DESIGN OF ROBUST REPETITIVE CONTROLLERS BASED ON CONVEX OPTIMIZATION WITH AN APPLICATION TO POWER ELECTRONICS

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***Abstract*— This paper provides a procedure to design repetitive controllers which can ensure the tracking of periodic reference signals in control systems whose plants are affected by structured uncertainty. The control design problem is expressed as a convex optimization problem whose objective is to maximize the bandwidth of a low-pass filter in the repetitive controller (leading to the optimization of the tracking performance) under matrix inequality constraints that guarantee the stability for the uncertain closed-loop system. The results rely on Lyapunov-Krasovskii functionals which ensure the stability of uncertain systems subject to delays. Available linear matrix inequality solvers allow to easily cope with the solution of the proposed conditions. An example of application to power electronics is presented to illustrate the efficiency of the proposed control design procedure.**

***Keywords* – Convex optimization; Delayed systems; Lyapunov-Krasovskii functionals; Structured uncertainty; Repetitive control; Robust tracking.**

I. INTRODUCTION

Repetitive control systems have been widely used to ensure good tracking of periodic reference signals and to reject periodic disturbances in several control applications (see, for instance, [5, 10, 11, 14, 16, 17]). It is well known that finding a good tradeoff between stability and tracking performance can be a difficult task when dealing with repetitive control design. Although many works propose heuristic formulations to choose the parameters of repetitive controllers, the determination of the control gains can be based on systematic procedures as for instance those from [9, 18], which do not take into account uncertainties in the model, and those from [8, 14], which cope with uncertain parameters.

However, it is apparent that few systematic studies of design of repetitive controllers based on linear matrix inequalities (LMIs – [2]) have been carried out. It is worth to mention the works in [4, 5]. This motivates deeper investigation on the subject, since the control design expressed in terms of LMIs is very attractive due to the possibility of finding the solution to the control problem in polynomial time by means of interior point based algorithms [6] and also due to the fact that robustness to uncertainties and to disturbances, specifications of bounds for the overshoot, pole location constraints, bounds for the control input, etc. can be easily handled in the LMI framework.

This paper provides sufficient LMI conditions to verify if a chosen bandwidth w_c for a low-pass filter in a repetitive controller ensures the stability of the closed-loop system for a plant with uncertain matrices in a polytope. It is known that, even in the case without uncertainties, as w_c increases, the tracking performance improves. Thus, to obtain the maximum value of w_c such that the uncertain closed-loop system remains stable is of great interest. In order to cope with this issue, a convex optimization problem is also proposed in the paper, given by the maximization of w_c subject to a finite set of matrix inequality constraints that, if satisfied, assure the existence of a Lyapunov-Krasovskii functional that certifies the stability of the uncertain polytopic system [7, 15]. The efficiency of the proposed conditions is illustrated by means of an example dealing with the maximization of w_c for a repetitive controller applied to a control system whose plant is an electrical circuit with an uncertain resistive load, used as a stage in many power converters.

II. PROBLEM FORMULATION

Consider the control system given in Figure 1.

The Laplace transforms of the reference, the system output, the control input and the tracking error are, re-

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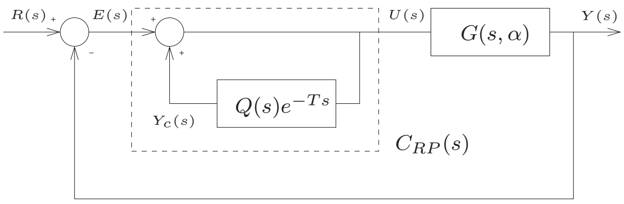

Fig. 1. Closed-loop system with a repetitive controller ($C_{RP}(s)$) and an uncertain plant ($G(s,\alpha)$).

