# Power Quality Evaluation Based On Optimum Filtering Theory for Time-Varying Frequencies

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***Abstract*— This paper proposes the kernel algorithm for power quality monitoring. The algorithm is based on optimum filtering theory and are capable to provide several information of the power system grid voltage. It provides information about the fundamental component, harmonics and total harmonic distortion. It also identifies the positive, negative and zero sequence components of the voltages. The instantaneous values of these information and their magnitudes are available. Additionally, instantaneous phase for fundamental voltages, harmonics, positive, negative and zero sequence components are provided. Finally, frequency identification is also focused.**

**Keywords - Harmonic identification, Kalman filter, power quality analysis, phase/magnitude extraction, sequence components identification, time-varying frequency**

## I. Introduction

The continuously use of power electronic devices connected to the power system grid has contributed to the grown of harmonic pollution. This harmonic content constitutes a serious problem to several sensitive loads. Another aspect of the power system is the possibility of existence of voltage unbalance. The unbalance usually appears due the occurrence of unsymmetrical faults or to not perfect balancing current flow in the line leading to voltage components other than the positive sequence. Frequency can also vary over the time due continuous changes in the dynamics of the power systems related to load changes or faults, for instance. Thus, monitoring the status of the power system voltages is a necessity to maintain the power quality within the tolerable limits of pollution or even to carry out studies of the behavior of a line or to compensate perturbations.

Concerning harmonics, the most common approach to detect them is the use of the Fast Fourier Transform (FFT) [1], [2]. This is a popular approach because its simple structure. However, the FFT method relies on some well known assumptions that make it suffers from some shortcomings. For instance, it is sensitive to the variation of the fundamental frequency or sampling rate and during transients, it loses its capacity to decompose adequately the signal in the frequency domain [3], [4].

Some authors has addressed the problem of harmonic detection [5], [6], [7] using a Kalman filter [8], [9]. This method is an optimum method concerning several criteria [10]. However, they considered a fixed grid frequency that are prone to vary in real situations compromising the performance of the filter.

The problem of frequency identification and harmonic detection is considered in [11], [12] where good results are achieved. The proposed method is based on a nonlinear adaptive filter responsible to identify the amplitude, frequency and phase of a sinusoidal signal.

The voltage unbalance detection is usually based on the Symmetrical Components method [13] which is applicable to sinusoidal signals. Some authors address the problem of detecting the sequences under distorted signals [14] and [15] which consider the a constant grid frequency. The positive sequence extraction under time-variable frequency is discussed in [16]. The detection of the steady-state and instantaneous symmetrical components are addressed in [17].

This paper proposes a signal processing algorithm useful for real-time (on-line) power quality evaluation capable to extract as many features as possible from distorted voltage signals. The algorithm is based on the optimum filtering theory. More precisely it relies on a Kalman filter. It minimizes the impact of measurement noise on the signals of interest while keeping the best transient performance. The proposed scheme provides information about the fundamental component of the voltage, the harmonics presented in the signal measured and the total harmonic distortion (THD). Since the unbalance is an important information, the positive, negative and zero sequence components are also extracted. Peak detectors are easily obtained for any voltage, fundamental, harmonics or symmetrical components as will be shown. Instantaneous phase information about the fundamental voltage, harmonics, positive, negative, and zero sequences can also be obtained. For purposes of connection of any device that must be properly synchronized with the grid, synchronization signals are also available. Finally, frequency identification is addressed.

This paper is organized as follow: Section II summarizes the Kalman filter, presents the mathematical model of a signal with harmonics and describes the extraction of information about the components of a distorted signal at the same time that the THD is evaluated. Section III describes the obtainment of the symmetrical components. The generation of synchronization signals and instantaneous phase detection are described in section IV. The frequency identification procedure used is described in section V. Finally, experimental results are given in section VI.

## II. Optimum Extraction of the Harmonic Content of a Signal and THD Determination

The harmonic components of a signal can be optimally extracted using a Kalman filter with an appropriate mathematical model that describes the evolution of that signal. The Kalman filter algorithm is well known and is summarized below.

