FLATNESS-BASED CONTROL OF A LINE CONDITIONER FOR A SINGLE PHASE AC SYSTEM

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***Abstract*— This paper presents the design of a flatness-based controller for a line conditioner for a single phase AC system. Some basics will be given on the concept of flatness-based control. Reconstructors for model parameters will be derived utilizing an algebraic parameter identification algorithm. A flatness-based controller for the capacitor's voltage of the line conditioner will be compared with a linear proportional-integral-derivative controller with a first-order delay element (PIDT$_1$) for the same purpose by simulation.**

***Keywords*— AC/AC, algebraic parameter identification algorithm, flatness-based control, line conditioner**

I. INTRODUCTION

The growing amount of non-resistive, nonlinear loads leads to a couple of problems. For instance, the weaker the supply grid the larger the voltage's distortions caused by a non-sinusoidal load current. More and more, passive and active power filters compensate nonlinear loads in order to fulfill the increasingly strict regulations. Nonetheless, the supplied voltage deviates more and more from the sinusoidal shape. Therefore, so called line conditioners were supposed to improve the total harmonic distortion (THD) of the supplied voltage for sensitive loads.

One topology was published in [6]. It uses the line voltage directly as input voltage of a buck topology in order to generate the necessary voltage for improving the THD of the voltage provided to the load. This topology and a modification was further studied e. g. in [7] and [8], respectively. The analog control of a demonstrator at the Power Electronics Institute is currently replaced by a digital one. Other control algorithms can easily be applied to improve the dynamics of the voltage regulation. One possible approach—the flatness-based control—is introduced in this paper and a controller is designed in order to show the attainable improvements of the control by using this control method.

The concept of differential flatness is explained in [1]. It is based on a mathematical property of the system. The flatness based control uses it to calculate analytically the system's input needed to follow a desired trajectory for the chosen (flat) output of the system. A feedback controller has to be designed to stabilize the trajectory around the nominal trajectory. Already used in the field of mechanical engineering, this approach is still quite unknown in electrical engineering and especially in power electronics. A more comprehensive publication in this field is [4]. There this method is applied to basic DC/DC converters.

The paper is organized as follows: Necessary basics of the flatness based control will be given and a controller will be designed in Section III after the modeling in Section II. The knowledge of main parameters of the system's mathematical model is essential for the quality of the control which can be attained. Therefore, an algebraic identification algorithm is used in Section IV to calculate the unknown parameters. Simulation results from a switched model under MATLAB/Simulink/PLECS are finally presented in Section V.

II. CIRCUIT AND MODEL

The following Fig. 1 shows the ideal circuit of the indirect line conditioner used for the investigations. The output of a

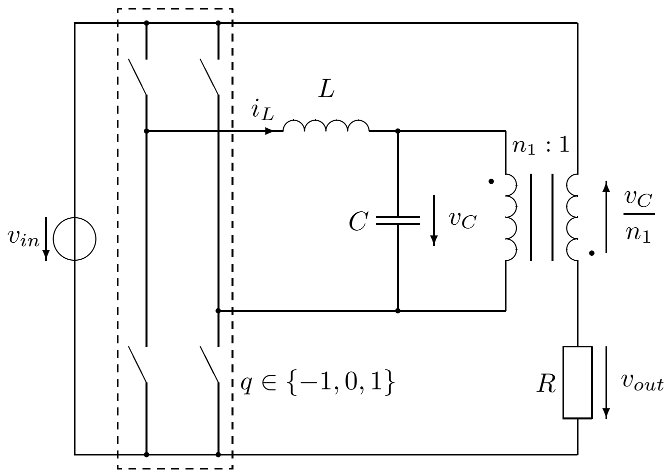

Fig. 1. Lossless electric circuit of the indirect line conditioner

buck converter is connected via a transformer in series between source and load. The input voltage of the buck is the time-varying line voltage v_{in}. It is connected by a full bridge allowing the generation of positive and negative voltages across the output capacitor independently of the present polarity of v_{in}. The buck converter shall provide the necessary voltage to cancel the harmonic content of v_{in}. The control goal is a sinusoidal voltage v_{out} across the load R.