spectively, $R(s)$, $Y(s)$, $U(s)$ and $E(s)$. $G(s,\alpha)$ is the transfer function of a stable SISO compensated uncertain plant with state space representation given by

$$\dot{x}_p(t) = A_p(\alpha)x_p(t) + B_p(\alpha)u(t) \tag{1}$$

$$y(t) = C_p x_p(t) \tag{2}$$

where $A_p(\alpha)$ and $B_p(\alpha)$ belong to the polytope [2]

$$\mathcal{P} = \{(A_p, B_p)(\alpha) \ : \ (A_p, B_p)(\alpha) = \sum_{i=1}^{N} \alpha_i (A_p, B_p)_i \ , \\ \sum_{i=1}^{N} \alpha_i = 1 \ , \ \alpha_i \geq 0 \ , \ i = 1, \ldots, N\} \tag{3}$$

with vertices $(A_p, B_p)_i$, $i = 1, \ldots, N$ known *a priori*. $C_{RP}(s)$ is the transfer function of a repetitive controller, which includes a low-pass filter $Q(s)$ and a time-delay T. Without loss of generality [4], the low-pass filter is assumed to be of first-order, given by

$$Q(s) = \frac{w_c}{s + w_c} \tag{4}$$

The aim of this paper is to provide a convex optimization solution for the following problem.

Problem 1: Determine the maximum value of the bandwidth w_c for the low-pass filter in the repetitive controller such that the closed-loop system is stable for any uncertain plant matrices $(A_p, B_p)(\alpha)$ belonging to the polytope $\mathcal{P}$ (robust stability problem).

It is known that even in the case of the plant without uncertainties, if the bandwidth of $Q(s)$ increases, the ability of the closed-loop system to track periodic references also improves, but the stability margin decreases and *vice-versa*. In order to rewrite Problem 1 as a convex optimization problem, a state space model of the closed-loop system is obtained following similar steps to those in [4], but taking into account that here one deals with structured uncertainty. The equations of the closed-loop system can be written as

$$\dot{\xi}(t) = A(\alpha, w_c)\xi(t) + A_d(\alpha, w_c)\xi(t - T) \tag{5}$$

where

$$\xi(t) = \begin{bmatrix} x_p(t) \\ x_c(t) \end{bmatrix} ,$$

$$A(\alpha, w_c) = \begin{bmatrix} A_p(\alpha) - B_p(\alpha)C_p & \mathbf{0} \\ -C_p & -w_c \end{bmatrix} ,$$

$$A_d(\alpha, w_c) = \begin{bmatrix} \mathbf{0} & w_c B_p(\alpha) \\ 0 & w_c \end{bmatrix}$$

Thus, Problem 1 can be regarded as a problem of computation of the maximum value of w_c such that the system with state delay (5) is stable for any $(A_p, B_p)(\alpha) \in \mathcal{P}$. This problem can be tackled using the approach based on Lyapunov-Krasovskii functionals to assess the stability of systems subject to delay (for details, see for instance [7, 15]).

III. MAIN RESULTS

The next theorem provides an LMI condition to verify if the uncertain closed-loop system with state delay (5) is stable for a given value of w_c.

Theorem 1: Given $w_c > 0$, if there exist symmetric matrices P and S of appropriate dimensions such that

$$P > 0 \tag{6}$$

$$\mathcal{M}_i \triangleq \\ \begin{bmatrix} A_i(w_c)'P + PA_i(w_c) + S & PA_{di}(w_c) \\ A_{di}(w_c)'P & -S \end{bmatrix} < 0 \ , \\ i = 1, \ldots, N \tag{7}$$

where

$$A_i(w_c) = \begin{bmatrix} A_{pi} - B_{pi}C_p & \mathbf{0} \\ -C_p & -w_c \end{bmatrix} ,$$

$$A_{di}(w_c) = \begin{bmatrix} \mathbf{0} & w_c B_{pi} \\ 0 & w_c \end{bmatrix}$$

then the closed-loop system (5) is stable for any $T > 0$, for any $(A_p, B_p)(\alpha) \in \mathcal{P}$.

