TWO PHASE VOLTAGE INVERTER WITH THREE LEGS OPERATING IN THE OVERMODULATION RANGE

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***Abstract* – In this paper a two-phase voltage inverter with three legs operating in the overmodulation range is presented. The performance of the system can be improved by increasing the output voltage through the operation in the non linear area. Two solutions are studied and it is also proposed a new one to operate in the non linear area: the elliptical locus. The new solution is able to reduce the harmonic content. All solutions are analyzed, but only the elliptical locus is implemented and the experimental results are shown.**

KEYWORDS

Two-phase voltage inverter, vector modulation, overmodulation.

I. INTRODUCTION

Two-phase voltage inverter with three legs (Fig. 1) has two regions of operation: the linear and the non linear area. The linear area is that the output voltage is equal to the reference voltage multiplied by a constant, i.e., it is defined by a circumference (because it is used two sinusoidal waveforms as references) circumscribed in the non regular hexagon described by the output voltage locus of the inverter. The non linear area is outside this circumference and inside the non regular hexagon, as well.

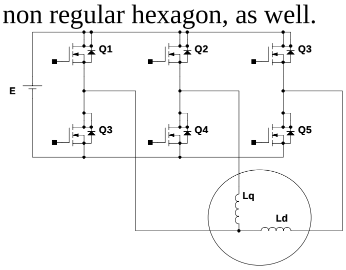


Fig. 1. Two-phase voltage inverter with three legs.

In Fig. 2, it can be noticed the linear and the non linear area. The output voltage available in the linear region decreases when compared with the three phase operation. It is reasonable to say that the non linear region is bigger, in this mode of operation, and the inverter does not use all the voltage capacity installed in the dc bus. So the utilization factor is lower, when compared with the three phase inverter. If it is limited the operation to the linear area, the maximum fundamental output voltage is limited to $1/\sqrt{2}$pu (normalized by the mean dc link voltage). As example, if it is used a single-phase full wave rectifier the maximum dc link voltage is about $\sqrt{2}Vin_{ef}$. So, the maximum output voltage, per phase, is about $(1/\sqrt{2})Vin_{ef}$. In this manner, the driver is not using all the capacity of the line voltage. If it is used a motor with nominal voltage equal to line voltage, it is observed that it is no possible to obtain the maximum output torque in all the frequency range, when it is using the two phase voltage inverter. This loss decreases the dynamic performance of the system. So, the study of the inverter operation in the non linear area is important for some industrial applications, because it is possible to increase the average torque. It is important to note that a torque ripple is also presented when the inverter works in the non linear area. This happens because the phase difference between the two output phase voltages decreases. If the load is a two-phase symmetrical induction motor this means that it works as a single-phase induction motor. It is important to note that it is impossible to keep the phase difference of the output voltage in 90 degrees with this topology due to the output voltage locus.

Analyzing the output voltage locus (Fig. 2), it can see two modes of operation in the non linear area.

The first mode consists in the operation on the non linear area using the technique proposed to three-phase voltage inverter [1] with some modifications (in this paper this technique is referred as first technique). In this case the locus of the output voltage vector is modified to track the non regular hexagon when the amplitude of the reference vector passes outside of the non regular hexagon. It is important to say that, with this technique, low order harmonics will be inserted to the output voltage, due to amplitude distortion. The second mode, proposed here, is based on modifying the phase difference of the two reference voltages. With this situation, the amplitude is not distorted and low order harmonics will not be presented in the output voltage.

It is important to note that the ripple on the torque, in both techniques, comes from the fact that the phase difference of the output voltages decreases. So the best way to work in the non linear area is using the elliptical locus. Both techniques are analyzed using vector modulation.

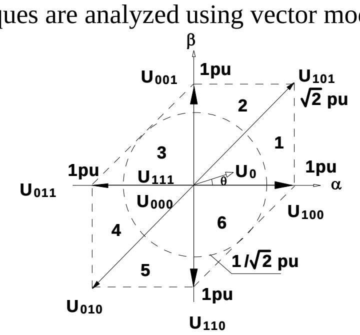


Fig. 2 – Basic vectors and switching patterns with normalized values.