A. Switched model

The system is governed by the following system of ordinary differential equations using the definitions of the currents and voltages as shown in Fig. 1

$$L\frac{\mathrm{d}i_L}{\mathrm{d}t} = -v_C + q v_{in} \,, \tag{1a}$$

$$C\frac{\mathrm{d}v_C}{\mathrm{d}t} = i_L - \frac{v_{in} + \frac{v_C}{n_1}}{n_1 R} \,, \tag{1b}$$

with the switching function—and input of the system—$q \in \{-1, 0, 1\}$ and the winding's ratio $n_1 = v_{prim}/v_{sec}$ of the transformer. The quantities v_{in}, $v_{out} = (v_{in} + v_C/n_1)$ and i_L are assumed to be measured.

B. Continuous model

A continuous model is needed in order to be able to use the property of differential flatness. The model (1) will be averaged over one switching period T_s by leaving T_s tend to zero. It is shown in [9] that there is always a sufficient small switching period, for which the deviations between the responses of the original (switched) system, and those of an averaged or continuous model, under identical initial conditions, remain arbitrarily close to each other.

The equivalent switching function d can take values in the interval $[-1, 1]$. It will be used as the time varying duty ratio in the implementation with finite switching frequency in order to generate the switching signals for the full bridge. The model used for the investigations reads

$$L\frac{\mathrm{d}i_L}{\mathrm{d}t} = -v_C + d\, v_{in} \,, \tag{2a}$$

$$C\frac{\mathrm{d}v_C}{\mathrm{d}t} = i_L - \frac{v_C}{n_1^2 R} - \frac{v_{in}}{n_1 R} \tag{2b}$$

using the same notation for the averaged system variables as in (1).

III. Flatness-based control

A dynamical system is (differentially) flat if:

1) Every component of $\mathbf{y}$ may be expressed as a function of the system variables and of a finite number of their time derivatives.
2) The components of $\mathbf{y}$ are differentially independent, i. e., there is no differential equation of the form $R(\mathbf{y}, \dot{\mathbf{y}}, \ddot{\mathbf{y}}, \ldots, \mathbf{y}^{(\chi)}) = 0$ only in $\mathbf{y}$.
3) Every system variable may be expressed as a function of the components of a finite set $\mathbf{y} = (y_1, \ldots, y_m)$ and of a finite set of their time derivatives. The tuple $\mathbf{y}$ is called a flat output.

The number of components of the flat output is equal to the number of input variables of the system.

As already mentioned, the switched part of the topology can be compared with a buck converter. Its flat output is the capacitor voltage [4]. Therefore the flat output is defined as $y = v_C$ here too. Its time derivatives read as

$$\dot{y} = \frac{i_L}{C} - \frac{v_C}{n_1^2 RC} - \frac{v_{in}}{n_1 RC} \tag{3a}$$

$$\begin{aligned} \ddot{y} &= \frac{1}{C}\frac{\mathrm{d}i_L}{\mathrm{d}t} - \frac{1}{n_1^2 RC}\frac{\mathrm{d}v_C}{\mathrm{d}t} - \frac{1}{n_1 RC}\frac{\mathrm{d}v_{in}}{\mathrm{d}t} \\ &= -\frac{1}{n_1^2 RC}\frac{\mathrm{d}v_C}{\mathrm{d}t} - \frac{v_C}{LC} + \frac{d\,v_{in}}{LC} - \frac{1}{n_1 RC}\frac{\mathrm{d}v_{in}}{\mathrm{d}t} \end{aligned} \tag{3b}$$

If the defined output is a flat output, all system variables and the system's input can be expressed in terms of y, $\dot{y}$ and $\ddot{y}$. This is the case for the model and the output y

$$v_C = y \tag{4a}$$

$$i_L = C\dot{y} + \frac{y}{n_1^2 R} + \frac{v_{in}}{n_1 R} \tag{4b}$$

$$d = \frac{LC}{v_{in}}\ddot{y} + \frac{L}{n_1^2 R v_{in}}\dot{y} + \frac{1}{v_{in}}y + \frac{\dot{v}_{in} L}{n_1 R v_{in}} \,. \tag{4c}$$

Hence, the capacitor voltage v_C is one possible flat output of the system in the sense of [1]. Notice that $v_{in} = 0$ is a singular point for the chosen system's input. The system is not controllable at this point. However, the system is only for a very short time at this point in regular operation and passes it without getting unstable.

