$\mathcal{H}_\infty$ SWITCHED CONTROL WITH POLE LOCATION CONSTRAINTS APPLIED TO A UPS SYSTEM

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***Abstract*— The aim of this paper is to present a design strategy to improve the performance of a UPS system with arbitrary switched loads by means of discrete-time switched state feedback $\mathcal{H}_\infty$ control with pole location constraints. Sufficient linear matrix inequalities conditions are given to determine controller gains that guarantee: i) transient responses within specified bounds; ii) attenuation of disturbances that are energy signals; iii) overall stability of the closed-loop system for any arbitrary switching rule. Numerical results show how the proposed scheme can improve the overall performance of the UPS system under analysis.**

***Keywords* – Discrete-time switched linear systems; H-infinity control; Linear matrix inequalities; Pole location; Switched Lyapunov functions; Uninterruptible power supplies.**

I. INTRODUCTION

Switched linear systems are an important class of dynamical systems defined by several subsystems and a rule (switching function) that determines which subsystem is active at each instant of time [1]. Many engineering applications present switching behaviors, rising for example from the use of switching control techniques [2], with special attention to switched electrical circuits [3]. Among these circuits, an important class is the one which includes the uninterruptible power supply (UPS) systems, used to feed critical loads as medical and telecommunication equipment (see [4], [5] for details). UPS performances can be measured in terms of transient response and total harmonic distortion of the output voltage waveform. A very important feature of the system is the capacity to assure a sinusoidal output voltage with low distortion, even in the presence of nonlinear loads, as switched cyclic loads that are common for this application. Several techniques have been used to improve the system robustness and performance, with special attention to discrete-time control strategies, since they allow to implement sophisticated control schemes in low cost digital processors. For instance, deadbeat control was applied to UPS systems in [6], [7], a predictive control was used in [8] and a robust model reference adaptive controller was used in [9]. Repetitive controllers, whose objective is to reduce errors caused by periodical disturbances, have shown good results for UPS applications, reducing the distortion caused by cyclic loads, as reported in [10], [11], [12], [13]. The aim here is to address the control of a UPS system with an arbitrary switching load by means of a discrete-time switched $\mathcal{H}_\infty$ state feedback control with pole location constraints. Differently from the previous mentioned works and from most of the papers dealing with control of UPS in the literature, the proposed approach is based on Lyapunov functions and uses the linear matrix inequality (LMI - see [14] for details) framework to solve the design problem efficiently.

It is well known that quadratic Lyapunov functions have been widely used for robust stability, filtering and control of several classes of systems [14], [15], [16]. This class of functions could be used to address the problem under consideration here, providing a fixed control gain to achieve the design specifications, but since the same matrix must be used to assess the stability of the entire domain, this condition can be frequently unfeasible or can lead to conservative results. Less conservative results can be obtained through piecewise Lyapunov functions, whenever the transitions between two subsystems of the switched system are described by some rule on the state space [17], [18]. However, this class of Lyapunov functions often produces tests of high numerical complexity, not suitable to cope with systems subject to arbitrary switching rules.

Recently, LMI conditions for the analysis and control synthesis of discrete-time linear uncertain time-varying systems have been given, based on parameter dependent Lyapunov functions, including $\mathcal{H}_\infty$ norm specifications [19], [20]. The present paper extends the results of [20] to the context of discrete-time switched systems with arbitrary switching rules, taking into account pole location specifications inside circles with given centers and radii, lying inside the unit circle. The proposed technique guarantees transient responses within prespecified bounds and minimizes the effect of energy signal disturbance inputs in the output of the closed-loop system. The application in a UPS system with an arbitrary switched load shows that the performance of the system can be significantly improved with the control strategy proposed here, illustrating the efficiency and usefulness of the $\mathcal{H}_\infty$ switched control with pole location.