Consider a dynamic system represented by the following stochastic model

$$x_{k+1} = \Phi_k x_k + \Gamma_k \gamma_k, \tag{1}$$

$$y_k = F_k x_k + \nu_k, \tag{2}$$

$$dim\ x_k = n \times 1,\ dim\ y_k = r \times 1,\ dim\ \gamma_k = p \times 1, \tag{3}$$

where $\gamma_k$ and $\nu_k$ are uncorrelated Gaussian white-noise sequences with means and covariances as follows

$$E\{\gamma_i\} = 0,\ E\{\gamma_i \gamma_j^T\} = Q_i \delta_{ij}, \tag{4}$$

$$E\{\nu_i\} = 0,\ E\{\nu_i \nu_j^T\} = R_i \delta_{ij}, \tag{5}$$

$$E\{\gamma_i \nu_j^T\} = 0,\ E\{\gamma_i x_j^T\} = 0,\ E\{\nu_i x_j^T\} = 0,\ \forall i, j, \tag{6}$$

where $E\{\cdot\}$ denotes the expectation operator and $\delta_{ij}$ denotes the Kronecker delta function. Matrices $\Phi_k$, $\Gamma_k$ and $F_k$ have adequate dimensions.

Denoting by $\hat{x}_{k+1|k}$ the estimate of the state vector $x_{k+1}$, evaluated at the time $t_k$, the filtering equation is [18]:

$$\hat{x}_{k+1|k} = \Phi_k \hat{x}_{k|k-1} + K_k \left( y_k - F_k \hat{x}_{k|k-1} \right) \tag{7}$$

where

$$K_k = \Phi_k P_{k|k-1} F_k^T \left( F_k P_{k|k-1} F_k^T + R_k \right)^{-1} \tag{8}$$

and

$$P_{k+1|k} = \Phi_k P_{k|k-1} \Phi_k^T - K_k F_k P_{k|k-1} \Phi_k^T + \Gamma_k Q_k \Gamma_k^T \tag{9}$$

with given initial conditions $\hat{x}_{0|-1}$ and $P_{0|-1}$. Additionally,

$$P_{k+1|k} \triangleq E\left\{ (x_{k+1} - \hat{x}_{k+1|k})(x_{k+1} - \hat{x}_{k+1|k})^T \right\} \tag{10}$$

represents the covariance of the estimation error of the vector $x_{k+1}$, evaluated at time $t_k$.

The next step to use the Kalman filter, is to model the dynamics of the signal of interest in an appropriated form suited for the filter. As presented in [19] a signal $S_k$ with $n$ harmonic components, that is,

$$S_k = \sum_{i=1}^{n} A_{i_k} \sin(i\omega_k t_k + \theta_{i_k}), \tag{11}$$

where $A_{i_k}$, $i\omega_k$ and $\theta_{i_k}$ are the amplitude, angular frequency and phase of each harmonic component $i$ at the time instant $t_k$, can be modeled in state-space as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix}_{k+1} = \begin{bmatrix} M_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & M_n \end{bmatrix}_k \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix}_k + \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_{2n-1} \\ \gamma_{2n} \end{bmatrix}_k, \tag{12}$$

$$y_k = \begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix}_k + \nu_k, \tag{13}$$

where

$$M_i = \begin{bmatrix} \cos(i\omega_k T_s) & \sin(i\omega_k T_s) \\ -\sin(i\omega_k T_s) & \cos(i\omega_k T_s) \end{bmatrix}, \tag{14}$$

$$x_{(2i-1)_k} = A_{i_k} \sin(i\omega_k t_k + \theta_{i_k}), \tag{15}$$

and

$$x_{2i_k} = A_{i_k} \cos(i\omega_k t_k + \theta_{i_k}). \tag{16}$$

Additionally, $T_s$ is a constant sampling frequency. In (12) it is considered a perturbation vector $[\gamma_1\ \gamma_2\ \cdots\ \gamma_{2n-1}\ \gamma_{2n}]_k^T$ that models amplitude or phase changes in the signal. In (13) $\nu_k$ represents the measurement noise. At the same time that the mathematical model (12)-(14) describes a signal with harmonics, it has the appropriate form necessary for the use in the Kalman filter. Note that the mathematical model needs the knowledge of the fundamental grid frequency ($\omega_k$) that can deviates from its nominal value. If the grid frequency considered in the mathematical model differs from its true value, the estimates provided by the filter will not be accurate. Therefore, its value will be updated by an identification algorithm described later.