The control goal is a sinusoidal voltage across the load $v_{out}(t) = v_{ref}(t) = \hat{V}_{ref} \sin(\omega_N t)$. The desired trajectory y_d of the flat output can easily be formulated $y_d = n_1(v_{ref} - v_{in})$. The time derivatives are given by $\dot{y}_d = n_1(\dot{v}_{ref} - \dot{v}_{in})$ and $\ddot{y}_d = n_1(\ddot{v}_{ref} - \ddot{v}_{in})$. The derivatives of v_{ref} can analytically be calculated, but $\dot{v}_{in}$ and $\ddot{v}_{in}$ are unknown quantities. An algebraic approach will be used in Subsection IV-B to estimate them.

A. Exact feedforward linearization

Assuming that $\dot{v}_{in}$ and $\ddot{v}_{in}$ are estimated correctly, exact feedforward linearization becomes possible [5]. Asymptotic convergence towards the desired trajectory can be forced on the system by introducing a stable error dynamics—the following oscillator (for $(y_d - y)$):

$$0 = \ddot{y}_d - \ddot{y} + k_d(\dot{y}_d - \dot{y}) + k_p(y_d - y) \,, \tag{5}$$

with the constant parameters $k_d, k_p \in \mathbb{R}$, $k_d, k_p > 0$. This equation is solved for $\ddot{y}$ and the result is used in (4c) where y and $\dot{y}$ are substituted by their desired values y_d and $\dot{y}_d$, respectively. The variable now gives information how to choose the equivalent switching function to obtain trajectory tracking

$$\begin{aligned} d =& \frac{LC}{v_{in}}\ddot{y}_d + \frac{L}{n_1^2 R v_{in}}\dot{y}_d + \frac{1}{v_{in}}y_d + \frac{\dot{v}_{in} L}{n_1 R v_{in}} \\ &+ k_d^* \frac{\dot{y}_d - \dot{y}}{v_{in}} + k_p^* \frac{y_d - y}{v_{in}} \,, \end{aligned} \tag{6}$$

with $k_d^* = LC k_d$ and $k_p^* = LC k_p$.

B. Stability

The stability of the proposed controller has to be shown. The differential equations of the system in its desired operating point read

$$L\frac{\mathrm{d}i_{L,d}}{\mathrm{d}t} = -v_{C,d} + d_d\, v_{in}\,, \tag{7a}$$

$$C\frac{\mathrm{d}v_{C,d}}{\mathrm{d}t} = i_{L,d} - \frac{v_{C,d}}{n_1^2 R} - \frac{v_{in}}{n_1 R}\,, \tag{7b}$$

with

$$d_d = \frac{LC}{v_{in}}\ddot{y}_d + \frac{L}{n_1^2 R v_{in}}\dot{y}_d + \frac{1}{v_{in}}y_d + \frac{\dot{v}_{in}L}{n_1 R v_{in}}\,.$$

The error is defined by $\tilde{y} = (y_d - y)$. Subtracting (2) from (7) results in the error system

$$L\frac{\mathrm{d}\tilde{i}_L}{\mathrm{d}t} = -\tilde{v}_C + \tilde{d}\, v_{in}\,, \tag{8a}$$

$$C\frac{\mathrm{d}\tilde{v}_C}{\mathrm{d}t} = \tilde{i}_L - \frac{\tilde{v}_C}{n_1^2 R}\,. \tag{8b}$$

The method of Ljapunov is used to show the stability. The following positive definite Ljapunov-function is chosen

$$\mathcal{V} = \frac{1}{2}\left(L\tilde{i}_L^2 + C\tilde{v}_C^2\right)\,. \tag{9}$$

After one differentiation with respect to time of (9) and substituting (8) it follows

$$\dot{\mathcal{V}} = -\frac{\tilde{v}_C^2}{n_1^2 R} + v_{in}\tilde{d}\,\tilde{i}_L\,, \tag{10}$$

which has to be negative definite.

The input voltage v_{in} can take positive and negative values over a wide range. Therefore it is useful to cancel it. The controller $\tilde{d}$ is chosen $\tilde{d} = -k\tilde{i}_L/v_{in}$ leading to a stable error system.

The current's error $\tilde{i}_L$ can be expressed using (8b) as $\tilde{i}_L = C\dot{\tilde{y}} + \tilde{y}/n_1^2 R$. This leads to

$$\tilde{d} = -\left(kC\frac{\dot{\tilde{y}}}{v_{in}} + \frac{k}{n_1^2 R}\frac{\tilde{y}}{v_{in}}\right) \tag{11}$$

which are the two assumed feedback parts in (6). Now conditions are available to choose the values of k_d^* and k_p^*.