II. Problem formulation

Consider the discrete-time switched linear system

$$x_{k+1} = A_{\alpha(k)}x_k + B_{1\alpha(k)}w_k + B_{2\alpha(k)}u_k \tag{1}$$

$$z_k = C_{\alpha(k)}x_k + D_{1\alpha(k)}w_k + D_{2\alpha(k)}u_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state, $w_k \in \mathbb{R}^r$ is an exogenous input, $u_k \in \mathbb{R}^m$ is the control input, $z_k \in \mathbb{R}^p$ is the controlled output and $A_{\alpha(k)} \in \mathbb{R}^{n\times n}$, $B_{1\alpha(k)} \in \mathbb{R}^{n\times r}$, $B_{2\alpha(k)} \in \mathbb{R}^{n\times m}$, $C_{\alpha(k)} \in \mathbb{R}^{p\times n}$, $D_{1\alpha(k)} \in \mathbb{R}^{p\times r}$, $D_{2\alpha(k)} \in \mathbb{R}^{p\times m}$ are the subsystems matrices. The switching function $\alpha(k)$ is not known *a priori* but it is available at each sample k, being described as

$$\alpha(k) : \mathbb{N} \longrightarrow \mathcal{I} \ , \ \mathcal{I} = \{1,\ldots,N\} \tag{3}$$

The design problem addressed here is to find, if possible, a state feedback control law

$$u_k = K_{\alpha(k)}x_k \tag{4}$$

with $K_{\alpha(k)} \in \mathbb{R}^{m\times n}$ such that the following properties are verified:

P1) The poles of each linear subsystem of the closed-loop switched system, that is, the eigenvalues of $(A_j + B_{2j}K_j)$, $j = 1,\ldots,N$, are located in the circle with center at σ_j, with $-1 < \sigma_j < 1$ and radius r_j, with $0 < r_j \le 1$, placed inside the unit circle in the complex plane, depicted in Figure 1.

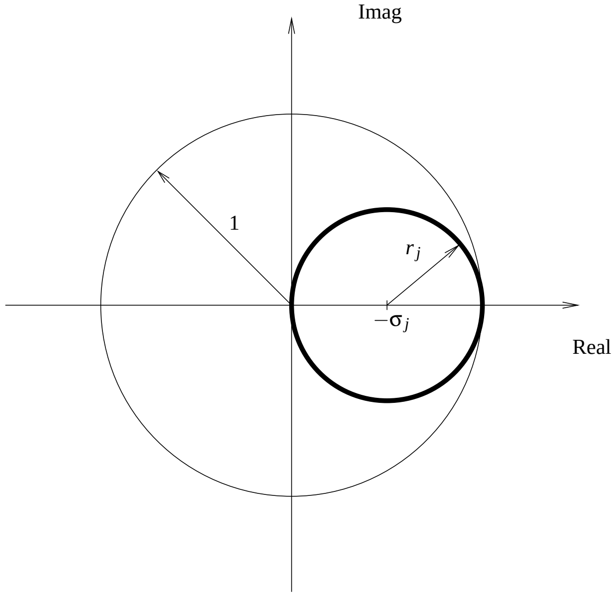

Fig. 1. Circular region with center σ_j and radius r_j for pole location of each subsystem $(A_j + B_{2j}K_j)$, $j = 1,\ldots,N$.

P2) For any input $w_k \in \ell_2$, the output $z_k \in \ell_2$ is such that

$$\|z_k\|_2 < \gamma\|w_k\|_2 \tag{5}$$

for some $\gamma > 0$. Any value of γ satisfying (5) is called an $\mathcal{H}_\infty$ guaranteed cost of the closed-loop switched system and it is of great interest to determine the switched control gains that provide the smallest attenuation level.

Property P1) can assure to each linear subsystem of the system the desired rate of asymptotic damping and frequency of oscillation, when the switching sequence allows the accommodation of all the system modes, as well as the closed-loop stability under any switching sequence, when $w_k = 0$. Property P2) guarantees that the closed-loop system can reject disturbances in ℓ_2 for any arbitrary switching sequence.

III. Preliminary results

Consider system (1)-(2) with $w_k = \mathbf{0}$ and $u_k = \mathbf{0}$. A sufficient LMI condition for the stability in this case is reproduced in the next lemma (quadratic stability, [14]).

Lemma 1 If there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ such that

$$A_j' P A_j - P < 0 \quad , \quad j = 1,\ldots,N \tag{6}$$

then system (1)-(2) with $w_k = \mathbf{0}$ and $u_k = \mathbf{0}$ is stable for any arbitrary switching function.

