MODELING OF NON-LINEAR LOADS BY SYNCHRONIZED CURRENT SOURCES AND ZIP MODEL.

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***Abstract* – Two distinct types of representation can be suggested in order to model non-linear loads: through reproduction of the original circuits or by parallel current harmonic sources. In this work, the modeling by current sources will be used following the recommendation of the IEEE 519 standard.**

The calculation of the contributions of the current harmonic sources can be made through a single mathematical expression, implemented on a digital program. This strategy allows efficient and fast simulations. The exact model of the original circuits can be impracticable, in general requiring a deep knowledge of the internal components of the equipment.

***Keywords* – Power Quality, Non-linear Load, ZIP MODEL, PLL**

INTRODUCTION

In electrical distribution systems, the significant growth of non-linear loads, mainly observed during the last two decades, has preoccupied several agents of the electrical sector. Especially, the distributors of the electrical energy have tried to analyze the impacts on the quality of its supplied energy with an always crescent number of connections of this kind of load. The availability of digital signal processing platforms with enough accuracy and the development of several user-friendly digital simulators, allow a more detailed representation of the electrical plants, and have favored the researches on power quality.

In this work is proposed a mathematical approach, in order to derive consistent digital models for non-linear electrical loads. The model is based on the IEEE 519 standard [1], and it can be used in systems where the total voltage harmonic distortion is less than 10%. Additionally, a PLL (*Phase Locked Loop*) will be employed as a synchronizing circuit. Furthermore, this work presents the ZIP model as capable of reproducing correctly any type of load.

I. THE LOAD MODEL

Modeling non-linear loads by reproducing the original electrical and electromechanical circuit leads to a significant increase in the complexity of the digital simulation, and the analyses of the results. Moreover, there is a great difficulty in obtaining the original corresponding circuit of an equipment, for the manufacturers do not have commercial interest in spreading this kind of information. Additionally, the application of this strategy in the representation of composed loads, such as commercial offices, residences and small industries can be extremely laborious.

Alternatively, the representation of non-linear loads by equivalent harmonic current sources can be adopted. This strategy can be used when some accurate information is available regarding the harmonic content of the current drained by the load (magnitude and phase of the main harmonic components) in nominal operation conditions. The IEEE 519 standard (utility version) [1] recommends the use of this methodology, in cases for which the maximum harmonic distortion on the voltage bus of interest is equal or inferior to 10%. Equipments capable to directly register the harmonic content of the drained current can be employed. Another possibility would be the application of FFT (Fast Fourier Transform), for an isolated equipment or for a group of equipments. Nevertheless, once the current spectrum is determined, a digital model of the load can be obtained.

Fig. 1 shows the representation of non-linear loads by harmonic current sources, as described previously.

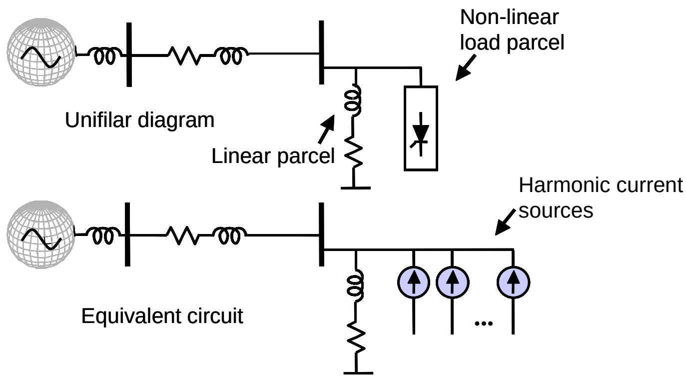

Fig. 1: Non-linear load representation

II. CHARACTERIZATION OF THE DISTURBANCES

For a non-linear load, and considering that the bus voltage is a pure sine wave, the drained current can be decomposed as:

$$v_s(t) = V_s\sqrt{2}\,\sin(2\pi f_o t) \tag{1}$$

$$i_c(t) = i_{cf}(t) + i_{ch}(t) \tag{2}$$

$$i_{cf}(t) = I_{cf}\sin(2\pi f_0 t + \varphi_1) \tag{3}$$

$$i_{ch}(t) = \sum_{h=2}^{\infty} I_{ch}\sin(2\pi h f_0 t + \varphi_h) \tag{4}$$

where:

f_0: fundamental frequency,

V_s: voltage RMS value,

i_{cf}: load fundamental current,

i_{ch}: load harmonic current,
φ: phase angle for each current component.

Fig. 2 shows the simplified electric circuit with the adopted variables.