Proof: Notice that the feasibility of Theorem 1 ensures the existence of $P = P' > 0$ and $S = S' > 0$ such that the functional

$$v = \xi(t)'P\xi(t) + \int_{t-T}^{t} \xi(\theta)'S\xi(\theta)d\theta \tag{8}$$

is positive $\forall T > 0$, $\forall \xi(\theta) \neq \mathbf{0}$, $\theta \in [t - T, t]$ and that its time-derivative along the system trajectories

$$\dot{v} = \dot{\xi}(t)'P\xi(t) + \\ \xi(t)'P\dot{\xi}(t) + \xi(t)'S\xi(t) - \xi(t-T)'S\xi(t-T) \\ = \begin{bmatrix} \xi(t)' & \xi(t-T)' \end{bmatrix} \mathcal{M}(\alpha) \begin{bmatrix} \xi(t) \\ \xi(t-T) \end{bmatrix} \tag{9}$$

with

$$\mathcal{M}(\alpha) = \sum_{i=1}^{N} \alpha_i \mathcal{M}_i \ ,$$

$$\sum_{i=1}^{N} \alpha_i = 1 \ , \ \alpha_i \geq 0 \ , \ i = 1, \ldots, N$$

and $\mathcal{M}_i$ given by (7), is negative for the given value of $w_c > 0$, $\forall [\xi(t)' \ \xi(t-T)']' \neq \mathbf{0}$, $\forall T > 0$, for any $(A_p, B_p)(\alpha) \in \mathcal{P}$. Thus, (8) is a Lyapunov-Krasovskii functional ensuring the stability of the closed-loop system (5) for the given value of w_c, for any arbitrary time-delay $T > 0$, for any $(A_p, B_p)(\alpha) \in \mathcal{P}$. ■

Corollary 1: The global maximum value of w_c such that the closed-loop system has its stability ensured by the conditions from Theorem 1 is obtained by means of the following generalized eigenvalue problem:

$$\begin{aligned} & w_c^* \triangleq \max_{P=P', S=S'} w_c \ \text{s.t.} \\ & w_c > 0 \ , \ (6) \ , \ (7) \end{aligned} \tag{10}$$

Proof: See [2] for generalized eigenvalue problem (convex optimization problem). ■

Remark 1: It is important to emphasize that if the conditions from Theorem 1 have a solution, then the closed-loop system is stable for the given value w_c, for any arbitrary value of $T > 0$, for any $(A_p, B_p)(\alpha) \in \mathcal{P}$ since the solution of Theorem 1 (i.e. matrices P and S) allows to construct a Lyapunov-Krasovskii functional which ensures closed-loop stability independently of the value of delay introduced by the repetitive controller. Notice that a necessary condition for the feasibility of Theorem 1 is that $A_{pi} - B_{pi} C_p$, $i = 1, \ldots, N$ are Hurwitz matrices.

Remark 2: The control design formulation given above is applicable also for uncertain MIMO plants with transfer matrix $G(s, \alpha)$ of dimension $m \times m$. In this case, one has that the low-pass filters in the repetitive loops are described by

$$q(s) = \frac{w_c}{s + w_c} \mathbf{I}_{m \times m}$$

and the proposed conditions can cope with the computation of w_c using matrix variables of appropriate dimension.

IV. DESIGN EXAMPLE

To illustrate the application of the proposed conditions for a plant commonly used in power electronics, the following example is given. The plant to be controlled is the circuit in Figure 2. This circuit is used, for instance,

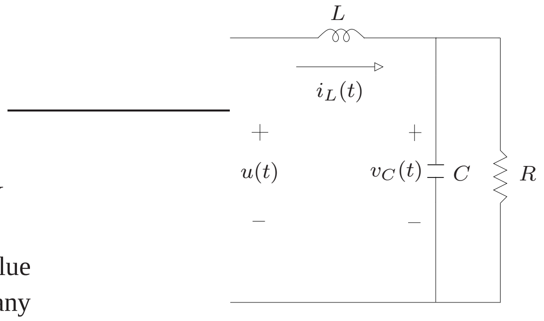

Fig. 2. Electrical circuit used as a stage in UPS and ACPS systems.

as a final stage in uninterruptible power supply (UPS) systems and in AC power source (ACPS) systems. Here, the parameters $L = 5 \ mH$ and $C = 80 \ \mu F$ are borrowed from [3] and R is supposed as an uncertain parameter with $\pm 50\%$ deviation from the nominal value $R = 24 \ \Omega$ given in [3] (i.e. here $12 \ \Omega \leq R \leq 36 \ \Omega$).