Proof: Consider the Lyapunov function

$$v(x_k) = x_k' P x_k, \ P = P' > 0 \tag{7}$$

The difference function

$$\Delta v(x_k) = v(x_{k+1}) - v(x_k) \tag{8}$$

is given by

$$\Delta v(x_k) = x_k' \big(A_{\alpha(k)}' P A_{\alpha(k)} - P\big) x_k \tag{9}$$

The feasibility of Lemma 1 assures that (9) is negative for all $x_k \neq \mathbf{0}$, thus guaranteeing the system stability. □

A less conservative condition to assess the stability of the system under consideration is reproduced in the next lemma.

Lemma 2 The following statements are equivalent [21]:

i) If there exist symmetric positive definite matrices $P_j \in \mathbb{R}^{n\times n}$, $j = 1,\ldots,N$ such that

$$A_j' P_i A_j - P_j < 0 \quad , \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \tag{10}$$

then system (1)-(2) with $w_k = \mathbf{0}$ and $u_k = \mathbf{0}$ is stable for any arbitrary switching function;

ii) If there exist symmetric positive definite matrices $P_j \in \mathbb{R}^{n\times n}$, $j = 1,\ldots,N$ such that ($\star$ represents symmetrical blocks in the LMIs)

$$\begin{bmatrix} P_j & \star \\ P_i A_j & P_i \end{bmatrix} > 0 \quad , \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \qquad (11)$$

then system (1)-(2) with $w_k = \mathbf{0}$ and $u_k = \mathbf{0}$ is stable for any arbitrary switching function;

iii) If there exist symmetric positive definite matrices $S_j \in \mathbb{R}^{n\times n}$, $j = 1,\ldots,N$ such that

$$\begin{bmatrix} S_j & \star \\ A_j S_j & S_i \end{bmatrix} > 0 \quad , \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \qquad (12)$$

then system (1)-(2) with $w_k = \mathbf{0}$ and $u_k = \mathbf{0}$ is stable for any arbitrary switching function;

iv) If there exist symmetric positive definite matrices $S_j \in \mathbb{R}^{n\times n}$, and matrices $G_j \in \mathbb{R}^{n\times n}$, $j = 1,\ldots,N$ such that

$$\begin{bmatrix} G_j + G_j' - S_j & \star \\ A_j G_j & S_i \end{bmatrix} > 0 \quad , \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \qquad (13)$$

then system (1)-(2) with $w_k = \mathbf{0}$ and $u_k = \mathbf{0}$ is stable for any arbitrary switching function.

Proof: The equivalence between i) and ii) is straightforward, obtained using Schur complement. To prove the equivalence between ii) and iii), multiply at left and at right (11) by

$$T = \begin{bmatrix} P_j^{-1} & \mathbf{0} \\ \mathbf{0} & P_i^{-1} \end{bmatrix}$$

and use the variable transformation $S_j = P_j^{-1}$, $S_i = P_i^{-1}$ to obtain (12) and vice-versa. Equivalence between iii) and iv): if (12) has a solution S_j, $j = 1,\ldots,N$, then (13) will be feasible for $G_j = G_j' = S_j$. Conversely, if (13) is feasible, then multiplying (13) at left by

$$T = \begin{bmatrix} -A_j & \mathbf{I} \end{bmatrix}$$

and at right by T', one has $S_i - A_j S_j A_j' > 0$, which is Schur complement of (12). □

Note that condition i) of Lemma 2 contains the quadratic stability condition of Lemma 1 as a special case ($P_j = P_i = P$). Thus, Lemma 2 produces less conservative evaluations of stability of the system. Notice also that although conditions i) to iv) of Lemma 2 are equivalent in the case of stability analysis, matrices G_j in (13) provides an extra degree of freedom in the design of state feedback control gains, when the problem of structural constrained control is addressed (see, for instance, [22]).

The extension of condition iv) of Lemma 2 for synthesis is reproduced in the next lemma (see [21] for details).

Lemma 3 If there exist symmetric positive definite matrices $S_j \in \mathbb{R}^{n\times n}$, and matrices $G_j \in \mathbb{R}^{n\times n}$ and $F_j \in \mathbb{R}^{m\times n}$, $j = 1,\ldots,N$ such that

$$\begin{bmatrix} G_j + G_j' - S_j & \star \\ A_j G_j + B_2 F_j & S_i \end{bmatrix} > 0 \quad , \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \qquad (14)$$

the the stability of system (1)-(2) with $w_k = \mathbf{0}$ is assured by the state feedback control law (4), where the switching function $\alpha(k)$ (3) selects the active control gain in the set $K_j = F_j G_j^{-1} \quad , \quad j = 1,\ldots,N$.