IV. Observer design

A. An algebraic reconstructor for the load

It is possible to design an algebraic reconstructor [2], [3] for the load which calculates the resistance of the load in an algebraic manner. The used algebraic identification algorithm is still quite unknown in power electronics. It enables a fast on-line identification of parameters without the necessity of a discussion of the stability as for asymptotic observers.

It can be done e. g. by using (2b) and integrate it once over the interval $[t - T_B, t]$. The capacitor voltage has still to be substituted by measured quantities $v_C = n_1(v_{out} - v_{in})$. The equation now reads

$$\begin{aligned}\int_{t-T_B}^{t} Cn_1(\dot{v}_{out}(\tau) - \dot{v}_{in}(\tau))\,\mathrm{d}\tau =\\ \int_{t-T_B}^{t} i_L(\tau)\,\mathrm{d}\tau - \int_{t-T_B}^{t}\frac{v_{out}(\tau)}{n_1 R}\,\mathrm{d}\tau\,.\end{aligned} \tag{12}$$

It can now be solved for the unknown parameter R

$$R = \frac{\displaystyle\int_{t-T_B}^{t}\frac{v_{out}(\tau)}{n_1}\,\mathrm{d}\tau}{\displaystyle\int_{t-T_B}^{t} i_L(\tau)\,\mathrm{d}\tau - Cn_1\Big[v_{out}(\tau) - v_{in}(\tau)\Big]_{t-T_B}^{t}}\,. \tag{13}$$

B. Estimation of the derivatives of the input voltage

The algorithm is based on the one described above. The signal (here v_{in}) whose derivatives shall be estimated is locally approximated by a polynomial of desired order

$$v_{in}(\tau) = a_0 + a_1\tau + a_2\frac{\tau^2}{2}\,.$$

Then the third derivative is $v_{in}^{(3)}(\tau) = 0$. This is multiplied by τ^2 and integrated twice for an estimate of the first derivative

$$\int_{t-T_B}^{t}\int_{t-T_B}^{\tau^*}\tau^2 v_{in}^{(3)}(\tau)\,\mathrm{d}\tau\,\mathrm{d}\tau^* = 0\,. \tag{14}$$

Using partial integration the estimate can be calculated using

$$\begin{aligned}\hat{\dot{v}}_{in}(t) = \frac{4v_{in}(t)}{T_B} + \frac{2v_{in}(t-T_B)}{T_B}\\ - \frac{6}{T_B^2}\int_{t-T_B}^{t} v_{in}(\tau^*)\,\mathrm{d}\tau^*\,.\end{aligned} \tag{15}$$

This solution can be used to calculate an estimation of the second derivative. The following equation has to be solved for this purpose

$$\int_{t-T_B}^{t}\tau v_{in}^{(3)}(\tau)\,\mathrm{d}\tau = 0\,. \tag{16}$$

Only one partial integration is needed to get the solution

$$\hat{\ddot{v}}_{in}(t) = \frac{\hat{\dot{v}}_{in}(t) - \hat{\dot{v}}_{in}(t-T_B)}{T_B}\,. \tag{17}$$

C. Implementation Issues

A 'sliding window' of the width $T_B = NT_a$ is used, with the sampling time T_a and the number $N \in \mathbb{N}$ of samples used. This allows to obtain an estimate for every integration step, or in other words, after every sampling step. Without loss of precision, only the values leaving and entering the 'sliding window' of the integral are considered. Therefore the computation time is independent of the number N of samples used. The integral is approximated by a sum using the trapezoidal rule. So, the values of the measured quantities inside the 'sliding window' have to be stored in separate ring buffers.

In (17) there is no need to use the time window T_B. An estimate $\hat{\dot{v}}_{in}(t)$ is available after every sampling step. In order to prevent the necessity of a ring buffer for $\hat{\dot{v}}_{in}(t)$ the approximation

$$\hat{\dot{v}}_{in}(t) = \frac{\hat{\dot{v}}_{in}(t) - \hat{\dot{v}}_{in}(t - T_a)}{T_a} \,. \tag{18}$$

is used requiring only one memory cell.