First step: plant model.

The uncompensated plant admits a state space representation with vertices given by

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2500000 & -1041.667 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -2500000 & -347.222 \end{bmatrix}$$

and

$$B_{p1} = B_{p2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ , \ C_p = \begin{bmatrix} 2500000 & 0 \end{bmatrix}$$

Second step: robust stabilization of the uncertain plant.

To ensure the fulfillment of the assumption that the SISO compensated uncertain plant is stable, the quadratic stabilizability [1, 2, 12, 13] is used here to obtain the state feedback gains

$$K = \begin{bmatrix} -0.701569 & -0.000082 \end{bmatrix}$$

which guarantee the stability of the plant for the entire domain of uncertain parameters, leading to the dynamic matrix vertices $A_{p1} = A_1 + B_{p1} K$ and $A_{p2} = A_2 + B_{p2} K$.

Third step: computation of w_c.

The convex optimization problem in Corollary 1 is used to compute the maximum value of w_c such that the uncertain closed-loop system with compensated plant has its stability ensured by the Lyapunov-Krasovskii functional (8), yielding $w_c^* = 679$ rad/s.

Fourth step: dynamic simulation of the uncertain closed-loop system.

The dynamic simulation of the closed-loop system for five values of R equally distributed in the interval

$12\ \Omega \leq R \leq 36\ \Omega$ and with the reference signal given by $r(t) = 100\sin(2\pi t)$ is shown in Figure 3. A good tracking performance can be observed for these five different load conditions (the five curves for $y(t)$ in Figure 3 are practically superposed). Other simulations using smaller grid steps in the interval $12\ \Omega \leq R \leq 36\ \Omega$ corroborate the good tracking capacity of the closed-loop system, illustrating the efficiency of the proposed design procedure.

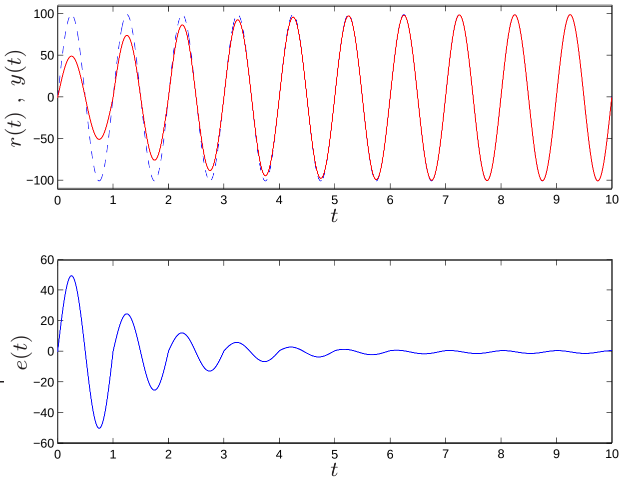

Fig. 3. Responses of the closed-loop system with repetitive controller designed by means of Corollary 1. Top: controlled output $y(t)$ (continuous line) and reference $r(t)$ (dashed line). Bottom: tracking error $e(t)$.

V. CONCLUSION

The main contribution of this paper is to provide a convex optimization problem to compute the maximum value of the bandwidth of a low-pass filter in a repetitive controller applied to a control system with a plant affected by structured uncertainties. The constraints in the optimization problem rely on the existence of a Lyapunov-Krasovskii functional ensuring the closed-loop stability for the entire domain of uncertain parameters. The solution for the proposed control design problem is easily obtained by means of available LMI solvers, without iterative procedures and heuristic methods. An example of application in power electronics illustrates the synthesis procedure.

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