II. TWO-PHASE VOLTAGE INVERTER OPERATING IN THE SIX STEP MODE

This mode of operation is the limit of operation on the non linear area. In this case the switching vector, which is closest to the reference vector, is holding on. Thus, the output voltages have a rectangular waveform. The output vector has only discrete states. To obtain the output voltages it is needed to draw the locus of a circumference of radius equal to the vertices who presents the biggest amplitude (Fig. 2). It is used only the discrete state closed to reference vector. So, from -45° to 22.5° it is used the state 100, from 22.5° to 67.5° 101, from 67.5° to 135° 001, from 135° to -157.5° 011, from -157.5° to -112.5° 010 and from -112.5° to -45° 110.

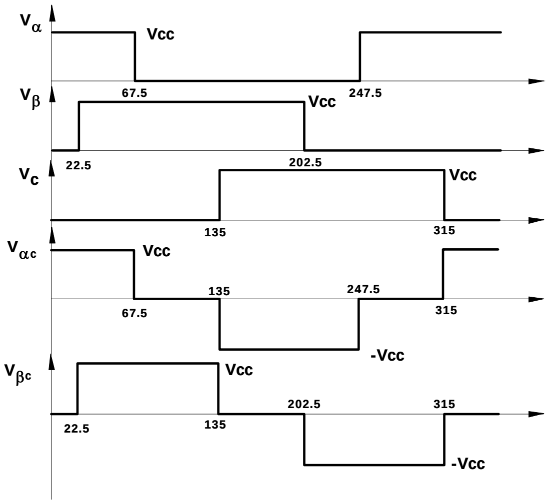


Fig. 3 – Output waveforms in the six step mode.

So, the output voltages could be determined as the differential voltages (Fig. 3). Appling the Fourier Analyses, we have:

$$v_{\alpha c}(t) = \frac{2V_{cc}}{\pi}\sum_{k=1}^{\infty}\frac{\mathrm{sen}(0.982k)+\mathrm{sen}(2.16k)}{k}\cos(k\omega t-12.25) \quad (1)$$

For the other differential output voltage, just the phase is changed. For k=1, the fundamental amplitude is about 1.059V, this means that the RMS output voltage is the same of the RMS voltage of the line. It is important to observe that low order harmonics are presented, so the torque ripple and machine losses increase in this mode of operation. The total harmonic distortion (THD), in this situation, is 0.334.

The modulation index (M) is defined as the relation between the RMS fundamental output voltages of the PWM method used (V_{ef1}) and the RMS fundamental output voltages in the six step mode (V_{efqua}) [2]. This index gives a good approximation of how the voltage capacity of the dc bus is being used.

$$M = V_{ef1} / V_{efqua} \quad (2)$$

III. OVERMODULATION OPERATION – FIRST TECHNIQUE

In this case the amplitude and the angle of the reference vector are modified. So, the reference vector:

$$\underline{U}_0^* = U_0 e^{j\alpha} \quad (3)$$

is modified to:

$$\underline{U}_p^* = U_p e^{j\alpha_p} \quad (4)$$

In this manner, the locus of the reference vector is modified to track the non regular hexagon and after it is used in the PWM modulator. There are three operations mode in this technique: conventional modulation, overmodulation mode I and overmodulation mode II [1].

A. *Conventional modulation*

In all the quadrants the reference vector is used in the PWM modulator [1, 3, 4].

B. *Overmodulation Mode I*

When the reference vector reaches the modulation index of 1/√2pu, in the quadrants II and IV, it is modified in magnitude. For the others two quadrants the reference vector is unaltered. So, the first step is to study the operation in this mode.

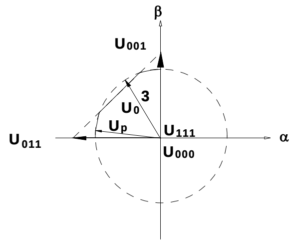


Fig. 4 – Operation on the second quadrant in the overmodulation mode I.

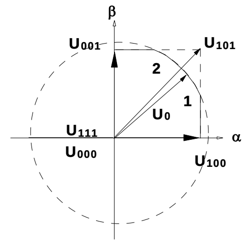


Fig. 5 – Operation on the first quadrant in the overmodulation mode I..