Proof: Replace A_j in the expression (13) by $A_j + B_2 K_j$ and use the change of variables $F_j = K_j G_j$ to obtain (14). □

The extension of the stabilization problem of Lemma 3 to deal with pole location and $\mathcal{H}_\infty$ control is presented in the next session.

IV. Main Result

Theorem 1 For given values of $-1 < \sigma_j < 1$, $0 < r_j \leq 1$, $j = 1,\ldots,N$, with $|\sigma_j| + r_j \leq 1$, if there exist positive definite matrices $S_j \in \mathbb{R}^{n\times n}$ and matrices $Z_j \in \mathbb{R}^{m\times n}$ and $G_j \in \mathbb{R}^{n\times n}$, $j = 1,\ldots,N$ such that

$$\begin{bmatrix} r_j(G_j + G_j' - S_j) & \star & \star & \star \\ \mathbf{0} & \gamma(r_j \mathbf{I}) & \star & \star \\ A_j G_j + B_{2j} Z_j + \sigma_j G_j & r_j B_{1j} & S_i & \star \\ r_j(C_j G_j + D_{2j} R_j) & r_j D_{1j} & \mathbf{0} & \gamma(r_j \mathbf{I}) \end{bmatrix} > 0 \ , \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \qquad (15)$$

then the switched control law

$$u_k = K_j x_k \quad , \quad K_j = Z_j G_j^{-1} \quad , \quad j = 1,\ldots,N \qquad (16)$$

stabilizes the switched system (1)-(2) with an $\mathcal{H}_\infty$ guaranteed cost γ and imposes to each linear mode the pole location given in Figure 1.

Proof: Condition (15) comes from [20], replacing A by $(A + \sigma\mathbf{I})/r$ and B_2 by B_2/r, taking into account the respective indices. Thus, condition (15) guarantees the stability of

$$x_{k+1} = \Big(\frac{A_j + B_{2j} K_j + \sigma_j \mathbf{I}}{r_j}\Big) x_k \qquad (17)$$

with an $\mathcal{H}_\infty$ guaranteed cost γ. □

Theorem 1 deserves some remarks.

- i) Theorem 1 is a sufficient LMI condition to compute switched state feedback gains that ensure properties P1) and P2) to the closed-loop switched linear system. Notice that the matrices $(A_j, B_{1j}, B_{2j}, C_j, D_{1j}, D_{2j})$, $j = 1,\ldots,N$ are the data of the problem. The parameters σ_j and r_j, $j = 1,\ldots,N$ are chosen by the control designer and the matrices G_j, S_j, Z_j, $j = 1,\ldots,N$ and γ are the variables of the

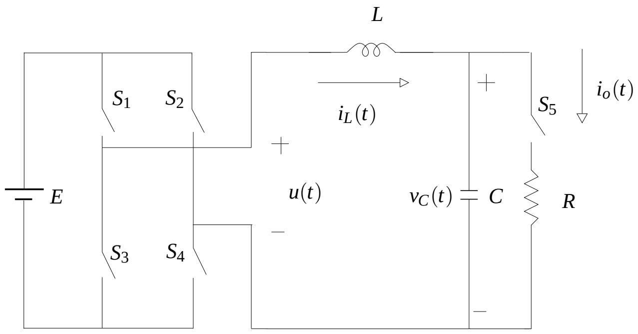

Fig. 2. Basic topology of a single-phase UPS with an arbitrary switching load.

problem, which can be automatically determined in polynomial time using LMI solvers, as for instance, the LMI Control Toolbox of Matlab [23].

- ii) Conditions of Theorem 1 can also provide a robust gain K, by using fixed matrices $Z_j = Z$, $G_j = G$.
- iii) Differently from [20], B_{2j} is supposed a switched matrix (i.e. each linear subsystem has its own control input matrix).
- iv) Structural constrained control can be addressed by Theorem 1, by simply imposing specific structures (as block diagonal matrices, for instance) to the matrices Z_j and G_j, $j = 1,\dots,N$. Notice that in this case, it is not necessary to impose any constraint to the matrices S_j, $j = 1,\dots,N$, that are the matrices of the Lyapunov function that ensures stability. This allows to reduce the conservatism in problems of structured constrained control.