From a Kalman filter with the model (12)-(13) it is possible to optimally extract the harmonics of a signal and its in quadrature components. Supposing that $\hat{x}_{1_k|k-1}$ and $\hat{x}_{2_k|k-1}$ represent the estimates of the fundamental component and its in quadrature component then, their amplitude can be determined straightforwardly, that is,

$$v_k^f = \hat{x}_{1_{k|k-1}} = \|V_1\|_k \sin(w_k t_k + \theta_{1_k}), \tag{17}$$

$$v_k^{fq} = \hat{x}_{2_{k|k-1}} = \|V_1\|_k \cos(w_k t_k + \theta_{1_k}), \tag{18}$$

$$\|V_1\|_k = \sqrt{\hat{x}_{1_k|k-1}^2 + \hat{x}_{2_k|k-1}^2}. \tag{19}$$

This approach can be extended to other harmonic components considered in the model (12)-(13). Once the amplitudes of the harmonics are determined, the Total Harmonic Distortion (THD) can also be evaluated in real-time, that is,

$$THD_k = \frac{\sqrt{\sum_{i=2}^{n} \|V_i\|_k^2}}{\|V_1\|_k}. \tag{20}$$

## III. Symmetrical Components Detection

The proposed scheme can also be useful to extract the symmetrical components of an unbalance three-phase grid voltage. To extract the components of positive sequence, the following set of equations is used [20],

$$\begin{bmatrix} v_{a_k}^{+} \\ v_{b_k}^{+} \\ v_{c_k}^{+} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha^2 & 1 & \alpha \\ \alpha & \alpha^2 & 1 \end{bmatrix}\begin{bmatrix} v_{a_k}^{f} \\ v_{b_k}^{f} \\ v_{c_k}^{f} \end{bmatrix} \tag{21}$$

where superscript $f$ stands for the fundamental component and $\alpha = e^{j120^\circ}$. Defining the $90^\circ$ phase-shift operator $S_{90} = e^{j90^\circ}$ and considering $e^{\pm j120^\circ} = -(1/2) \pm (\sqrt{3}/2)e^{j90^\circ}$, then equation (21) becomes

$$\begin{aligned} v_{a_k}^{+} &= \tfrac{1}{3}v_{a_k}^{f} - \tfrac{1}{6}\left(v_{b_k}^{f} + v_{c_k}^{f}\right) + \tfrac{\sqrt{3}}{6}S_{90}\left(v_{b_k}^{f} - v_{c_k}^{f}\right), \\ v_{b_k}^{+} &= -v_{a_k}^{+} - v_{c_k}^{+}, \\ v_{c_k}^{+} &= \tfrac{1}{3}v_{c_k}^{f} - \tfrac{1}{6}\left(v_{a_k}^{f} + v_{b_k}^{f}\right) + \tfrac{\sqrt{3}}{6}S_{90}\left(v_{a_k}^{f} - v_{b_k}^{f}\right). \end{aligned} \tag{22}$$

Since the Kalman filter provides the fundamental and in quadrature components of the signal, the values of $v_{a_k}^{f}$, $v_{b_k}^{f}$, $v_{c_k}^{f}$ and their $90^\circ$ shifted values $S_{90}v_{a_k}^{f}$, $S_{90}v_{b_k}^{f}$ and $S_{90}v_{c_k}^{f}$ are easily obtained without the use of additional filters to shift the fundamental signal.

The amplitude of the positive sequence components can be obtained from

$$\left\|V^{+}\right\|_k = \sqrt{2/3 * \left(\left(v_{a_k}^{+}\right)^2 + \left(v_{b_k}^{+}\right)^2 + \left(v_{c_k}^{+}\right)^2\right)}. \tag{23}$$

