A VARIABLE STRUCTURE ADAPTIVE POLE PLACEMENT CONTROL APPLIED TO THE SPEED CONTROL OF A THREE-PHASE INDUCTION MOTOR

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Abstract – **In this paper, a Variable Structure Adaptive Pole Placement Control (VS-APPC) is applied to the speed control of a three-phase induction motor. Due to its flexibility in choosing the controller design methodology (state feedback, compensator design, linear quadratic, etc.) and the adaptive law (least squares, gradient, etc.), the adaptive pole placement control (APPC) is the most general type of adaptive control. Traditionally, it has been developed in an indirect approach and, as an advantage, it may be applied to non-minimal phase plants. The combination of this strategy with the variable structure systems allows to aggregate fast transient and robustness to parametric uncertainties and disturbances. Therefore, new switching laws were proposed, instead of using the traditional integral adaptive laws. In this paper, preliminary simulation and experimental results for a speed control of a three-phase induction motor are shown.**

Keywords – **Adaptive Control; Variable Structure Systems; Induction Motor Driver; Nonlinear Systems.**

I. INTRODUCTION

In recent days the induction motors have been increasingly taking place of the DC motors in high performance electrical motor drives [1]. In the case of motors with squirrel's cage rotor, its main advantage is the elimination of all sliding electrical contacts, resulting in an exceedingly simple and rugged construction. Induction machines are made in a variety of designs with ratings of a few watts to several megawatts. The induction motors can be used in adverse atmospheres since that they don't have commutator and, consequently, there isn't a possibility of sparking. With the progress of the power electronics and the appearance of low cost and very fast microprocessors, the induction motor drives have reached a competitive position compared to DC machines. For the DC motors, the speed control can be carried out in a simple way, since the torque and the flux can be decoupled. The technique of vectorial control based on the rotor field orientation applied to the induction motors [1,2,3], when the motor is fed by ideal current sources, provides the decoupling between the torque and flux in a similar way to the DC machine. This technique is known as Field Orientation Control (FOC). The choice of the rotor flux as reference for the **d** axis facilitates the decoupling between motor torque and flux [1,3,4]. In this control strategy, an important element of uncertainty is the value of the rotor time constant that varies with the operation conditions, changing the system behavior. Then, there is the necessity of methods of adaptive and/or robust control, which can be applied to systems that present parametric uncertainties.

A class of control schemes that is popular in the known parameter case are those that change the poles and do not involve plant zero-pole cancellations. These schemes are referred as pole placement schemes and are applicable to both minimum and nonminimal phase linear time invariant (LTI) plants. The combination of a pole placement control law with a parameter estimator or an adaptive law leads to an adaptive pole placement control (APPC) scheme that can be used to control a wide class of LTI plant with unknown parameters. Such technique was developed based on the indirect adaptive control schemes, where the control signal is a function of the plant parameters estimates.

On the other hand, the variable structure control (VSC) approach has its roots in relay control, and consists of using a switching control law as a function of system state variables, and, in its common configuration, in order to restrict the system dynamics to a surface referred as a sliding surface. The variable structure systems have as main characteristics the fast transient and robustness to parameter changes and disturbances (in a range stipulated on project), although measurements of all states variables be necessary, what may be undesirable or even not possible in some cases [5].

Thereby, a control technique that inherit the VSC qualities was developed, but with only input/output measurements, which was named VS-MRAC (Variable Structure Model Reference Adaptive Control) [6,7,8], where the MRAC integral adaptation laws [9] were replaced by switching laws. This algorithm was based on the direct approach of MRAC, being thus restricted to minimal phase plants. In order to simplify the controller design, a new controller was proposed, named indirect VS-MRAC [10,11,12], which makes use of the plant nominal parameters for the relays amplitude calculation, since they are related with physical parameters, such as resistances, capacitances, inertia moments, etc. The VS-MRAC controller, in its direct and indirect approaches, have been successfully applied on control of DC machines [13] as well as on control of induction machines [11,14,15].

