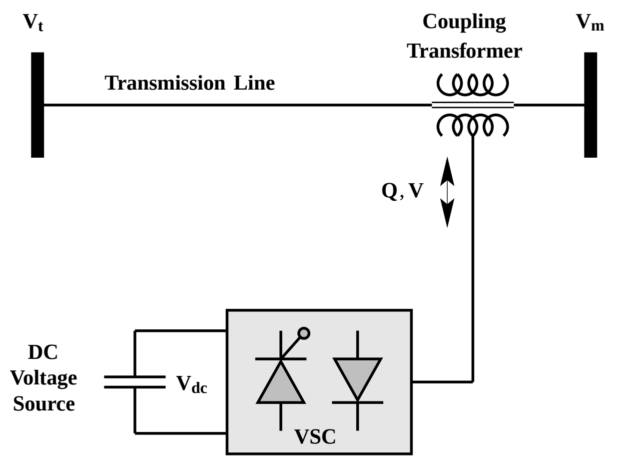

Fig. 3. Circuit for a Static Synchronous Series Compensator (SSSC).

The SSSC can be described by the following dynamic equation:

$$V_s = \frac{1}{1 + sT_s} V_0 \tag{14}$$

where V_0 is the injected series voltage of SSSC in steady-state. The active and reactive power flow equations after the inclusion of the SSSC, can be written as [10]:

$$P_{tm} = \left[1 + \frac{1}{\sqrt{u}}\right] \frac{V_t V_m}{X} sin(\theta t - \theta m) \tag{15}$$

$$Q_{tm} = \left[1 + \frac{1}{\sqrt{u}}\right] \left[\frac{V_t^2}{X} - \frac{V_t V_m}{X} cos(\theta t - \theta m)\right] \tag{16}$$

where u is described below:

$$u = V_t^2 + V_m^2 - 2 V_t V_m cos(\theta t - \theta m) \tag{17}$$

The active and reactive power power flow equations in the opposite sense, P_{mt} and Q_{mt}, are written analogously as the equations above.

For the SSSC case, the state equations can be obtained as:

$$\begin{bmatrix} \Delta\dot{E}'_q \\ \Delta\dot{E}'_d \\ \Delta\dot{E}_{FD} \\ \Delta\dot{x}_p \\ \Delta\dot{x}_q \\ \Delta\dot{V}_s \end{bmatrix} = \begin{bmatrix} -\frac{x_d}{x'_d T'_{d0}} & 0 & \frac{1}{T'_{d0}} & 0 & 0 & 0 \\ 0 & -\frac{x_q}{x'_q T'_{q0}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_e} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{P_t}{T_p} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{Q_t}{T_q} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_s} \end{bmatrix} \cdot$$

$$\begin{bmatrix} \Delta E'_q \\ \Delta E'_d \\ \Delta E_{FD} \\ \Delta x_p \\ \Delta x_q \\ \Delta V_s \end{bmatrix} + \begin{bmatrix} \frac{K_a}{T'_{d0}} & \frac{K_v}{x'_d T'_{d0}} & 0 & 0 \\ -\frac{K'_a}{T'_{q0}} & \frac{K'_v}{T'_{q0}} & 0 & 0 \\ 0 & -\frac{K_e}{T_e} & 0 & 0 \\ 0 & 0 & 0 & K_{pd} \\ 0 & 0 & 0 & K_{qd} \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta\Theta_t \\ \Delta V_t \\ \Delta\Theta_m \\ \Delta V_m \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{2_g} & A_{4_g} & 0 & 0 & 0 & -A_{4_t} \\ R_{2_g} & R_{4_g} & 0 & 0 & 0 & -R_{4_t} \\ 0 & 0 & 0 & -A_{cd} & 0 & -A_{4_m} \\ 0 & 0 & 0 & 0 & -R_{cd} & -R_{4_m} \end{bmatrix} \cdot$$