To simplify the determination of the negative sequence components, it is used the following relation,

$$v_{i_k}^{-} = v_{i_k}^{f} - v_{i_k}^{+} - v_k^{0}, \quad i = a, b, c \tag{24}$$

where $v_k^0,\ i = a, b, c$ are the zero sequence components of the fundamental signal and can be obtained from

$$v_k^0 = 1/3 * \left(v_{a_k}^{f} + v_{b_k}^{f} + v_{c_k}^{f}\right). \tag{25}$$

The magnitudes of the negative and zero sequence components are given by

$$\left\|V^{-}\right\|_k = \sqrt{2/3 * \left(\left(v_{a_k}^{-}\right)^2 + \left(v_{b_k}^{-}\right)^2 + \left(v_{c_k}^{-}\right)^2\right)} \tag{26}$$

and

$$\left\|V^{0}\right\|_k = \sqrt{\left(v_k^0\right)^2 + \left(S_{90}v_k^0\right)^2} \tag{27}$$

respectively, where,

$$S_{90}v_k^0 = 1/3 * \left(S_{90}v_{a_k}^{f} + S_{90}v_{b_k}^{f} + S_{90}v_{c_k}^{f}\right). \tag{28}$$

## IV. Generation of Synchronization Signals and Instantaneous Phase Detection

Usually, synchronization signals are not so important for purposes of power quality. Nevertheless they are introduced to show that they can be obtained straightforwardly. Based on (17), (18) and (19), the single phase synchronization signals are given by

$$\sin(\phi_{V1_k}) = \frac{\hat{x}_{1_{k|k-1}}}{\|V_1\|_k}, \quad \cos(\phi_{V1_k}) = \frac{\hat{x}_{2_{k|k-1}}}{\|V_1\|_k} \tag{29}$$

where

$$\phi_{V1_k} = \omega_k t_k + \theta_{1_k}. \tag{30}$$

For three-phase systems, the synchronization signals can be obtained from the positive and negative sequence componentes, depending on the interest. To achieve this goals, the positive and negative components are represented in the stationary reference frame, since it provides directly two ortogonal components. These componentes are usually required in practical implementation on digital processors where it is intended to extract the instantaneous phase of the signal, as described below.

$$\begin{bmatrix} v_{\alpha}^{+} \\ v_{\beta}^{+} \end{bmatrix}_k = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a^{+} \\ v_b^{+} \\ v_c^{+} \end{bmatrix}_k \tag{31}$$

and

$$\begin{bmatrix} v_{\alpha}^{-} \\ v_{\beta}^{-} \end{bmatrix}_k = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a^{-} \\ v_b^{-} \\ v_c^{-} \end{bmatrix}_k. \tag{32}$$

Hence, the synchronization signals are

$$\sin(\phi_{Va_k}^{+}) = \frac{v_{\alpha_k}^{+}}{\|v_{\alpha\beta_k}^{+}\|}, \quad \cos(\phi_{Va_k}^{+}) = \frac{v_{\beta_k}^{+}}{\|v_{\alpha\beta_k}^{+}\|}, \tag{33}$$

$$\sin(\phi_{Va_k}^{-}) = \frac{v_{\alpha_k}^{-}}{\|v_{\alpha\beta_k}^{-}\|}, \quad \cos(\phi_{Va_k}^{-}) = \frac{v_{\beta_k}^{-}}{\|v_{\alpha\beta_k}^{-}\|}, \tag{34}$$

where

$$\|v_{\alpha\beta_k}^{+}\| = \left\|V^{+}\right\|_k = \sqrt{(v_{\alpha_k}^{+})^2 + (v_{\beta_k}^{+})^2} \tag{35}$$

and

$$\|v_{\alpha\beta_k}^{-}\| = \left\|V^{-}\right\|_k = \sqrt{(v_{\alpha_k}^{-})^2 + (v_{\beta_k}^{-})^2}. \tag{36}$$

From (29) it is also possible to obtain the instantaneous phase of a signal, that is,

$$\phi_{V1_k} = \arctan\left(\frac{\hat{x}_{1_{k|k-1}}}{\hat{x}_{2_{k|k-1}}}\right). \tag{37}$$

Similarly, based on (33) and (34), that is, the two orthogonal components provided by the $\alpha\beta$ transform, it is possible to extract the instantaneous phase of the positive and negative sequences,

$$\phi_{Va_k}^{+} = \arctan\left(\frac{v_{\alpha_k}^{+}}{v_{\beta_k}^{+}}\right), \tag{38}$$

$$\phi_{Va_k}^{-} = \arctan\left(\frac{v_{\alpha_k}^{-}}{v_{\beta_k}^{-}}\right). \tag{39}$$

It important to notice that the generation os synchronization signals and phase detection can be easily extended to any signal generated by the Kalman filter and even to the zero sequence symmetrical component.