In a recent work was presented a controller that aggregates the characteristics of both techniques, namely, APPC and VSC [16]. Thus, it's expected applicability to non-minimal phase plants, fast transient and robustness to parameter changes and disturbances. This controller was named VS-APPC, where, likewise VS-MRAC, the integral adaptive laws were replaced by switching laws. In this paper, in order to confirm its feasibility, an application on a three-phase induction motor is shown.

II. MODEL OF INDUCTION MOTOR

In this section we use a vectorial technique for modeling the induction motor, which is very important to study field orientation control [1,4]. We define a system of complex orthogonal axis, d and q, where the rotor flux is the reference for the **d** axis. The motor vectorial diagram is presented in Figure 1, where

δ - stator electrical current vector angle related to the rotor flux;

ρ - rotor flux angle related to stator phase 1 axis;

ω_S - stator electrical current vector angular speed;

$\psi_{Rd}(t)$ - rotor flux related to the **d** axis;

ε - angle between axis of stator phase 1 and rotor phase 1;

$i_S(t)$ - stator electrical current vector;

i_{Sd}, i_{Sq} - stator electrical current vector components on direct and quadrature axis, respectively;

$\omega(t) = \dfrac{d\varepsilon(t)}{dt}$ - rotor angular mechanical speed.

From Figure 1 we have

$$i_S(t) = \left(i_{Sd}(t) + j i_{Sq}(t)\right) e^{j\rho} \tag{1}$$

$$\psi_R(t) = \psi_{Rd}(t) e^{j(\rho - \varepsilon)} \tag{2}$$

Using the vectorial analysis with the rotor flux orientation [1,4], we obtain the following expression for the torque

$$T_e(t) = \frac{2}{3} P \frac{L_m}{L_r} \psi_{Rd}(t) i_{Sq}(t) \tag{3}$$

where

L_r - rotor inductance by phase;

L_m - magnetization inductance by phase;

P - number of poles pairs.

The Equation (3) describes the induction motor torque in a similar way to the DC machine. The component of the rotor flux vector on direct axis is equivalent to the field flux in DC machine and the component of stator electrical current vector on the quadrature axis is equivalent to the armature current in a DC machine. Additionally, if the component of the rotor flux is kept constant, the torque can be controlled only by the component of the stator electrical current vector on the quadrature axis.

III. POLE PLACEMENT CONTROL (PPC)

Considering the single input/single output (SISO) LTI plant

$$y = G(s)u,\ G(s) = \frac{Z(s)}{R(s)} = \frac{b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} \tag{4}$$

there are, as plant parameters, $2n$ elements, which are the coefficients of the numerator and denominator of $G(s)$. Therefore, we can define the vector θ^* as

$$\theta^* = [b_{n-1} \ \ . \ . \ . \ \ b_1 \ \ b_0 \ \ a_{n-1} \ \ . \ . \ . \ \ a_1 \ \ a_0]^T$$

S1. *R(s)* is a monic polynomial whose degree *n* is known.

S2. *Z(s)*, *R(s)* are coprime and degree(*Z*) < *n*.

Assumptions (S1) and (S2) allow *Z, R* to be non-Hurwitz in contrast to the MRC case where *Z* is required to be Hurwitz.

We can also extend the PPC objective to include tracking, where *y* is required to follow a certain class of reference signals *r*, by using the internal model principle [17]. The uniformly bounded reference signal is assumed to satisfy

$$Q_m(s) r = 0 \tag{5}$$

where $Q_m(s)$, the internal model of *r*, is a known monic polynomial of degree *q* with non-repeated roots on the $j\omega$-axis and satisfies

S3. $Q_m(s)$, *Z(s)* are coprime.

We consider the control law

$$Q_m(s) L(s) u = -P(s) y + M(s) r \tag{6}$$

where $P(s)$, $M(s)$, $L(s)$ are polynomials (with $L(s)$ monic) of degree $q + n - 1$, $q + n - 1$ e $n - 1$, respectively, to be found and $Q_m(s)$ satisfies (5) and assumption (S3).