V. UPS APPLICATION

Consider the circuit of the single-phase UPS system with a switched load, shown in Figure 2. Notice that the resistor R in series with the ideal switch S_5 is an arbitrary switching load.

Defining the state variables $x(t) = [v_C(t) \quad i_L(t)]'$, the UPS system of Figure 2 can be modeled as

$$\dot{x}(t) = A_{cj}x(t) + B_{cj}u(t) \quad ; \quad j = 1,2 \tag{18}$$

with

$$A_{c1} = \begin{bmatrix} 0 & 1/C \\ -1/L & 0 \end{bmatrix} \; ; \; B_{c1} = \begin{bmatrix} 0 \\ 1/L \end{bmatrix} \; ; \; S_5 \text{ off} \tag{19}$$

and

$$A_{c2} = \begin{bmatrix} -1/(RC) & 1/C \\ -1/L & 0 \end{bmatrix} \; ; \; B_{c2} = \begin{bmatrix} 0 \\ 1/L \end{bmatrix} \; ; \; S_5 \text{ on} \tag{20}$$

In order to apply a discrete-time control scheme, a zero-order-hold method with sampling period T_s has been used. Then, system (18) becomes

$$x_{k+1} = A_j x_k + B_j u_k \tag{21}$$

with

$$x_k = [v_{C_k} \quad i_{L_k}]' \tag{22}$$

and

$$A_j = \exp(A_{cj}T_s) \quad ; \quad B_j = (\exp(A_{cj}T_s) - \mathbf{I})A_{cj}{}^{-1}B_{cj} \tag{23}$$

Define the tracking error e_k as

$$e_k = r_k - x_{1_k} \tag{24}$$

with r_k being a sinusoidal reference signal and

$$x_{3_{k+1}} = x_{3_k} + e_k \tag{25}$$

represents an integral action over the tracking error to be incorporated to system (21) to improve its steady state response. Including also the effects of disturbances $w(t)$, the augmented system is given by

$$\bar{x}_{k+1} = \bar{A}_j\bar{x}_k + B_{1j}w_k + \bar{B}_{2j}u_k + B_r r_k \tag{26}$$

$$z_k = \bar{C}_j\bar{x}_k + D_{1j}w_k + D_{2j}u_k \tag{27}$$

with

$$\bar{A}_j = \begin{bmatrix} A_j & \mathbf{0} \\ H & 1 \end{bmatrix} \quad ; \quad H = \begin{bmatrix} -1 & 0 \end{bmatrix} \tag{28}$$

$$\bar{B}_{1j} = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} \; ; \; \bar{B}_{2j} = \begin{bmatrix} B_j \\ 0 \end{bmatrix} \; ; \; B_r = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{29}$$

$$\bar{C}_j = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \; ; \; D_{1j} = \begin{bmatrix} d_{11} \end{bmatrix} \; ; \; D_{2j} = \begin{bmatrix} 0 \end{bmatrix} \tag{30}$$

with the parameters given in Table I. The output filter has been designed in order to exhibit a natural frequency 20 times lower than the switching frequency, which is usual in the literature.

TABLE I
NOMINAL PARAMETERS FOR THE UPS SYSTEM.

Nominal output voltage	V_o	$110V_{RMS}$, $60Hz$
Input voltage	E	$250V$
Switching frequency	f_{sw}	$10800Hz$
Sampling period	T_s	$1/10800s$
Output filter inductor	L	$1mH$
Output filter capacitor	C	100μF
Load	R	24Ω

Matrices $(\bar{A},\bar{B})_j$ in (29), determined by (19), (20) and (23) with the parameters of Table I, are given by

$$\bar{A}_1 = \begin{bmatrix} 0.9574 & 0.9128 & 0 \\ -0.0913 & 0.9574 & 0 \\ -1 & 0 & 1 \end{bmatrix} \; ;$$

$$\bar{B}_{21} = \begin{bmatrix} 0.0426 \\ 0.0913 \\ 0 \end{bmatrix} \; ; \; S_5 \text{ off} \tag{31}$$

$$\bar{A}_2 = \begin{bmatrix} 0.9207 & 0.8954 & 0 \\ -0.0895 & 0.9580 & 0 \\ -1 & 0 & 1 \end{bmatrix};$$

$$\bar{B}_{22} = \begin{bmatrix} 0.0420 \\ 0.0913 \\ 0 \end{bmatrix}; \; S_5 \text{ on} \tag{32}$$