V. Simulation results

The parameters of the system for the simulations are $\hat{V}_{ref} = 311\,\mathrm{V}$, $f_N = 60\,\mathrm{Hz}$, $f_s = 20\,\mathrm{kHz}$, $L = 570\,\mu\mathrm{H}$, $C = 13.3\,\mu\mathrm{F}$, $T_a = T_s/2$, $T_B = 16\,T_a$ and $P_{out,n} = 10\,\mathrm{kW}$ corresponding to $R_n = 4.84\,\Omega$. The focus lies on the following cases:

1) as reference the "fast" PIDT$_1$ controller from [7]
2) $d = y_d/v_{in}$,
3) $d = y_d/v_{in} + k_p^*(v_{ref} - v_{out})/v_{in}$, $k_p^* = 21$,
4) Equ. (6) with $\ddot{y}_d = k_d^* = k_p^* = 0$ and
5) Equ. (6) with $\ddot{y}_d = 0$, $k_d^* = 0.2\,\mathrm{ms}$, $k_p^* = 21$.

Fig. 2 gives an overview of the simulated scenario. The input voltage $v_{in}(t) = v_{ref}(t) + \tilde{v}_3(t)$ has a third harmonic $\tilde{v}_3(t) = 20\,\mathrm{V}\sin(3\omega_N t - \pi/2)$ from the beginning. A load step occurs at $t \approx 16.6\,\mathrm{ms}$ from $R = 2R_n$ to R_n . At $t \approx 25\,\mathrm{ms}$ a step of the maximum input voltage takes place. Now the input voltage is $v_{in}(t) = 1.2\,v_{ref}(t) + \tilde{v}_3(t)$.

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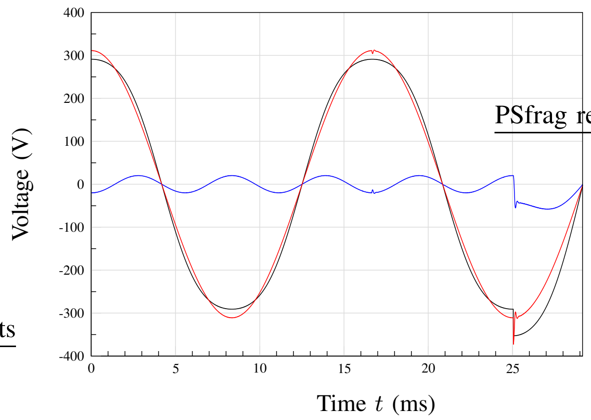

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Fig. 2. v_{in} (black), v_{out} (red) and v_C (blue) of case 5

First, the quality of the feedback control for sudden changes is demonstrated in Fig. 3 for cases 1 and 5 with the step of the input voltage at $t \approx 25\,\mathrm{ms}$. Then, Fig. 4 compares the tracking error in steady state of cases 1 and 5 for the interval $0 < t < 16.5\,\mathrm{ms}$. At last, Fig. 5 looks more closely at the tracking behavior of different flatness-based controllers for the interval $8.3\,\mathrm{ms} < t < 16.5\,\mathrm{ms}$.

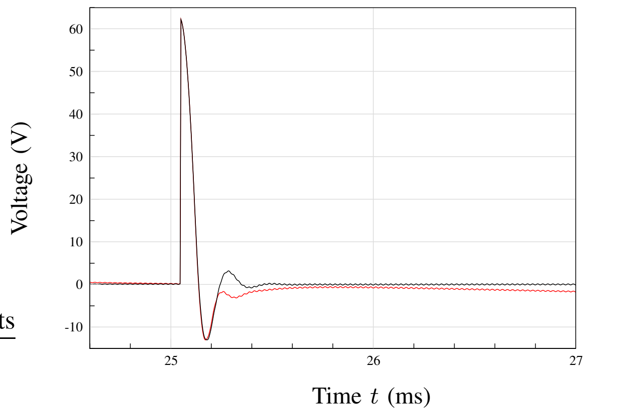

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Fig. 3. Transient of the error voltage $(v_{ref} - v_{out})$ for a $20\,\%$ input voltage step of case 1 (red) and case 5 (black)

As one can see by the error voltage $(v_{ref} - v_{out})$ in Fig. 3, the parameters of the flatness-based controller were chosen to closely follow the waveform of the PIDT$_1$ controller. Therefore, both approaches show nearly the same dynamical response. To be honest, it is very hard to beat a good PID control for the linear buck converter which is the switching part of this topology. But a difference can be seen after the transients are settled. This becomes more obvious if one looks at Fig. 4.