The analysis is made just in the second quadrant. The results could be extended to the fourth quadrant. The hatched circumference is the locus of the reference vector wanted and the solid line is the locus of the reference vector modified (Fig. 4). It can be noticed that in some regions the output voltage is in the linear area and the time calculations of the vectors applied to output are made in the same method for the conventional operation. When the locus of the reference vector is outside of the non regular hexagon, just the two adjacent vectors are consecutively used, without the null vectors. So, in this situation, the locus tracks the non regular hexagon. The times associated to the adjacent are obtained from:

$$t_1 = T_s \frac{\cos(\theta-90)}{\mathrm{sen}(\theta)-\cos(\theta)} \quad (5)$$

$$t_2 = T_s - t_1 \quad (6)$$

On the fourth quadrant the expressions are still valid, but the equation (5) is used to compute t_2 and (6) to calculate t_1. Fig. 6 presents the normalized times for the vectors as a function of the angle of reference vector. The expression (5) is simplified:

$$t_1 = T_s\left(1 - \frac{2\left(\theta - \pi/2\right)}{\pi}\right) \quad (7)$$

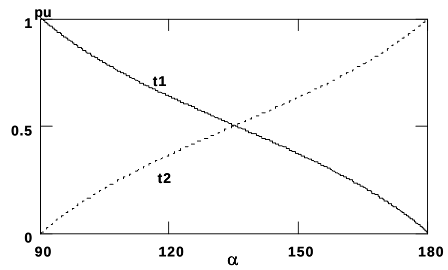


Fig. 6 – Normalized times for the vectors as a function of the angle of reference vector.

The switching period, T_s, must be corrected so that the number of commutations is kept constant. This mode of operation reaches its limit when the reference vector tracks all the side of the non regular hexagon in the second (and fourth) quadrant. In this manner the maximum output voltage is obtained in this mode.

The locus of the modified reference vector for different values of the reference vector is shown in Fig. 7. The normalized fundamental output voltage as a function of the reference voltage is presented in Fig. 8. With this technique the output voltage increases from 0,707pu to about 0,91pu.

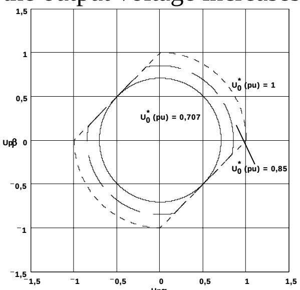


Fig. 7 – Locus of the modified reference vector for different values of the reference vector.

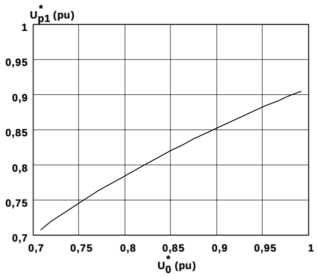


Fig. 8 – Normalized fundamental output voltage as a function of the normalized reference voltage.

When the reference vector reaches the magnitude of 1pu, it has its magnitude modified on quadrants I and III. In the quadrants II and IV the reference vector tracks the sides of the non regular hexagon (Fig. 5). In this case the operation is identical to the operation in the quadrants II and IV. The same considerations adopted there are valid here, but here there are two sectors in each quadrant. The times associated to the adjacent vectors in the first sector are computed by:

$$t_1 = T_s\left(1-\tan(\theta)\right) \tag{8}$$

$$t_2 = T_s - t_1 \tag{9}$$

For the second sector:

$$t_1 = T_s\left(1-\frac{1}{\tan(\theta)}\right) \tag{10}$$

$$t_2 = T_s - t_1 \tag{11}$$

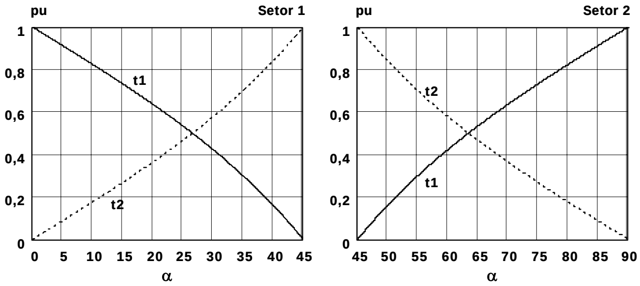


Fig. 9 – Normalized times for the vectors as a function of the angle of reference vector for sectors one and two.

Fig. 9 present the normalized times for the vectors as a function of the angle of reference vector for sectors one and two. On the third quadrant the functions are identical to the first, so that the fourth sector is equal to the first and the fifth is equal to the second. The expression could be simplified:

$$t_1 = T_s\left(1-\frac{2\theta}{\pi}\right) \tag{12}$$

When the locus of modified reference vector tracks all sides of non regular hexagon, this mode of operation reaches its limits. The locus of the modified reference vector for different values in reference vector is shown in Fig. 10.