Assume that

$$\bar{B}_{11} = \bar{B}_{12} = \begin{bmatrix} 0.2 \\ 0.5 \\ 0 \end{bmatrix} \tag{33}$$

and that

$$D_{11} = D_{12} = \begin{bmatrix} 0.2 \end{bmatrix} \tag{34}$$

$$D_{21} = D_{22} = \begin{bmatrix} 0 \end{bmatrix} \tag{35}$$

Choosing the design parameters $\sigma_1 = \sigma_2 = 0$, $r_1 = r_2 = 0.5$, Theorem 1 yields the controller with gains

$$K_1 = \begin{bmatrix} -28.5637 & -18.8443 & 10.9834 \end{bmatrix} \tag{36}$$

$$K_2 = \begin{bmatrix} -28.1810 & -18.7150 & 11.0902 \end{bmatrix} \tag{37}$$

and $\mathcal{H}_\infty$ attenuation level $\gamma = 0.8812$. Figure 3 shows the response of the discrete-time model of the closed-loop UPS system when the energy signal disturbance w_k is applied. The switch S_5 is on for $60 \leq k < 120$ and it is off otherwise. Observe the fast transient responses at the switching instants $k = 60$ and $k = 120$ and the good attenuation of the disturbance w_k.

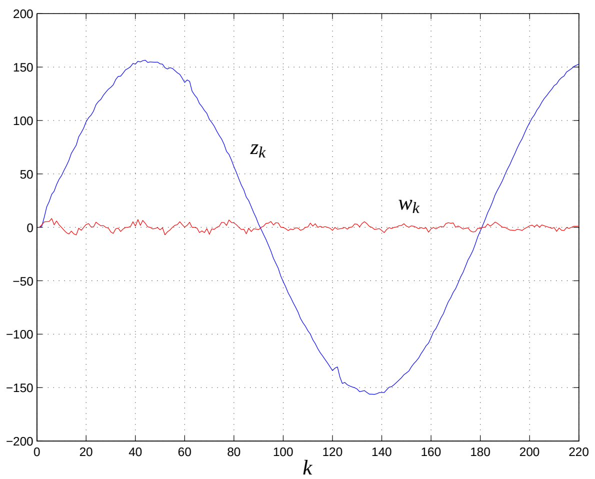

Fig. 3. Response of the UPS system (29)-(35) with switched controller gains (36)-(37). The switch S_5 is on for $60 \leq k < 120$ and it is off otherwise. The design parameters for pole location are $\sigma_1 = \sigma_2 = 0$ and $r_1 = r_2 = 0.5$, and the attenuation level obtained is $\gamma = 0.8812$.

If a less stringent pole location specification is used, as for instance $\sigma_1 = \sigma_2 = 0$, $r_1 = r_2 = 0.9$, Theorem 1 yields the gains

$$K_1 = \begin{bmatrix} -33.7598 & -20.4074 & 4.0156 \end{bmatrix} \tag{38}$$

$$K_2 = \begin{bmatrix} -33.3636 & -20.2708 & 4.0636 \end{bmatrix} \tag{39}$$

which assure a higher attenuation of disturbances measured by $\gamma = 0.3615$, at the price of a slower transient response, as shown in Figure 4.

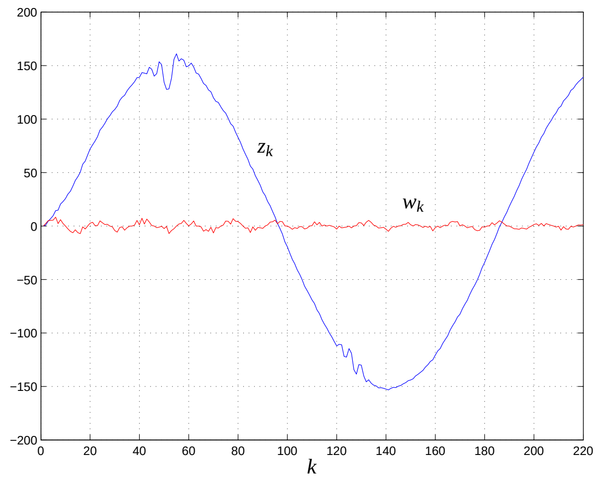

Fig. 4. Response of the UPS system (29)-(35) with switched controller gains given by Theorem 1. The switch S_5 is on for $60 \leq k < 120$ and it is off otherwise. The design parameters for pole location are $\sigma_1 = \sigma_2 = 0$ and $r_1 = r_2 = 0.9$, and the attenuation level obtained is $\gamma = 0.3615$.