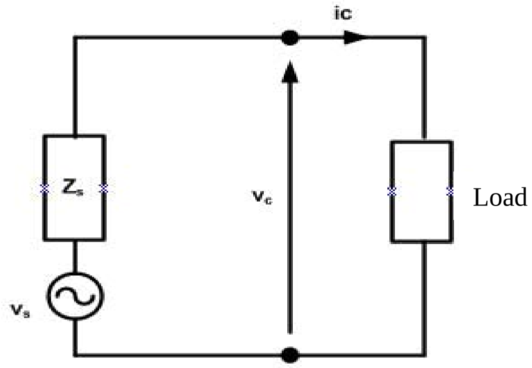

Fig. 2: Modeling example of the electrical network and the load.

$$v_c(t) = v_{c1}(t) + \sum_{h=2}^{\infty} v_{ch}(t), \tag{5}$$

$$v_{c1}(t) = v_s(t) - \left|Z_s^1\right| I_{cf} sin(2\pi f_0 t + \varphi_1 - \varphi_s^1), \tag{6}$$

$$v_{ch}(t) = -\left|Z_s^h\right| I_h sin(2\pi h f_0 t + \varphi_h - \varphi_s^h), \tag{7}$$

v_{c1} are v_{ch} are the voltages for the fundamental and harmonic frequency, respectively. Fig. 3 shows an example of this phenomenon.

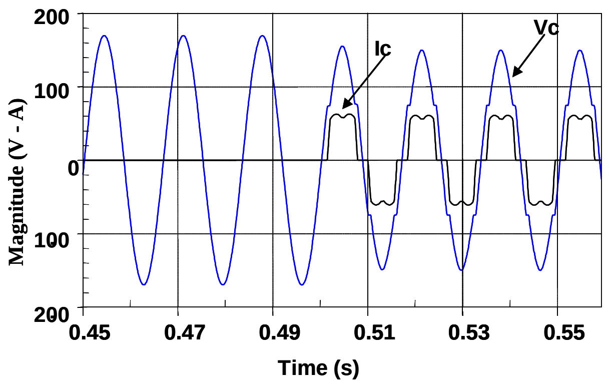

Fig. 3: Voltage and current for a non-linear load.

III. THE SYNCHRONIZATION CIRCUIT

One way to avoid the problem of synchronism is to assume, initially, that the phase angle of the voltage across the load is zero and then refer all the current harmonic components to it. After obtaining the first solution, the actual phases are upgraded in order to follow the new phase angle of the voltage. This process continues until the convergence is achieved.

This method does not lead to expressive discrepancy, but it is not optimized for simulations of larger systems. In those cases it may result in slow convergence rate.

To overcome this difficulty and optimize the simulation process, it is proposed here and additional component to the non-linear load model, which is a PLL (*Phase Locked Loop*), synchronization algorithm [2] for determining in real time the phase angle of the voltage at the load bus. Fig. 4 shows the scheme of a possible implementation of a three-phase PLL circuit. In this in case, the input signals are the phase voltages $v_a(t)$,$v_b(t)$ and $v_c(t)$.

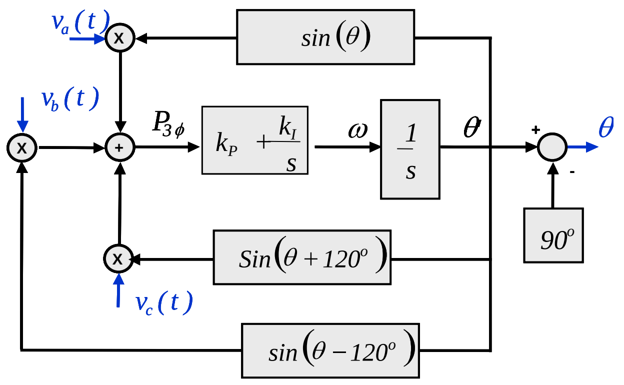

Fig. 4: Synchronized circuit.

Fig. 5 shows the proposed model comprising the PLL algorithm.

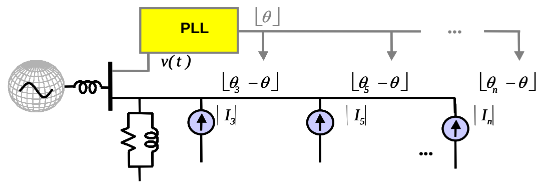

Fig. 5: Proposed non-linear load model.

IV. THE ZIP MODEL

The ZIP Model reproduces the variations of the load according to the voltage oscillations in the electrical network. In most situations, this refinement is enough for the representation of loads at the fundamental frequency.