$$\begin{bmatrix} \Delta E'_q \\ \Delta E'_d \\ \Delta E_{FD} \\ \Delta x_p \\ \Delta x_q \\ \Delta V_s \end{bmatrix} + \begin{bmatrix} -A_{1_g} - A_{1_t} & A_{3_g} - A_{2_t} & \cdots \\ -R_{1_g} - R_{1_t} & R_{3_g} - R_{2_t} & \cdots \\ A_{1_m} & -A_{3_m} & \cdots \\ R_{1_m} & -R_{3_m} & \cdots \end{bmatrix}$$

$$\begin{matrix} A_{1_t} & -A_{3_t} \\ R_{1_t} & -R_{3_t} \\ -A_{1_m} & -A_{2_m} - A_{1_c} \\ -R_{1_m} & -R_{2_m} - R_{1_c} \end{matrix} \Bigg] \cdot \begin{bmatrix} \Delta\Theta_t \\ \Delta V_t \\ \Delta\Theta_m \\ \Delta V_m \end{bmatrix} \tag{19}$$

Like the STATCOM case, the matrix J_{FC} can be derivative of the Jacobian matrix J_4 and can be expressed as:

$$J_{FC} = \begin{bmatrix} -A_{1_m} & -A_{2_m} - A_{1_c} \\ -R_{1_m} & -R_{2_m} - R_{1_c} \end{bmatrix} \tag{20}$$

C. UPFC

The UPFC is a power electronic controller which can be used to control active and reactive power flows in a transmission line by injection of variable series voltage and reactive shunt current. Like the others FACTS presented, it is based on VSC and it was proposed by Gyugyi in 1991 [11]. The UPFC consists of a SSSC and a STATCOM, connected in such a way that they share a common DC capacitor. The DC link provides a path to exchange active power between the converters. The UPFC makes the voltage regulator function through the shunt converter (STATCOM) and makes the line series compensation, injecting a series voltage in quadrature with the line current, where the series converter (SSSC) is connected. [12]. The schematic of the UPFC is shown in Fig. 4.

The UPFC can be represented adding the characteristics of the STATCOM and the SSSC. The dynamic equations of the UPFC may be described as follows [12]:

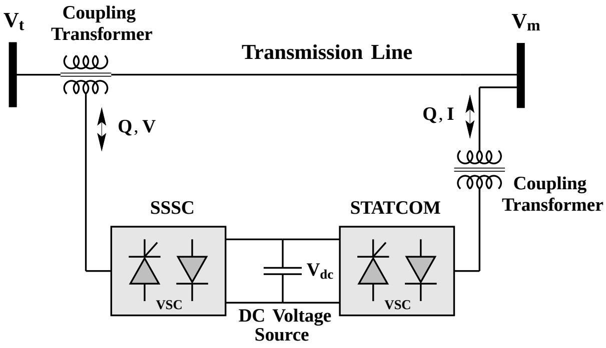

Fig. 4. Circuit for a Unified Power Flow Controller (UPFC).

$$V_s = \frac{1}{1+sT_{sh}} V_0 \tag{21}$$

$$I_{sh} = \frac{K_u}{1+sT_{sh}} (V_{m_{ref}} - V_m) \tag{22}$$

where V_0 is the injected series voltage of UPFC in steady-state. The shunt current I_{sh} has the role of maintenance of the bus voltage at the level specified, through of injection of reactive power. The series voltage V_s emulates a reactive series compensation.