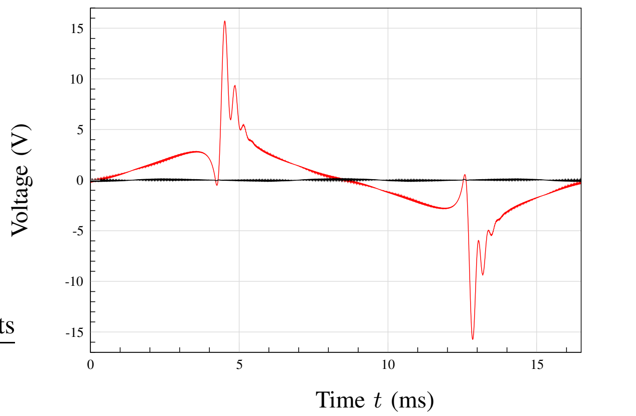

Fig. 4. Error voltage $(v_{ref} - v_{out})$ in steady state of case 1 (red) and case 5 (black)

The error voltage $(v_{ref} - v_{out})$ in steady state of case 1 is approximately 100-times larger then the error voltage of case 5. Especially, the PIDT$_1$ controller has a problem to follow the reference voltage after the zero crossings of the input voltage. This point is better treated by the flatness-based controller.

A flatness-based controller consists usually of two parts. First, a feedforward control which drives the system closely to a desired trajectory, taking the modeled system's dynamics into account. Second, a feedback control which stabilizes the system's trajectory around the desired one. Therefore the zero crossings are already taken into account at the calculation of the system's input from the desired trajectory. The consequences can be seen in Fig. 5.

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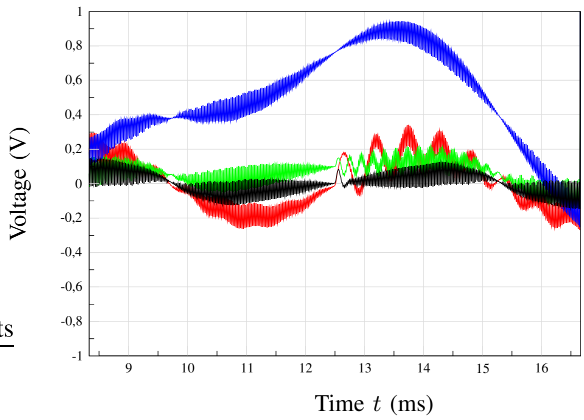

Fig. 5. Error voltage $(v_{ref} - v_{out})$ in steady state of case 2 (blue), 3 (green), 4 (red) and 5 (black)

The error voltage of case 2 (blue curve) can be interpreted as a small phase delay. But compared with the line voltage and the error voltage of case 1, the error of about 1 V is small. The simple usage of the proportional part of the feedforward control already improves the tracking significantly compared to case 1. The error can be decreased by a proportional feedback controller like the waveform of case 3 (green curve) shows, but the characteristic as a phase delay holds.

The feedforward control of case 4 (red curve) with proportional and differential parts centers the error voltage around zero—the tracking is very good. The maximum error is decreased again by the feedback of both parts of the feedforward control as the waveform of case 5 (black curve) shows. The maximum error voltage is only about 0.15 V for this case.

VI. Conclusions

The paper presented the design of a flatness-based controller for the capacitor's voltage of a line conditioner. The stability of the proposed controller was shown. Model-based control methods motivate the usage of estimated quantities for the uncertain main parameters of the system. An algebraic parameter identification algorithm was used in order to calculate this unknown parameters.

The simulation results showed the improvements in the tracking of the desired trajectory by using the flat output and its first derivative for feedforward and feedback control. In this simple case the flat output is directly measurable—it is the capacitor's voltage of the buck topology. Its first derivative is the capacitor's current. Therefore the PIDT$_1$ controller for the capacitor's voltage can do a good part of the job. It already handles sudden changes of the load or input voltage very well.

But it can not do the same as the flatness-based controller because first, it lacks the feedforward control and with it the inherent handling of the modeled system's dynamics. The consequence was visible at the error voltage in steady state. And second, the identification of two important parameters for the calculation of the desired trajectory provides knowledge to further improve the control.

Acknowledgement

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