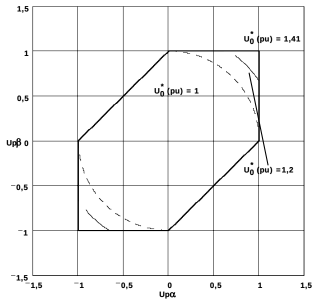


Fig. 10 – Locus of the modified reference vector for different values in reference vector.

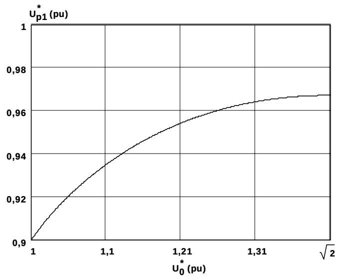


Fig. 11 – Normalized fundamental output voltage as a function of the reference voltage.

The normalized fundamental output voltage as a function of the reference voltage is presented in Fig. 11. With this technique the output voltage increases from 0.91pu to about 0.967pu.

C. Overmodulation mode II

In this mode of operation, the reference vector is modified in angle and amplitude in a manner that a transition between the continuous loci (the locus is the sides of the non regular hexagon) to the discrete locus occurs. So, in the discrete locus the modified reference vector change its value by steps. These steps are the vertices of non regular hexagon. When the reference vector reaches √2pu, the inverter passes from mode I to mode II. In mode II, the modified vector is holding over the vertices by a period defined by angle α_h. After this, the modified reference vector tracks the side of the non regular hexagon. In other words, during the time defined by the angle α_h, the modified reference vector is discrete. With this technique a soft transition is obtained between the operation as PWM inverter to step inverter.

$$\begin{cases} \alpha_p = 0 & 0 \le \alpha \le \alpha_h \\ \alpha_p = \dfrac{\alpha-\alpha_h}{\pi/8-\alpha_h}\,\pi/8 & \alpha_h \le \alpha \le \pi/4-\alpha_h \quad \text{Para os setores 1, 2, 4 e 5} \\ \alpha_p = \pi/4 & \pi/4-\alpha_h \le \alpha \le \pi/4 \end{cases}$$
$$\begin{cases} \alpha_p = 0 & 0 \le \alpha \le 2\alpha_h \\ \alpha_p = \dfrac{\alpha-\alpha_h}{\pi/4-\alpha_h}\,\pi/4 & 2\alpha_h \le \alpha \le \pi/2-2\alpha_h \quad \text{Para os setores 3 e 6..} \\ \alpha_p = \pi/2 & \pi/2-2\alpha_h \le \alpha \le \pi/2 \end{cases} \tag{13}$$

It is important to note that the maximum value to α_p is half of the angle between two adjacent vectors in this sector and that it is applied the active vector closed to the reference vector. As the hexagon is non regular, it is defined that for sectors II and IV its value is 2 α_h. So, the magnitude of the modified reference vector is defined by the sides of the non regular hexagon and the angle is defined by(13).

When the angle is equal to π/8 the inverter is operating in step mode. Fig. 12 shows the locus for the modified reference vector. It can be noticed that the existence of line segments, that show the operation in mode 1. It is important to remember that the modified reference vector uses just discrete values in mode 2; this is the reason of the points in the figure. The denomination discontinuous is used in this case due to these discontinuities.

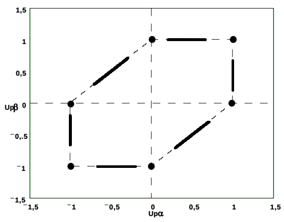


Fig. 12 – Locus of the modified reference vector for different values in reference vector.

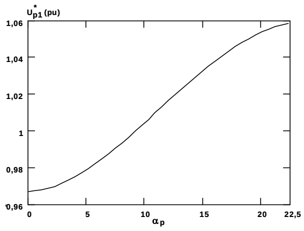


Fig. 13 – Normalized fundamental output voltage as a function of the angle α_h.

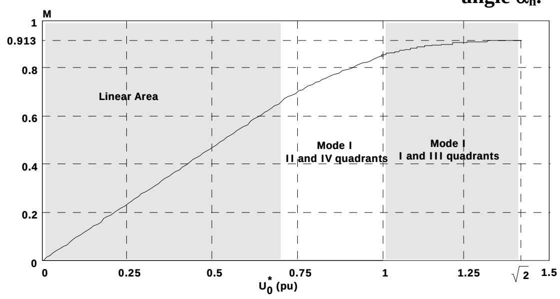


Fig. 14 – Modulation index in function of the reference vector.