For the UPFC case, the following state equations can be obtained:

$$\begin{bmatrix} \Delta\dot{E}'_q \\ \Delta\dot{E}'_d \\ \Delta\dot{E}_{FD} \\ \Delta\dot{x}_p \\ \Delta\dot{x}_q \\ \Delta\dot{I}_{sh} \\ \Delta\dot{V}_s \end{bmatrix} = \begin{bmatrix} -\frac{x_d}{x'_d T'_{d0}} & 0 & \frac{1}{T'_{d0}} & \cdots \\ 0 & -\frac{x_q}{x'_q T'_{q0}} & 0 & \cdots \\ 0 & 0 & -\frac{1}{T_e} & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{P_t}{T_p} & 0 & 0 & 0 \\ 0 & -\frac{Q_t}{T_q} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{sh}} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_s} \end{matrix} \Bigg] \cdot \begin{bmatrix} \Delta E'_q \\ \Delta E'_d \\ \Delta E_{FD} \\ \Delta x_p \\ \Delta x_q \\ \Delta I_{sh} \\ \Delta V_s \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{K_a}{T'_{d0}} & \frac{K_v}{x'_d T'_{d0}} & 0 & 0 \\ -\frac{K'_a}{T'_{q0}} & \frac{K'_v}{T'_{q0}} & 0 & 0 \\ 0 & -\frac{K_e}{T_e} & 0 & 0 \\ 0 & 0 & 0 & K_{pd} \\ 0 & 0 & 0 & K_{qd} \\ 0 & 0 & 0 & -\frac{K_u}{T_{sh}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta\Theta_t \\ \Delta V_t \\ \Delta\Theta_m \\ \Delta V_m \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{2_g} & A_{4_g} & 0 & 0 & 0 & 0 & -A_{4_t} \\ R_{2_g} & R_{4_g} & 0 & 0 & 0 & 0 & -R_{4_t} \\ 0 & 0 & 0 & -A_{cd} & 0 & 0 & -A_{4_m} \\ 0 & 0 & 0 & 0 & -R_{cd} & V_m & -R_{4_m} \end{bmatrix} \cdot$$

$$\begin{bmatrix} \Delta E'_q \\ \Delta E'_d \\ \Delta E_{FD} \\ \Delta x_p \\ \Delta x_q \\ \Delta I_{sh} \\ \Delta V_s \end{bmatrix} + \begin{bmatrix} -A_{1_g} - A_{1_t} & A_{3_g} - A_{2_t} & \cdots \\ -R_{1_g} - R_{1_t} & R_{3_g} - R_{2_t} & \cdots \\ A_{1_m} & -A_{3_m} & \cdots \\ R_{1_m} & -R_{3_m} & \cdots \end{bmatrix}$$

$$\begin{matrix} A_{1_t} & -A_{3_t} \\ R_{1_t} & -R_{3_t} \\ -A_{1_m} & -A_{2_m} - A_{1_c} \\ -R_{1_m} & -R_{2_m} - R_{1_c} + I_{sh} \end{matrix} \Bigg] \cdot \begin{bmatrix} \Delta\Theta_t \\ \Delta V_t \\ \Delta\Theta_m \\ \Delta V_m \end{bmatrix} \tag{24}$$

Considering the UPFC, the Jacobian J_{FC} in this case is given by:

$$J_{FC} = \begin{bmatrix} -A_{1_m} & -A_{2_m} - A_{1_c} \\ -R_{1_m} & -R_{2_m} - R_{1_c} + I_{sh} \end{bmatrix} \tag{25}$$

IV. SIMULATIONS AND RESULTS

The system of Figure 1 is simulated over a range of operating points. The small signal voltage stability assessment is performed by monitoring the critical eigenvalues of matrix A and the determinants of the Jacobians J_4, J_{FC} as the system loading is increased. The singularities of these Jacobians represent the singularity induced and saddle-node bifurcations, respectively.

The steady-state components (P_s,Q_s) and the transient components (P_t,Q_t) are modelled as constant power and constant impedance, respectively. The results were obtained considering the load time constant as $T_p = T_q = 30s$.

For the simulations realized were considered a nominal loading factor ($\mu = 1.0$ p.u.) of $100 + j48.7$ MVA. The parameters of the AVR, transmission line and the FACTS are given in Appendix A.

A. System without FACTS

Figs. 5 illustrates the critical eigenvalues loci of system state matrix without FACTS. As can be noted, the critical eigenvalue crosses the imaginary axis characterizing the saddle-node bifurcation.