Applying (6) to the plant (4), we obtain the closed-loop plant equation

$$y = \frac{Z(s)M(s)}{Q_m(s)L(s)R(s) + P(s)Z(s)} r \tag{7}$$

whose characteristic equation

$$Q_m(s)L(s)R(s) + P(s)Z(s) = 0 \tag{8}$$

has order $2n + q - 1$. The objective now is to choose *P*, *L* such that

$$Q_m(s)L(s)R(s) + P(s)Z(s) = A^*(s) \tag{9}$$

is satisfied for a given monic Hurwitz polynomial $A^*(s)$ of degree $2n + q - 1$. Because of assumptions S2 e S3 guarantee that Q_m, *R*, *Z* are coprime, there is a solution so that *L* and *P* satisfy (9) and this solution is unique [17].

Using (6), the closed-loop is described by

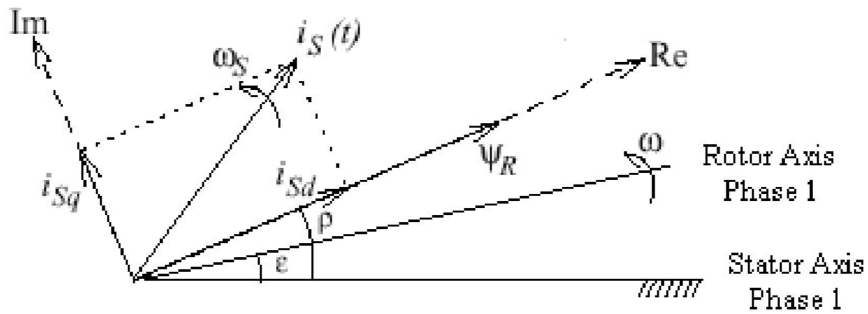

Fig. 1. Motor Vector Diagram.

$$y = \frac{ZM}{A^*} r \tag{10}$$

Similarly, from the plant in (4) and the control law in (6) and (9), we obtain

$$u = \frac{RM}{A^*} r \tag{11}$$

Because r is uniformly bounded and $\frac{ZM}{A^*}, \frac{RM}{A^*}$ are proper with stable poles, y and u remain bounded whenever $t \rightarrow \infty$ for any polynomial $M(s)$ of degree $n + q - 1$ [17]. Therefore, the pole placement objective is achieved by the control law (6) without having to put any additional restrictions on $M(s)$, $Q_m(s)$. When $r = 0$, (10), (11) imply that y, u converge to zero exponentially fast.

When $r \neq 0$, the tracking error $e = y - r$ is given by

$$e = \frac{ZM - A^*}{A^*} r = \frac{Z}{A^*}(M - P)r - \frac{LR}{A^*} Q_m r \tag{12}$$

In order to obtain zero tracking error, (12) suggests the choice of $M(s) = P(s)$ to cancel the first term in (12). The second term in (12) is canceled by using $Q_m r = 0$. Therefore, the pole placement and tracking objective are achieved by using the control law

$$Q_m L u = -P(y - r) \tag{13}$$

which is implemented as shown in Figure 2 using $n + q - 1$ integrators to realize $C(s) = \frac{P(s)}{Q_m(s)L(s)}$. Because $L(s)$ is not necessarily Hurwitz, the realization of (13) with $n + q - 1$ integrators may have a transfer function, namely C(s) unstable. An alternative realization of (13) is obtained by rewriting (13) as

$$u = \frac{\Lambda - LQ_m}{\Lambda} u - \frac{P}{\Lambda}(y - r) \tag{14}$$

where Λ is any monic Hurwitz polynomial of degree $n + q - 1$.

IV. VARIABLE STRUCTURE ADAPTIVE POLE PLACEMENT CONTROL

In this section is showed the development of the variable structure adaptive pole placement controller proposed in [16]. The uncertainty in the plant parameters can be known easier, since they represent physical parameters such as, resistances, capacitances, inertia moments, friction coefficients, etc. In this paper, it will be covered the first order plant case.