Notice that as matrices of the discrete-time model (31)-(32) are similar for the parameters used in this application, the gain (36) is similar to the gain (37) and the same is valid for (38) and (39). As stated in remark ii) of Theorem 1, it is possible to use condition (15) to compute a fixed gain for the same specification of pole location ($\sigma_1 = \sigma_2 = 0$ and $r_1 = r_2 = 0.9$), but the resulting cost is $\gamma_f = 2.1909$, which represents an increase of 24% on the $\mathcal{H}_\infty$ guaranteed cost. Thus, the switched control strategy allows to improve the performance of the system in this case.

Finally, it is interesting to notice that the value of the $\mathcal{H}_\infty$ guaranteed cost given by Theorem 1 depends on the pole location specification. For instance, choosing $\sigma_1 = \sigma_2 = 0$ and $r_1 = r_2 = r$, Figure 5 shows the value of the guaranteed cost γ as a function of the radius r. Observe that the $\mathcal{H}_\infty$ guaranteed cost increases as the pole location becomes more restrictive (i.e., for smaller values of r) and the desired tradeoff must be investigated by the control designer.

VI. Conclusion

This paper presented sufficient LMI conditions for the design of a state feedback controller with switched gains suitable to cope with pole location specifications and to guarantee the rejection of disturbances that are energy signals. Numerical results show a good response for a closed-loop UPS system with an arbitrary switched load, providing a fast transient response and good attenuation of the disturbance, illustrating that the proposed strategy

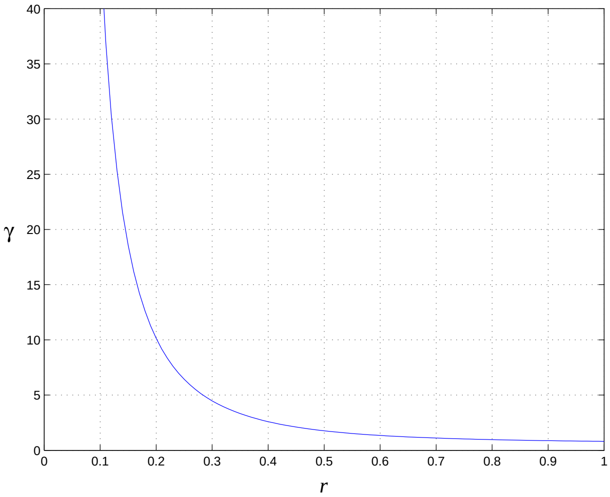

Fig. 5. Value of the $\mathcal{H}_\infty$ guaranteed cost of the UPS system as a function of the radius of a pole location with $\sigma_1 = \sigma_2 = 0$, $r_1 = r_2 = r$.

can be used to improve the closed-loop performance of this class of circuits.

Acknowledgments

The authors would like to thank the Brazilian agencies CAPES, CNPq, FAPEMIG and FAPESP for the financial support. The authors also thank the reviewers for their valuable comments and suggestions.

References

[1] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Systems Magazine*, vol. 19, no. 5, pp. 59–70, October 1999.

[2] A. V. Savkin and R. J. Evans, *Hybrid Dynamical Systems. Controller and Sensor Switching Problems*. Boston, MA: Birkhäuser, 2002.

[3] S. R. Sanders and G. C. Verghese, "Lyapunov-based control for switched power converters," *IEEE Transactions on Power Electronics*, vol. 7, no. 1, pp. 17–24, January 1992.

[4] S. B. Bekiarov and A. Emadi, "Uninterruptible power supplies: classification, operation, dynamics, and control," in *2002 IEEE Applied Power Electronics Conference and Exposition, APEC-2002*, vol. 1, Los Cabos, Mexico, October 2002, pp. 597–604.

[5] *IEEE Recommended Practice for Emergency and Standby Power Systems for Industrial and Commercial Applications:* IEEE-446, IEEE, USA, 1995.