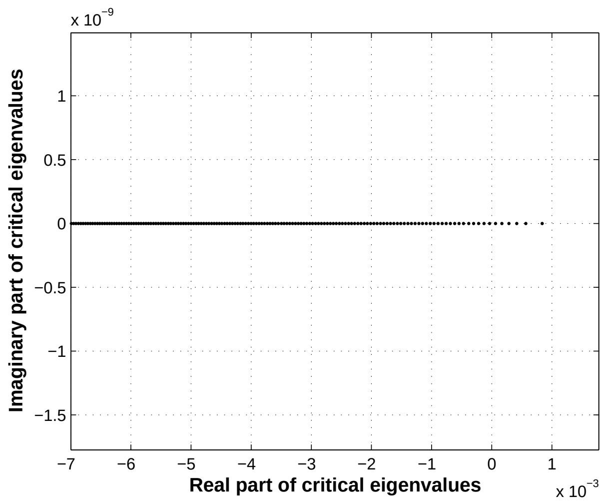

Fig. 5. Critical Eigenvalues loci.

Fig. 6 shows real eigenvalue loci as loading factor increase up to 1.042 p.u. The system instability occurs for a loading factor of 1.036 p.u.

B. System with STATCOM

The analysis of the eigenvalues of the system with STATCOM is made analogous as the previously. Fig. 7 shows the real part of the eigenvalues as the system loading is increased. In this case, the STATCOM keeps the system stable for a loading superior than 1.042 p.u.

Figs. 8 and 9 exhibit the determinants of Jacobian J_4 and the Jacobian J_{FC} have a tendency to be singular, occurring

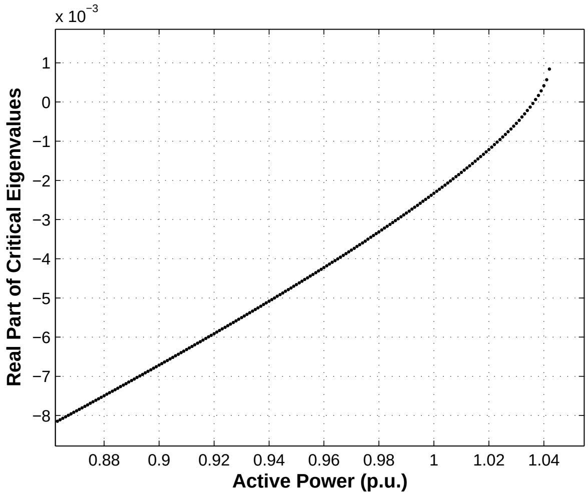

Fig. 6. Real Eigenvalues loci.

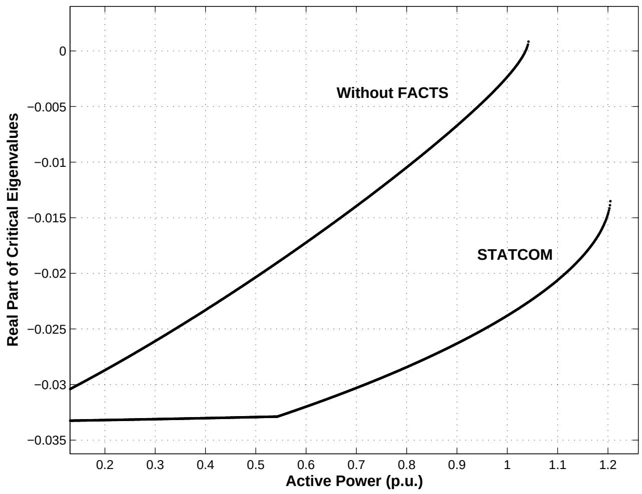

Fig. 7. Real Eigenvalues loci.

the singularity induced and the saddle-node bifurcations, respectively.