The normalized fundamental voltage as a function of angle α_h is from 0.967pu to 1.059pu; this value is obtained in the step mode operation of the inverter. The modulation index defined by (2), could be plotted as a function of the reference vector (

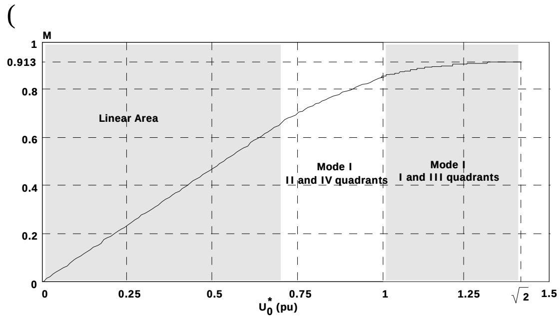


Fig. 14). In mode II the amplitude remains constant and the angle α_h is modified. Fig. 15 presents the value of α_h as a function of modulation index.

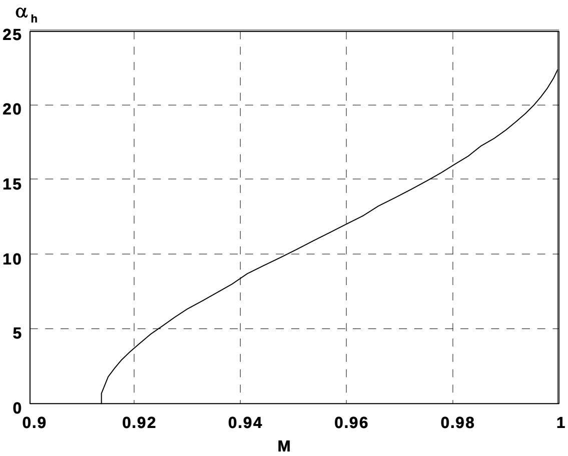


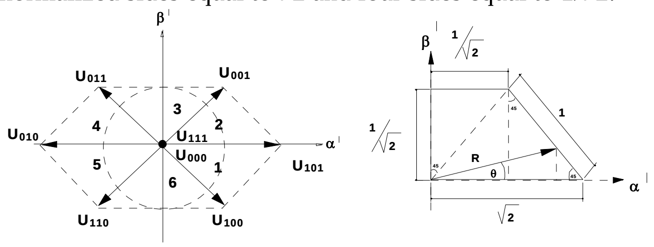
Fig. 15 – α_h as a function of modulation index.

IV. OVERMODULATIO OPERATION – SECOND TECHNIQUE

In the preceded section the reference vector was modified to operate in the non linear area. In this situation a harmonic content was added to the output voltage. To avoid this situation, it is used in this section the elliptical locus technique. In this it is changed the phase difference between the two output voltages. As consequence, the magnitude increases without insertion of low order harmonics. So it is possible distinguish two operations mode:

- Linear mode or circular locus – the reference is not modified and the phase displacement is constant and equal to 90 degrees.
- Elliptical locus – the phase displacement is modified in function of the desired output voltage. Remember that the ellipse is a Lissajous curve.

To study the limits of this strategy it is proposed a linear transformation of the coordinate system in which the new axes have 45 degrees angular displacement from their original position while the origin remains fixed (Fig. 16(a)). After this transformation the major axis is in parallel with the abscissa. In this new coordinates, it is simple to see that the minor axis is limited by the radius of the circumference that limits the linear area. The non regular hexagon has two normalized sides equal to $\sqrt{2}$ and four sides equal to $1/\sqrt{2}$.



(a) (b)

Fig. 16– (a). Basic vectors with a rotation of 45 degrees, (b) Representation of the first quadrant.