In electromechanical studies of load flow and transient, for example, equivalent representations are used for a determined consumers group.

Generally simplified models are used, based on the summation of the active and reactive power drained by typical consumers of a representative part of a city. In this case, the active and reactive load power can be divided in three parcels: constant Z, constant I or constant P and Q. Although the simulation will make use of each load model for a specific operating state, several operating conditions can be obtained for further simulations where those conditions would be applicable.

The power on load parcel of type Z varies with the square of the voltage. A variation on the voltage is followed by a corresponding variation on the current, keeping the impedance always constant. The coefficient of this variation is calculated as:

$$K_z = \frac{V_{RMS}}{V_{NOMINAL}}. \tag{8}$$

Concerning the load parcel of type I, the power varies directly with the voltage. In such a case, the current remains a constant (respecting the 5% limits for the voltage variation, as described in the IEEE 1159 standard) [3]. The coefficient of this variation is simply:

$$K_i = 1. \tag{9}$$

The power on load parcels of type P and Q are invariants. In this case, the current varies accordingly with the inverse on the voltage variation. The coefficient of this variation is calculated as:

$$K_p = \frac{V_{NOMINAL}}{V_{RMS}} = \frac{1}{K_z}. \tag{10}$$

Considering the load current as:

$$i(t) = K\left[I_1 sen(\omega t - \theta_1) + I_3.sen(3\omega t - \theta_3) + ... + I_n.sen(n\omega t - \theta_n)\right], \tag{11}$$

where K represent K_z, K_i or K_P according to the type of load. The proposed model takes into account harmonic components up to the 25th order, and it can be used for linear loads also. The linear parcel of the load is represented as a RL series circuit. For such a case, the resulting current has the same characteristics of the input voltage (same THD). Then, the equation (11) can be rewritten as composed by two parts:

$$i(t) = K_1 i_l(t) + K i_{nl}(t), \tag{12}$$

where $i_l(t)$ and $i_{nl}(t)$ represent respectively the linear and non-linear fractions of the current. The value of K_1 is calculated according to the type of load (Z, I, or P and Q).

$$K_1 = \begin{cases} 1 \quad \text{for load type Z,} \\ \dfrac{1}{K_z} \text{ for load type } I, \\ \dfrac{1}{K_z^2} \text{ for load type } P. \end{cases} \tag{13}$$

For a simple series RL load, the voltage is:

$$v(t) = R i_l(t) + L\frac{di_l(t)}{dt} \tag{14}$$

The Laplace transform is calculated as:

$$\frac{I_l(s)}{V(s)} = \frac{1/L}{s + R/L} \tag{15}$$

The transformation from the s domain to the z domain is made by means of the trapezoidal integration as:

$$s \longrightarrow \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right). \tag{16}$$

This approximation guarantees the preservation of the stability in the z domain (the poles are always inside the unit circle).

The discrete version of the equation (15) becomes:

$$\begin{aligned} \frac{I_l(z)}{V(z)} &= \frac{1/L}{\dfrac{2}{T}\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right) + R/L} \\ &= \frac{T(1+z^{-1})}{(2L+TL)-(2L-TR)z^{-1}}. \end{aligned} \tag{17}$$

The following difference equation can then be written:

$$i_l(n) = c_1\left(v(n) + v(n-1)\right) + c_2 i_l(n-1). \tag{18}$$

In (18), c1 and c2 are constant calculated as:

$$\left.\begin{aligned} c_1 &= \frac{T}{2L+RT} \\ c_2 &= \frac{2L-RT}{2L+RT} \end{aligned}\right\}, \tag{19}$$

where T is the sampling period.

Generally, transmission and distribution systems present an inductive behavior for the fundamental frequency. However, other types of load (RC, R, RLC) can be found. Studies for a future representation of these types of load are in progress.

For the computation of L and R, we can replace s by $j\omega$ in equation (15), and compute the voltage as:

$$V(j\omega) = (R + j\omega L)I(j\omega). \tag{20}$$

Considering the voltage phase angle as reference, the modulus and phase of (20) are:

$$\left.\begin{aligned} \left|\frac{V_{NOMINAL}}{I_1}\right|^2 &= R^2 + (\omega L)^2 \\ \frac{(\omega L)^2}{R^2} &= \tan^2(\theta_1) \end{aligned}\right\}. \tag{21}$$

where I_1 and θ_1 are respectively the modulus and phase angle of the fundamental current.