## V. Frequency Identification

Since the mathematical model used is frequency dependent and the grid frequency can vary, it is necessary to identify the fundamental grid frequency that is also an important information about the grid voltages.

The frequency identification method used was proposed in [19] and relies on the internal model principle [21]. The identifier can be represented as shown in Fig. 1.

In this figure, the Internal Model block is implemented by

$$\begin{bmatrix} x_{w1} \\ x_{w2} \end{bmatrix}_{k+1} = \begin{bmatrix} 0 & 1 \\ -1 & 2\cos(\omega_k T_s) \end{bmatrix} \begin{bmatrix} x_{w1} \\ x_{w2} \end{bmatrix}_k + \begin{bmatrix} 0 \\ K_w \end{bmatrix} e_k \quad (40)$$

and

$$y_{w_k} = \begin{bmatrix} -1 & \cos(\omega_k T_s) \end{bmatrix} \begin{bmatrix} x_{w1} \\ x_{w2} \end{bmatrix}_k + K_w e_k \quad (41)$$

where $\omega_k$ is considered at $k = 0$ as the nominal value of the angular frequency of the grid voltage. This value will be updated by the identification algorithm. This block provides the signals $x_{w2_k}$ and $y_{\omega_k}$.

The Error Evaluation block is responsible for the evaluation of the error $\varepsilon_k$ and is implemented by the following equation

$$\varepsilon_k = \frac{K_w \sin(\omega_k T_s) x_{w2_k} e_k}{[\sin(\omega_k T_s) x_{w2_k}]^2 + [y_{w_k}]^2}. \quad (42)$$

Using an integrator and the signal $\varepsilon_k$, $\omega_k$ can be updated by

$$\omega_{k+1} = \omega_k - K_u \varepsilon \quad (43)$$

where $K_u$ is a scalar gain that is responsible for the transient response of the identifier. Based on equation (43), $\omega_k$ in the mathematical model (12)-(14) is updated.

In Fig. 1,

$$e_k = r_{w_k} - y_{w_k} = \frac{r_{w_k} + x_{w1_k} - \cos(\omega_k T_s) x_{w2_k}}{1 + K_w} \quad (44)$$

and $r_{w_k}$ is a sinusoidal signal that is intended to have the frequency identified. This signal drives the identifier and can be obtained from the positive sequence components as follows,

$$r_{w_k} = \frac{v^+_{a_k}}{\|V^+\|_k}. \quad (45)$$

Since this signal is normalized, step changes in the measured signal will have almost no effect in the identifier behavior.

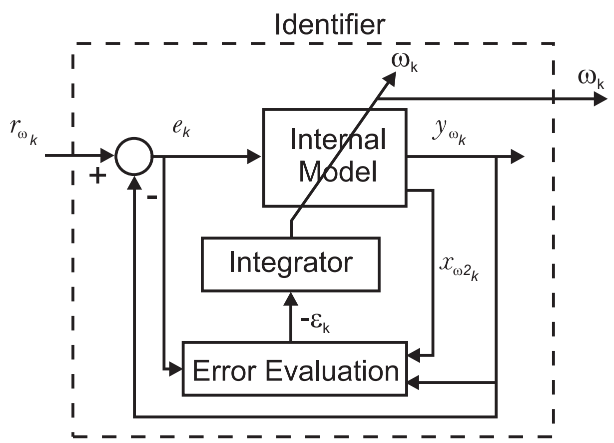


Fig. 1. Frequency identifier structure.

## VI. Experimental Results

To analyze the performance of the proposed method, some experimental results are presented. The proposed algorithm was implemented in a fixed point digital signal processor (TI-TMS320F2812). The measured grid voltages are presented in Fig. 2. The total harmonic distortion of the voltages is $THD = 34.7\%$ and the measurement noise has covariance $R = 20\ V^2$.