Let us consider the plant

$$y = \frac{b}{s + a} u \Rightarrow \dot{y} = -ay + bu \tag{15}$$

where the parameters a, b are constant and known with uncertainties. We shall treat a tracking problem.

Let be $a_m > 0$. Then, we may write (15) as

$$\dot{y} = -a_m y + (a_m - a)y + bu \tag{16}$$

A model for the plant may be written as

$$\dot{\hat{y}} = -a_m \hat{y} + (a_m - \hat{a})y + \hat{b}u \tag{17}$$

where $\hat{a}$, $\hat{b}$ are estimates for a,b, respectively [17].

We define the estimation error e_0 as

$$e_0 = y - \hat{y} \tag{18}$$

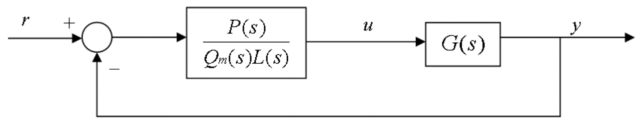

Fig. 2. Block diagram of pole placement control.

therefore,

$$\dot{e}_0 = -a_m e_0 + \tilde{a}y - \tilde{b}u \tag{19}$$

with

$$\tilde{a} = \hat{a} - a$$
$$\tilde{b} = \hat{b} - b \tag{20}$$

Because a, b are constant, by assumption, we have:

$$\dot{\tilde{a}} = \dot{\hat{a}}$$
$$\dot{\tilde{b}} = \dot{\hat{b}} \tag{21}$$

Now, we consider the following switching laws for $\hat{a}$ and $\hat{b}$

$$\hat{a} = -\bar{a}\,\mathrm{sgn}(e_0 y),\ \bar{a} > |a|$$
$$\hat{b} = \bar{b}\,\mathrm{sgn}(e_0 u),\ \bar{b} > |b| \tag{22}$$

Let us choose the Lyapunov function

$$V(e_0) = \frac{1}{2} e_0^2 > 0$$

therefore,

$$\begin{aligned}
\dot{V}(e_0) &= e_0 \dot{e}_0 \\
&= -a_m e_0^2 + \tilde{a} e_0 y - \tilde{b} e_0 u \\
&= -a_m e_0^2 + (\hat{a} - a) e_0 y - (\hat{b} - b) e_0 u \\
&= -a_m e_0^2 + [-\bar{a}\,\mathrm{sgn}(e_0 y) - a] e_0 y - [\bar{b}\,\mathrm{sgn}(e_0 u) - b] e_0 u \\
&= -a_m e_0^2 - (\bar{a}|e_0 y| + a e_0 y) - (\bar{b}|e_0 u| - b e_0 u)
\end{aligned}$$

Since $\bar{a} > |a|$ and $\bar{b} > |b|$, we have

$$\dot{V}(e_0) \le -a_m e_0^2 < 0$$

which guarantees that $e_0 = 0$ is a globally asymptotic stable equilibrium point. Moreover, if we follow a procedure similar to [6], one can prove that $e_0 = 0$ reaches the sliding surface in a finite time t_f ($e_0 = 0$, $\forall t \ge t_f$).

V. THE CONTROLLER APPLICATION

In this section will be presented the results of simulations obtained for the application of the technique described above to the speed control of a three-phase induction motor.

A. Driver System

The driver system that will be used to implement the variable structure adaptive pole placement control is shown in Figure 3. It is composed by an 0.25 HP induction motor fed by a three-phase VSI/PWM inverter with current control by hysteresis window. In the current control, Hall effect sensors are used to measure the currents of two phases of the motor. One microcomputer receives the motor speed using a tachometer and, by a control software in C language, sends the necessary signal to the inverter.