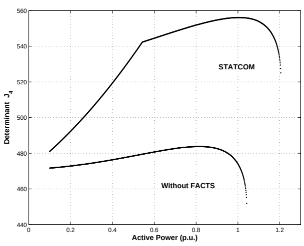

Fig. 8. Determinant of Jacobian J_4.

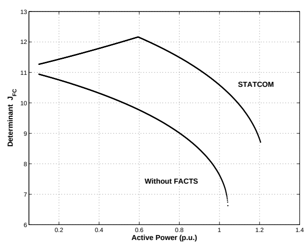

Fig. 9. Determinant of Jacobian J_{FC}.

Fig. 10 presents the PV curve of load bus where the STATCOM is installed. For the loading increase up to 1.205 p.u., the STATCOM keeps the bus voltage at the level specified until the limits of current injection did not have exceeded.

C. System with SSSC

Fig. 11 presents the real part of critical eigenvalues as the loading factor varies from 0.1 to 1.266 p.u. The instability of

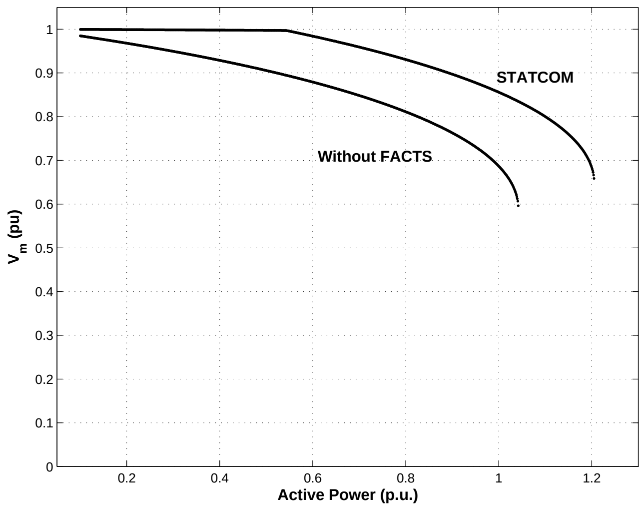

Fig. 10. P-V Curve.

the system with SSSC occurs for a loading factor at 1.203 p.u.

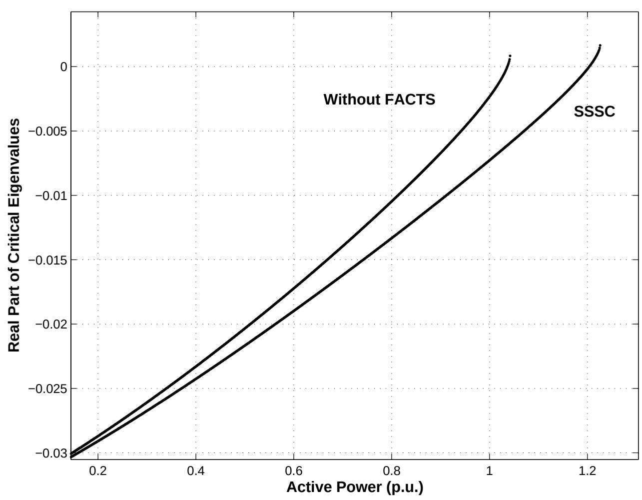

Fig. 11. Real Eigenvalues loci.

Fig. 12 illustrates that the system with SSSC have the voltage stability margin increased. Although the SSSC does not have the main function of regulate voltage, it could improve the bus voltage level through its reactive line compensation function.

Figs. 13 and 14 depicts the behavior of the determinants of Jacobian J_{FC} and J_4. The divergence of power flow of the system with SSSC occurs for a loading up to 1.266 p.u. Again, these figures show the occurrence of saddle-node and singularity-induced bifurcations, respectively.

D. System with UPFC

Fig. 15 shows the real part of the eigenvalues for the loading up to 1.388 p.u. The saddle-node bifurcation happens only for the system without FACTS.

The divergence of the power flow, for the system with UPFC, occurs for the loading up to 1.388 p.u. The determinants of the Jacobians J_4 and J_{FC} show the singularity induced and saddle-node bifurcations, respectively in Figs. 16 and 17.