The first step is to determine the amplitude of the vector who tracks the sides of the non regular hexagon. For this, it is divided in four identical parts (Fig. 16(b)) and if it is determined for the first, automatically the others are determined due to the symmetry. Using trigonometric relations it is calculated the amplitude (R) of the vector as a function of angle (θ) with the abscissa:

$$R(\theta)=\begin{cases} \dfrac{\sqrt{2}}{\cos(\theta)+\operatorname{sen}(\theta)} & 0\le\theta<\dfrac{\pi}{4} \\ \dfrac{1}{\sqrt{2\left(1-\cos^2(\theta)\right)}} & \dfrac{\pi}{4}\le\theta<\dfrac{3\pi}{4} \\ \dfrac{\sqrt{2}}{\operatorname{sen}(\theta)-\cos(\theta)} & \dfrac{3\pi}{4}\le\theta<\pi \\ -\dfrac{\sqrt{2}}{\cos(\theta)+\operatorname{sen}(\theta)} & \pi\le\theta<\dfrac{5\pi}{4} \\ \dfrac{1}{\sqrt{2\left(1-\cos^2(\theta)\right)}} & \dfrac{5\pi}{4}\le\theta<\dfrac{7\pi}{4} \\ \dfrac{\sqrt{2}}{\cos(\theta)-\operatorname{sen}(\theta)} & \dfrac{7\pi}{4}\le\theta<2\pi \end{cases} \tag{14}$$

Using (14) the curve R_{hex} is obtained (Fig. 17). As the radius of a circumference is constant, its maximum value is obtained direct from the Fig. 17 which is $1/\sqrt{2}$. If it is necessary values greater than this it is used an elliptical locus. As in the preceded technique the reference vector is modified to accomplish this. In polar coordinates the vector limited by the ellipse is determined by:

$$R_{elipse}\left(\theta\right)=\sqrt{\frac{a^2b^2}{a^2\,\mathrm{sen}^2\left(\theta\right)+b^2\cos^2\left(\theta\right)}} \qquad (15)$$

As the minor axis is constant and defined ($2b = \sqrt{2}$), the value of R_{elipse} is a function of the angle θ and of the major axis, 2a. In the modulation proposed, a = b until the reference vector reach the value $1/\sqrt{2}$, after this, *b* is fixed and *a* continuous to increase. It can be noticed that during the interval $\left[\pi/4, 3\pi/4\right] \cup \left[5\pi/4, 7\pi/4\right]$, the major axis can be any value between 0 and infinite. In the limit the ellipse track the sides of non regular hexagon. Due to the symmetry, to find the limit value of the major axis it is enough to analyze the interval $\left[0, \pi/4\right[$. Defining the variable ω, that represent the relation between the major axis and the minor axis (ω=a/b), and its minimum value is equal to one, replacing it on equation (15), subtracting from (14), equating to zero and isolating ω:

$$\omega=\sqrt{\frac{2\cos^2\left(\theta\right)}{b^2\left(1+2\cos\left(\theta\right)\mathrm{sen}\left(\theta\right)\right)-2\,\mathrm{sen}^2\left(\theta\right)}} \qquad (16)$$

This expression determines the maximum value for ω, as a function of θ. With this expression is possible define the maximum reference vector without it passes the non regular hexagon. Considering that the minor axis is equal to $1/\sqrt{2}$, the maximum value for major axis is $\sqrt{3}$. Replacing this value on (15) it is possible to obtain the curve R_{elipse} (Fig. 17).

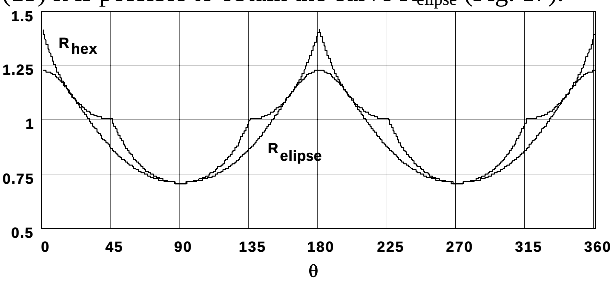


Fig. 17 – Vector amplitude for the elliptical locus (R_{elipse}) and for the non regular hexagon (R_{hex}).

In the preceded analysis the phase difference was replaced by an asymmetry in the amplitudes. In this moment it is used the inverse transform to return to original axis:

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=\begin{bmatrix}\mathrm{sen}\left(\phi\right) & -\cos\left(\phi\right)\\ \cos\left(\phi\right) & \mathrm{sen}\left(\phi\right)\end{bmatrix}\begin{bmatrix}\alpha'\\ \beta'\end{bmatrix} \qquad (17)$$

Using this transformation on the Cartesians representation of the ellipse and replacing ϕ (45 degrees), it is found the expressions:

$$\alpha\left(\theta\right)=b\sqrt{\tfrac{1}{2}\left(1+\omega^2\right)}\,\mathrm{sen}\left(-\theta+\tan^{-1}\left(\omega\right)\right) \qquad (18)$$

$$\beta\left(\theta\right)=b\sqrt{\tfrac{1}{2}\left(1+\omega^2\right)}\,\mathrm{sen}\left(-\theta+\tan^{-1}\left(\tfrac{1}{\omega}\right)+\tfrac{\pi}{2}\right) \qquad (19)$$

Using (18) and (19), one can conclude that the magnitudes of the reference voltages are equal, but the phase difference are not constant.