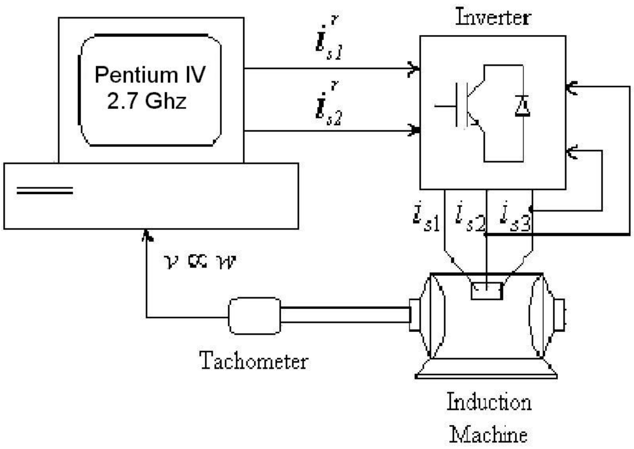

Fig. 3. Driver System.

B. Plant Model

The dynamic of the induction motor is represented by the following equation

$$J\frac{d\omega(t)}{dt} = T_e(t) - B\omega(t) - T_l(t) \quad (23)$$

where

J - moment of inertia of the rotational mass;

B - damping constant;

T_e - induction motor torque;

T_l - load torque.

The induction motor model introduced here ((3) and (23)), for a certain operating point, considering the rotor flux constant and the motor parameters given in [4], yields to a first order model given by

$$G(s) = \frac{3798}{s+11.3} = \frac{b}{s+a} \quad (24)$$

C. Calculation of Controller Parameters

The characteristic polynomial chosen was

$$A^*(s) = (s+12)^2 \quad (25)$$

Thus, we have

$$L(s) = 1, \quad P(s) = p_1 s + p_0 \quad (26)$$

In this paper, we use a constant reference signal r = 1000 rpm, $\forall t \geq 0$, then

$$Q_m(s) = s \quad (27)$$

According to (4) and (24)

$$Z(s) = b, \quad R(s) = s + a \quad (28)$$

and, by equation (6), we have

$$s(s+a) + (p_1 s + p_0)b = (s+12)^2 \quad (29)$$

which solution is

$$p_1 = \frac{24-a}{b}, \quad p_0 = \frac{144}{b} \quad (30)$$

When the plant parameters are known with uncertainties, the certainty equivalence principle suggests the using of the same control law, but with the controller polynomial $P(s) = p_1 s + p_0$ calculated by using the estimates of the parameters , and, therefore, we have

$$\hat{p}_1 = \frac{24-\hat{a}}{\hat{b}}, \quad \hat{p}_0 = \frac{144}{\hat{b}} \quad (31)$$

where $\hat{p}_1$ and $\hat{p}_0$ are the controller parameters estimates that must be generated on-line. In the traditional indirect APPC scheme adaptive laws driven by the error e_0 are used. To achieve this, it may be used the gradient method, the least squares method, etc. For the VS-APPC scheme the adaptive laws are replaced by switching laws as in (22).

Since the controller parameters can be functions of more than one plant parameter simultaneously, the signal may come undefined, due to high frequency switching signals. Besides this, the parameter $\hat{b}$, showed up in the denominator of the expressions, can cause divisions by zero. Thus, it's necessary the introduction of a nominal value of the parameter $\hat{b}$, in order to maintain the value with a defined signal. Rewriting the switching laws with a modification in $\hat{b}$ expression, we have

$$\begin{aligned} \hat{a} &= -\bar{a}\,\mathrm{sgn}(e_0 y), \ \bar{a} > |a| \\ \hat{b} &= \bar{b}\,\mathrm{sgn}(e_0 u) + b_{nom}, \ \bar{b} > |b - b_{nom}| \end{aligned} \quad (32)$$

where b_{nom} is a nominal value for the parameter b.

The control signal u is generated from equation (6).

D. Results

Before the implementation of VS-APPC, some simulations were carried out with an integration step of h = 0.01s and a reference signal r = 1000 rpm.

In the simulation (Figure 4), the conventional adaptive laws were replaced by the switching laws proposed in (32). We used the constants $b_{nom} = 3600$, $\bar{b} = 1200$ and $\bar{a} = 13$. The plant and reference model speeds are given in rpm and the control signal $u = i_{Sq}$ is given in mA. It was verified that the plant output matches in 0.53s. The Figure 5 shows the evolution of the plant parameters estimates.