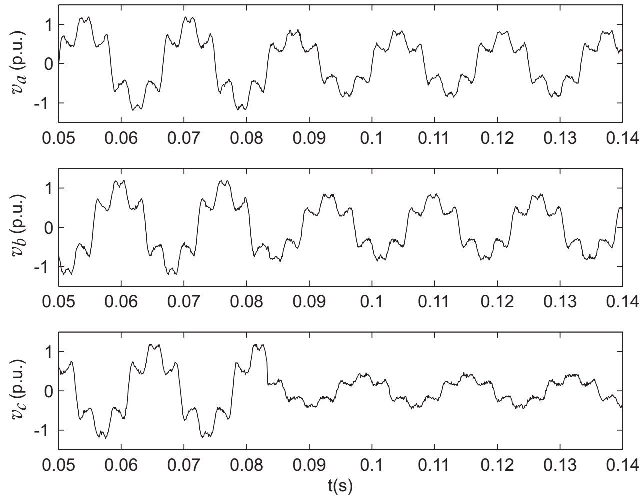


Fig. 2. Measured grid voltages.

The fundamental components are described by

$$v_a^f = \sqrt{2} * 127\angle 0^\circ\ V, \quad (46)$$
$$v_b^f = \sqrt{2} * 127\angle -120^\circ\ V, \quad (47)$$
$$v_c^f = \sqrt{2} * 127\angle 120^\circ\ V, \quad (48)$$

with nominal frequency $f = 60\ Hz$. The harmonics presented in the signal are: $5^{th}$, $7^{th}$ and $11^{th}$, with amplitudes given in p.u. by 0.3, 0.15 and 0.09, respectively. At $t = 0.0832\ s$ occurs a voltage drop of $30\%$ in all phases. Additionally, an unbalance of $50\%$ is considered in phase $c$. The value of the other gains of the algorithm are: $Q = 0.01 * I_{10\times 10} V^2$, $K_u = 20$ and $K_w = 0.052$.

Fig. 3 shows the extraction of the fundamental components of the grid voltages with the peak detection. The convergence of the estimator is fast and occur in less than one cycle.

Figs. 4 and 5 presents the extraction of the harmonics in the measured signal with their amplitudes evaluated in real-time. Note that the non-existent third harmonic is also correctly detected. The short transients that appear after the voltage drop fades away in less the one cycle of the fundamental component.

The detection of the symmetrical components are depicted in Fig. 6. Until $t = 0.0832\ s$ the grid voltages are balanced and the negative and zero sequence are zero. After $t = 0.0832\ s$ the voltages are unbalanced and the proposed method detects the presence of negative and zero sequences adequately. The amplitude of the positive, negative and zero sequences are also correctly identified.

Figs. 7 and 8 shows the generation of synchronization signals with the positive and negative sequence components of

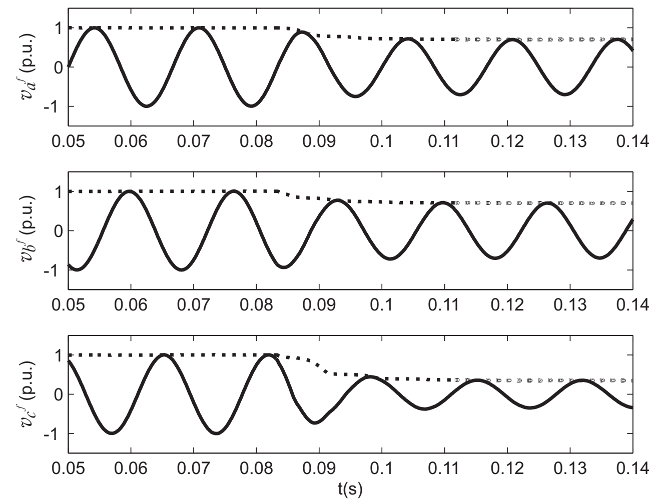


Fig. 3. Fundamental components.