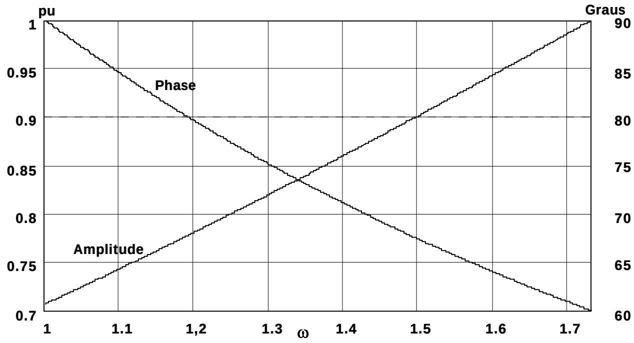


Fig. 18 – Normalized amplitude value and phase difference as a function of ω.

Fig. 18 presents the normalized amplitude voltage and the phase difference as a function of ω. Both could be approximated by an equation and with this one could determine the phase difference to a particular reference. Using Fig. 18 it is possible isolate ω as a function of the reference voltage by:

$$\omega\left(V_{REF}\right)=2.5V_{REF}-0.767 \qquad (20)$$

Using the equation above it is possible to determine an equation that gives the value for the phase displacement for a required reference voltage:

$$\gamma\left(V_{REF}\right)=\frac{\pi}{2}-0.715\left(\omega\left(V_{REF}\right)-1\right)=2.8342-1.7875V_{REF} \qquad (21)$$

This function gives the phase displacement (γ) between the two output voltages using as independent variable the amplitude (the value is in radians). Fig. 19 presents the curve.

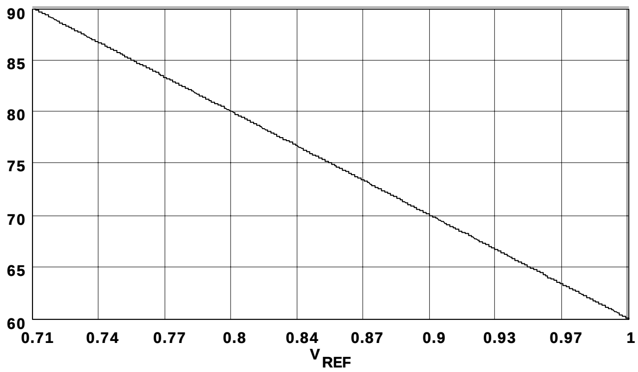


Fig. 19 – Phase relationship between the output sinusoids as function of the output voltage amplitude.

The maximum modulation index with this technique is about 0.944. This value could be obtained using the precede technique when the inverter operate in overmodulation region mode II with 10 degrees. To increase the index modulation it is necessary extend the mode II of operation to the elliptical locus.

V. EXPERIMENTAL RESULTS

A prototype was implemented to study the elliptical locus technique. The switching frequency used was 10kHz (a limitation imposed by the microcontroller). The modulation adopted was the asymmetrical continuous space vector modulation in an open loop fashion [3,4]. It was used a second order filter to observe just the fundamental of the output voltage. Some experimental waveforms are presented for different modulation indexes. In the linear region (Fig. 7

and Fig. 8) the trajectory is circular as expected. As the indexes increase the trajectory is elliptical (Fig. 9) and tends to track the sides of the non regular hexagon (Fig. 10). The dc link voltage, during the measures, is 30V. The load was a resistance in series with an inductance.