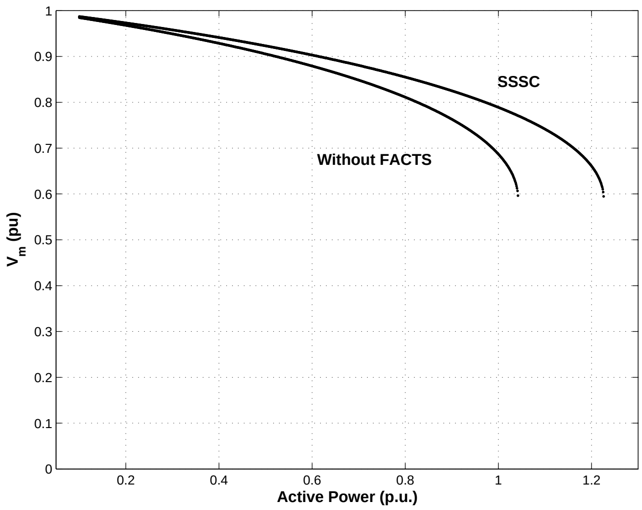

Fig. 12. P-V Curve.

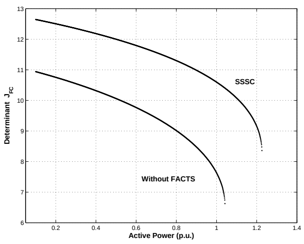

Fig. 13. Determinant of Jacobian J_4.

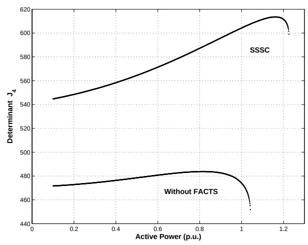

Fig. 14. Determinant of Jacobian J_{FC}.

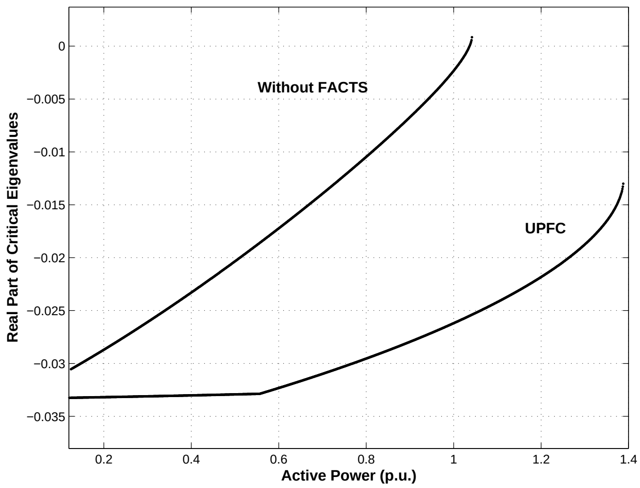

Fig. 15. Real Eigenvalues loci.

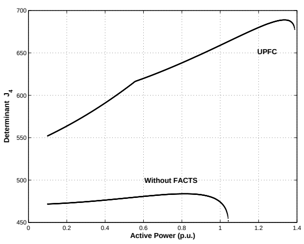

Fig. 16. Determinant of Jacobian J_4.

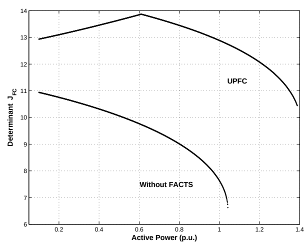

Fig. 17. Determinant of Jacobian J_{FC}.

Like the others FACTS presented, the UPFC enhance the voltage stability margin of the system. The Fig. 18 shows the PV curve of V_m bus.