The R and L values are calculated as:

$$\left.\begin{aligned} R &= \frac{V_{NOMINAL}}{I_1\sqrt{1+\tan^2(\theta_1)}} \\ L &= \frac{R\tan(\theta_1)}{\omega} \end{aligned}\right\}. \tag{22}$$

The three types of load are found often operating in the same environment. In order to accommodate this, the values of K and K_1 should be calculated as being the addition of the three parcels as:

$$K = \left(C_z K_z + C_i K_i + C_p K_p\right) \\ K_1 = \left(C_z + \frac{C_i}{K_z} + \frac{C_p}{K_z^2}\right) \Bigg\}, \qquad (23)$$

where *Cz*, *Ci* and *Cp* are coefficients that represent the weight of each load parcel. The summation of the three coefficients must equals 1.

V. EXPERIMENTAL RESULTS

Three types of loads had been implemented experimentally: a three-phase rectifier with RL load; a single-phase rectifier with RL load; and an induction motor. A combination of these three loads was also implemented. The meter ION 7600 by Power Measurement was employed for the data aquisition.

A. THREE-PHASE RECTIFIER

The three-phase rectifier with an RL load was modeled. The weight of each parcel was defined heuristically: 80% of constant impedance and 20% of constant current. The rectifier characteristics are:

$V_{LL} = 160\,V, R = 18,0\,\Omega, L = 44.0\,mH,$

$THDv = 4{,}11\%$ e THDi=26,18%.

Fig. 6 shows the measured spectrum, and Fig. 7 the experimental current and the current obtained from the simulation of the proposed model.

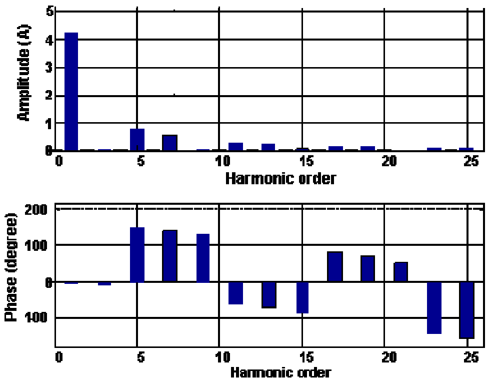

Fig. 6: Spectrum of the input current of a three-phase rectifier with constant load.

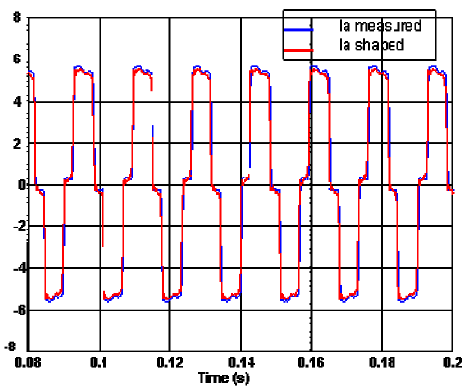

Fig. 7: Waveform of the measured and reconstructed current for a constant load Z-I (three-phase rectifier).

B. Single-phase rectifier

The model of the single-phase rectifier, similarly to the three-phase rectifier, can be represented by a Z+I type of load. Again, the weights have been chosen as 80% and 20% respectively. The rectifier characteristics are:

$V_{an} = 120,0\,V, R = 18,0\,\Omega, L = 44.0\,mH,$

$THDv = 2{,}21\%$ e $THDi$=32,44%.

Fig. 8 shows the measured spectrum, and Fig. 9 the experimental current and the current obtained from the simulation of the proposed model.

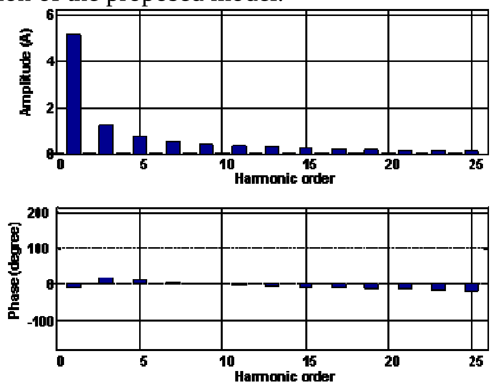

Fig. 8: Spectrum of the input current of a single-phase rectifier with constant load.

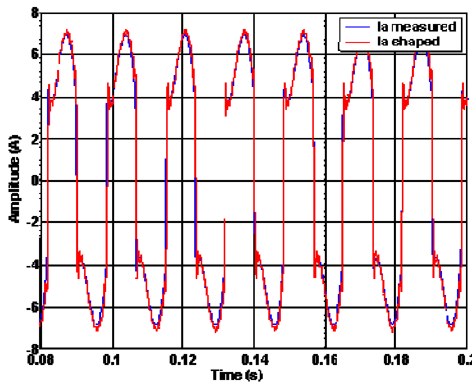

Fig. 9: Waveform of the original and reconstructed current for a constant load Z-I (single-phase).