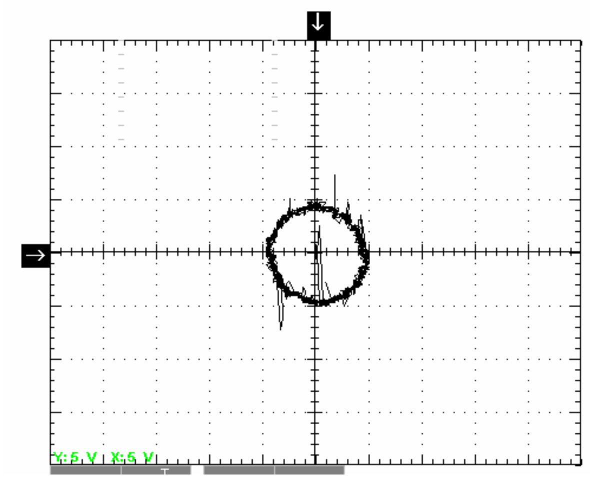


Fig. 20 – Output voltage on DQ plane with M = 0.16.

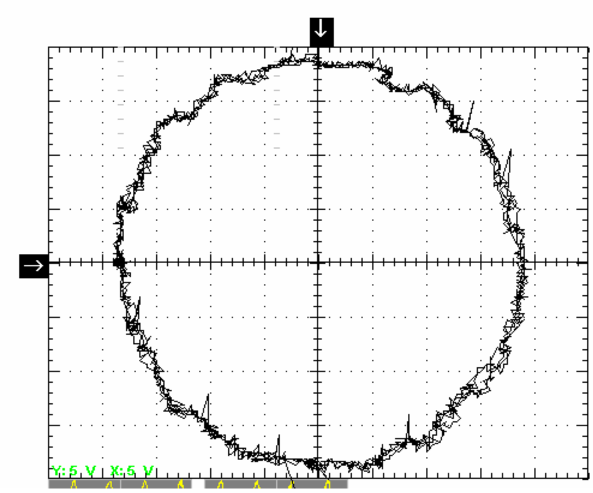


Fig. 21 - Output voltage on DQ plane with M = 0.64.

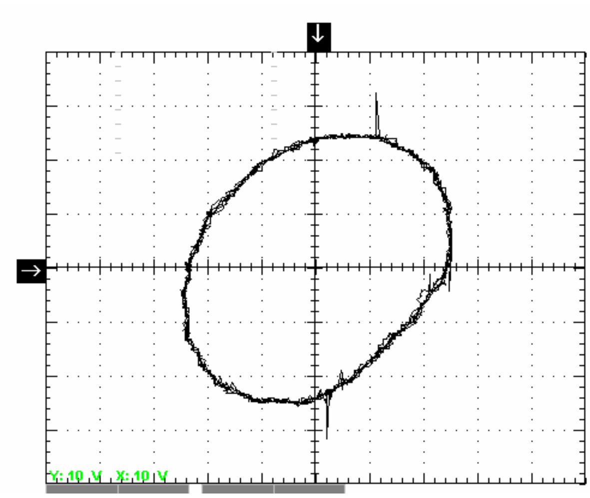


Fig. 22 - Output voltage on DQ plane with M = 0.8.

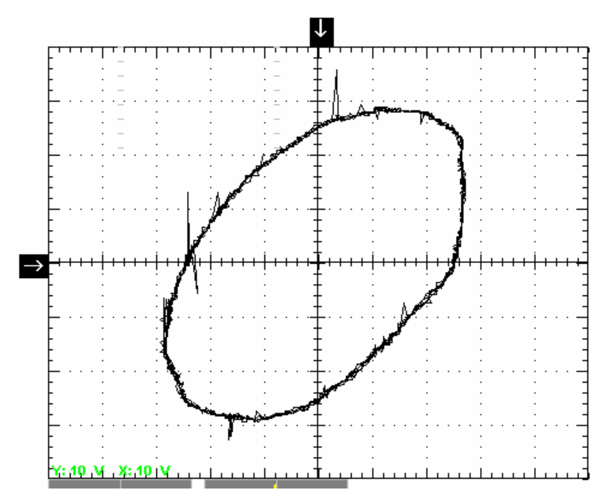


Fig. 23 - Output voltage on DQ plane with M = 0.944.

V. CONCLUSIONS

Both techniques are used to increase the output voltage of the inverter using the non linear region. To accomplish this it is modified the reference vector. The way how the reference vector is modified is the difference between the methods. To compare is used the continuous vector modulation.

The proposed strategy allows using the full voltage available in the dc bus. The main advantage of this method is that no lower order harmonics are inserted in the output voltage; however, the output power has an ac component because the phase displacement is not 90 degrees.

Both the techniques are similar. The difference is that in the technique one it is added low order harmonics.

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