Now, we have the practical implementation of VS-APPC for the speed control of a three-phase induction motor, using the same constants and reference signal of the simulations and h = 0.000140s. The Figure 6 shows an experimental result with a constant reference signal r = 1000 rpm. The Figure 7 shows a result with the reference signal being changed during the operation. The control signal is given in mA and is multiplied by 100 to be shown on the figure.

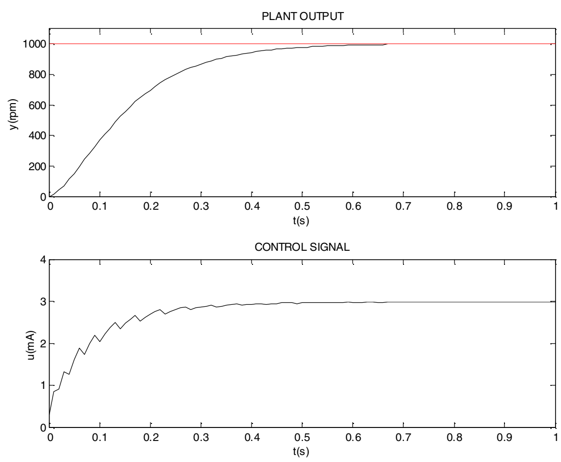

Fig. 4. Simulation: VS-APPC for a speed control of a three-phase induction motor.

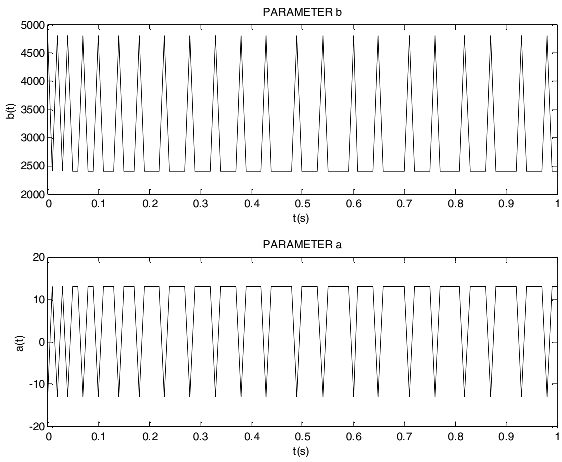

Fig. 5. Behavior of plant parameters using switching laws.

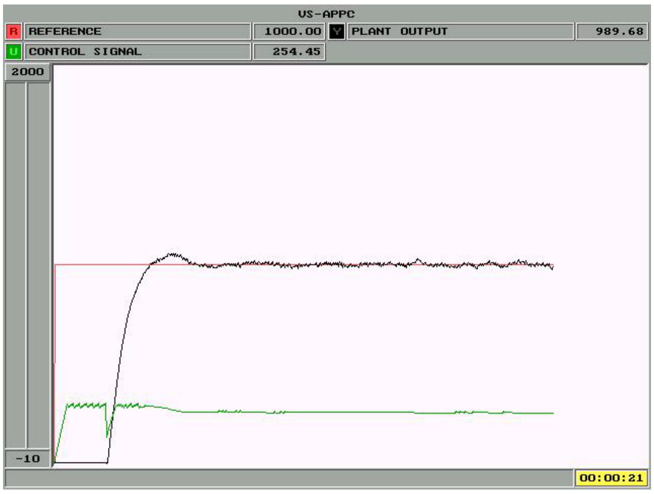

Fig. 6. Experimental result 1: VS-APPC for a speed control of a three-phase induction motor.

Fig. 7. Experimental result 2: VS-APPC for a speed control of a three-phase induction motor.

The indirect VS-APPC scheme is easy to design, with the relays amplitude calculation directly related to the plant physical parameters.

VI. CONCLUSION

In this paper, the new technique named VS-APPC was used to the speed control of a three-phase induction motor.

The proposed technique has presented a very fast transient.

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