C. Induction Motor

An induction motor in steady-state was modeled with the following characteristics:

$V_{LL} = 208,0\,V, I_{nom} = 1,5A,$

$THDv = 2{,}04\%$, $THDi$=1,97%.

The machine is modeled as a linear load. Fig. 10 shows the measured spectrum, and Fig.11 the experimental current and the current obtained from the simulation of the proposed model.

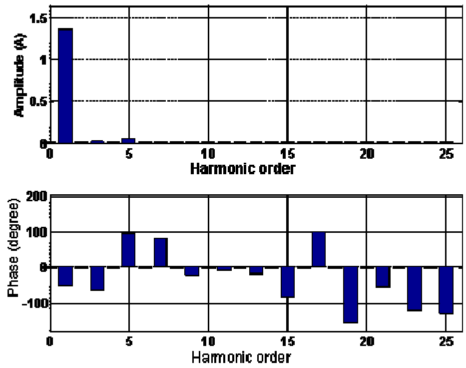

Fig. 10: Current spectrum of a three-phase induction motor.

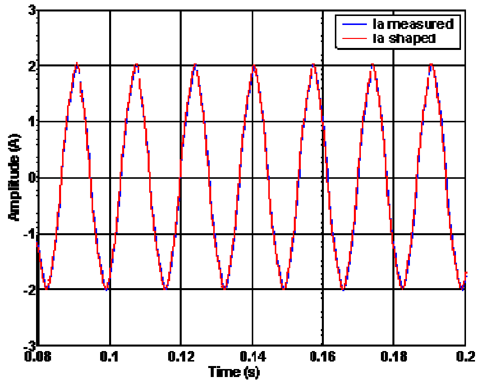

Fig. 11: Waveform of the original and reconstructed current for a three-phase induction machine.

D. Composed load

The results obtained so far, have addressed each load separately. But most of the time, the composed characteristics Z, I and P appear. The next example deal with a system having the three equipments analyzed before.

Fig 12 shows the measured spectrum, and Fig. 13 the experimental current and the current obtained from the simulation of the proposed model. In this study, the constant Z parcel represents 75%, the constant I parcel 15% and constant P parcel represents 10% of the load. As it can be seen in Fig. 13, the non-linear load representation by synchronized current sources can be applied successfully when all the components of the ZIP model should be taken into account.

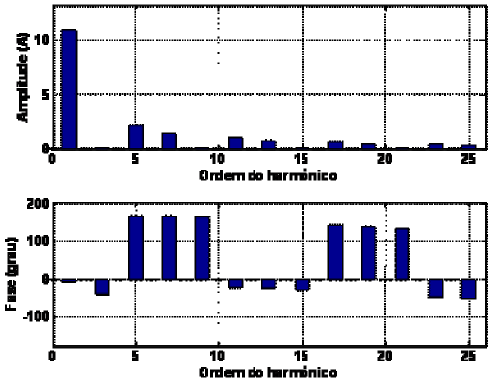

Fig. 12: Spectrum of the composed load.

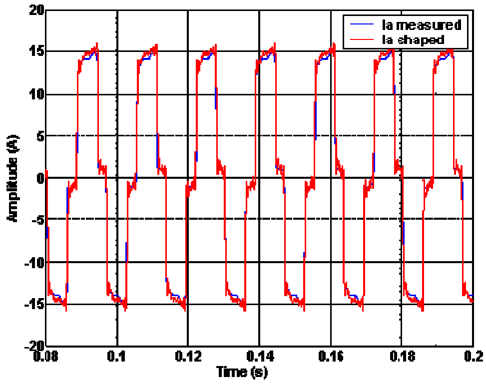

Fig. 13: Waveform of the original and reconstructed current for a ZIP load.

VI. CONLUSION

This work shows that the representation of non-linear loads by equivalents harmonic current sources proves to be an interesting alternative. This strategy can be used when we have information regarding the harmonic content of the current drained by determined load (harmonic magnitude and phase of the main ones).

The proposed non-linear load model employs PLL circuits in order to determine the reference for the phase angle at the load bus. This type of synchronization circuit can be used with three-phase or single-phase systems.

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[3] *IEEE Std 1159-1995: Recommended Practice for Monitoring Electric Power Quality”*, 1995.