[6] K. P. Gokhale, A. Kawamura, and R. G. Hoft, "Dead beat microprocessor control of PWM inverter for sinusoidal output waveform synthesis," *IEEE Transactions on Industry Applications*, vol. 23, no. 5, pp. 901–910, September/October 1987.

[7] L. Malesani, P. Matavelli, and S. Buso, "Robust dead-beat current control for PWM rectifiers and filters," *IEEE Transactions on Industry Applications*, vol. 35, no. 3, pp. 613–620, May/June 1999.

[8] S. Buso, S. Fasolo, and P. Mattaveli, "Uninterruptible power supply multiloop control employing digital predictive voltage and current regulators," *IEEE Transactions on Industry Applications*, vol. 37, no. 6, pp. 1846–1854, November/December 2001.

[9] H. A. Gründling, E. G. Carati, and J. R. Pinheiro, "A robust model reference adaptive controller for UPS applications," in *1997 IEEE Industry Applications Society Conference, IAS-1997*, vol. 2, 1997, pp. 901–905.

[10] T. Haneyoshi, A. Kawamura, and R. G. Hoft, "Waveform compensation of PWM inverter with cyclic fluctuating loads," *IEEE Transactions on Industry Applications*, vol. 24, no. 4, pp. 582–589, July/August 1988.

[11] V. F. Montagner, E. G. Carati, and H. A. Gründling, "Design and analysis of a linear quadratic regulator with repetitive controller for AC power supplies," in *2003 IEEE International Symposium on Industrial Electronics, ISIE-2003*, vol. 3, Rio de Janeiro, Brazil, June 2003, pp. 109–114.

[12] C. Rech, H. Pinheiro, H. A. Gründling, H. L. Hey, and J. R. Pinheiro, "Comparison of digital control techniques with repetitive integral action for low cost PWM inverters," *IEEE Transactions on Power Electronics*, vol. 18, no. 1, pp. 401–410, January 2003.

[13] Y. Y. Tzou, R. S. Ou, S. L. Jung, and M. Y. Chang, "High-performance programmable AC power source with low harmonic distortion using DSP-based repetitive control technique," *IEEE Transactions on Power Electronics*, vol. 12, no. 4, pp. 715–725, July 1997.

[14] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA: SIAM Studies in Applied Mathematics, 1994.

[15] B. R. Barmish, "Necessary and sufficient conditions for quadratic stabilizability of an uncertain system," *Journal of Optimization Theory and Applications*, vol. 46, no. 4, pp. 399–408, August 1985.

[16] J. Bernussou, P. L. D. Peres, and J. C. Geromel, "A linear programming oriented procedure for quadratic stabilization of uncertain systems," *Systems & Control Letters*, vol. 13, no. 1, pp. 65–72, July 1989.

[17] H. L. S. Almeida, A. Bhaya, D. M. Falcão, and E. Kaszkurewicz, "A team algorithm for robust stability analysis and control design of uncertain time-varying linear systems using piecewise quadratic Lyapunov functions," *International Journal of Robust and Nonlinear Control*, vol. 11, pp. 357–371, 2001.

[18] A. Rantzer and M. Johansson, "Piecewise linear quadratic optimal control," *IEEE Transactions on Automatic Control*, vol. 45, no. 4, pp. 629–637, April 2000.

[19] J. Daafouz and J. Bernussou, "Parameter dependent Lyapunov functions for discrete time systems with time varying parameter uncertainties," *Systems & Control Letters*, vol. 43, no. 5, pp. 355–359, August 2001.

[20] ——, "Poly-quadratic stability and $\mathcal{H}_\infty$ performance for discrete systems with time varying uncertainties," in *Proceedings of the 40th IEEE Conference on Decision and Control*, vol. 1, 2001, pp. 267–272.

[21] J. Daafouz, P. Riedinger, and C. Iung, "Stability analysis and control synthesis for switched systems: A switched Lyapunov function approach," *IEEE Transactions on Automatic Control*, vol. 47, no. 11, pp. 1883–1887, November 2002.

[22] V. F. Montagner, V. J. S. Leite, and P. L. D. Peres, "Robust control of discrete time switched systems with pole location and structural constraints," in *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, HI, USA, December 2003, pp. 6242–6247.

[23] P. Gahinet, A. Nemirovskii, A. J. Laub, and M. Chilali, *LMI Control Toolbox User's Guide*. The Math Works Inc., Natick, MA, 1995.