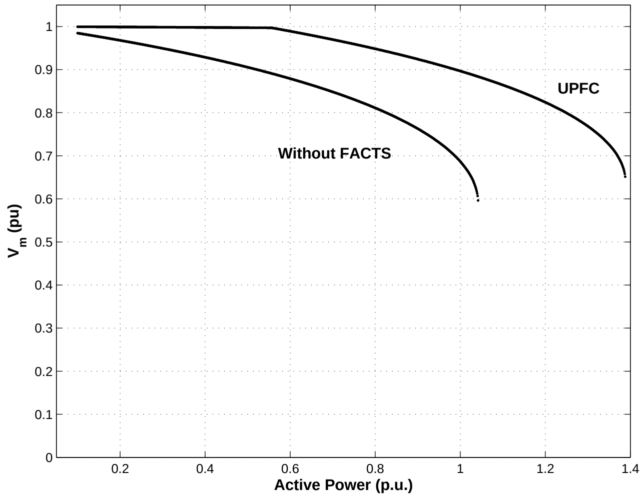

Fig. 18. P-V Curve.

Table I presents the maximum values reached over a range of operating points until the divergence of the power flow. All the FACTS have the ability to provide a greater margin stability for the system studied.

TABLE I

Stability Limits p.u.

without FACTS	with SSSC	with STATCOM	with UPFC
1.036	1.203	1.205	1.388

V. CONCLUSION

This work has examined the effects of the STATCOM, SSSC and the UPFC on small signal voltage stability. The voltage stability is evaluated by the analysis of the PV curve in conjunction with the trajectory of the critical eigenvalues of the state matrix of the system.

The simulation results presented show that these FACTS controllers have a good effectiveness to improve the stability of the system and thus the loadability margin of power systems.

APPENDIX A

TABLE II

Generator and Transmission line Parameters.

$H(s)$	D	$R_e(p.u.)$	$x_d(p.u.)$	$x'_d(p.u.)$	$x_q(p.u.)$
6.4	0.0	0.0	0.8958	0.1198	0.8645
$x'_q(p.u.)$	$T'_{d0}(s)$	$T'_{q0}(s)$	$K_e(p.u.)$	$T_e(p.u.)$	$X_L(p.u.)$
0.1969	6.0	0.535	20	0.2	0.1

TABLE III

STATCOM Parameters.

$K_u(p.u.)$	$T_{sh}(s)$	$I_{sh_{max}}(p.u.)$	$I_{sh_{min}}(p.u.)$
100	0.005	0.3	-0.3

TABLE IV

SSSC Parameters.

$T_s(s)$	$V_{s_{max}}(p.u.)$	$V_{s_{min}}(p.u.)$
0.005	0.2	-0.2

TABLE V

UPFC Parameters.

$K_u(p.u.)$	$T_{sh}(s)$	$I_{sh_{max}}(p.u.)$	$I_{sh_{min}}(p.u.)$
100	0.005	0.3	-0.3
$T_s(s)$	$V_{s_{max}}(p.u.)$	$V_{s_{min}}(p.u.)$	
0.005	0.2	-0.2	

APPENDIX B

Nomenclature

E	Generator voltage.
δ	Generator rotor angle.
ω	Generator rotor speed.
E'_q	Transitory voltage quadrature axis component.
E'_d	Transitory voltage direct axis component.
E_{FD}	Field voltage.
M	Inertia constant.
x_d	Direct axis reactance.
x'_d	Transitory direct axis reactance.
x_q	Quadrature axis reactance.
x'_q	Transitory quadrature axis reactance.
T'_{d0}	Transitory direct axis time constant.
T'_{q0}	Transitory quadrature axis time constant.
K_e	AVR static gain.
T_e	AVR time constant.
X	Transmission line reactance.
V_t	Bus t voltage.
V_m	Bus m voltage.
θ_t	Bus t angle.
θ_m	Bus m angle.
K_u	STATCOM or UPFC voltage regulator gain.
V_s	SSSC or UPFC voltage.
I_{sh}	STATCOM or UPFC current.
T_s	SSSC time constant.
T_{sh}	STATCOM or UPFC time constant.

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