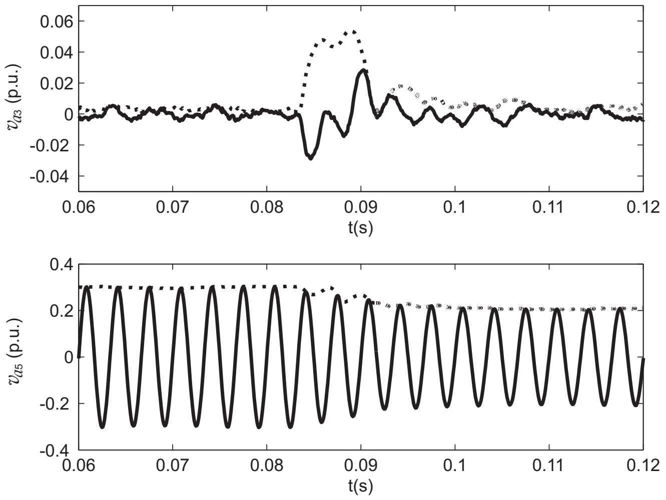


Fig. 4. Extraction of $3^{rd}$ (non-existent) and $5^{th}$ harmonics.

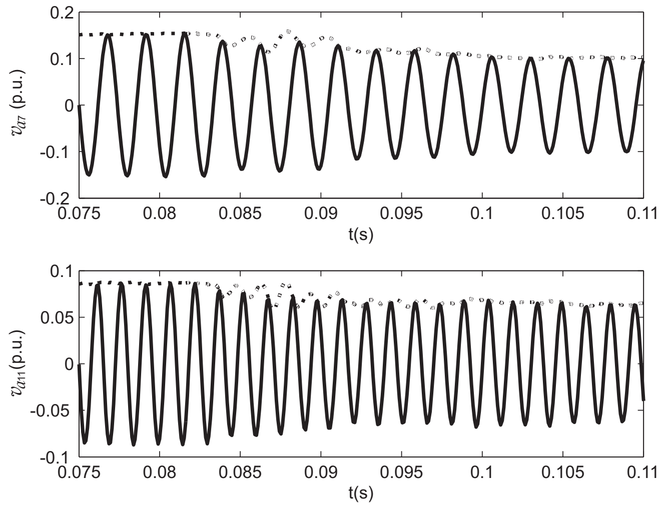


Fig. 5. Extraction of $7^{th}$ and $11^{th}$ harmonics.

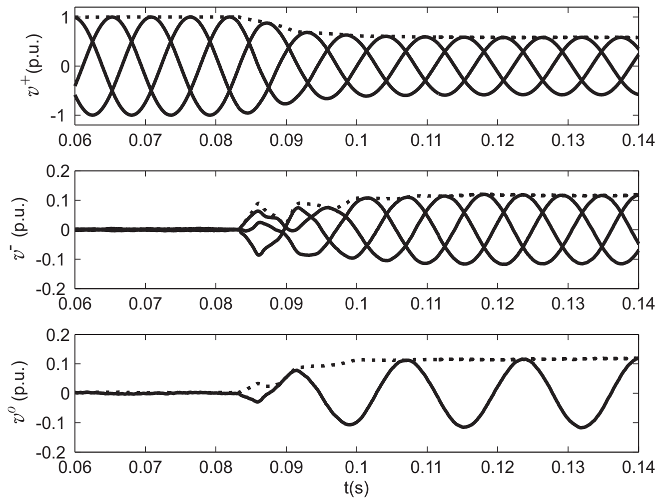


Fig. 6. Detection of positive, negative and zero sequences.

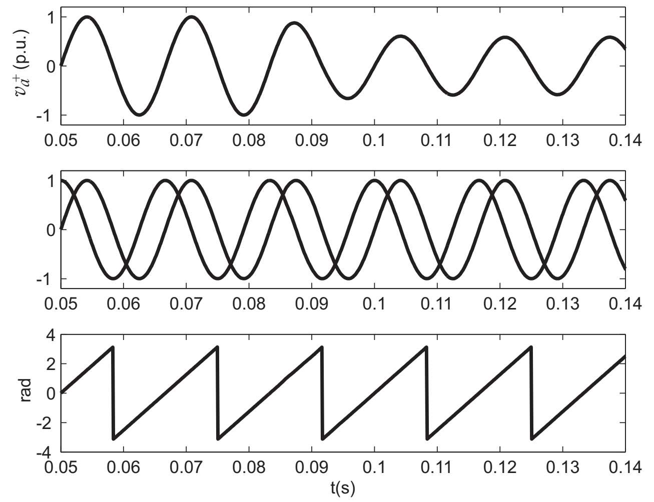


Fig. 7. Synchronism signals with positive sequence and instantaneous phase.

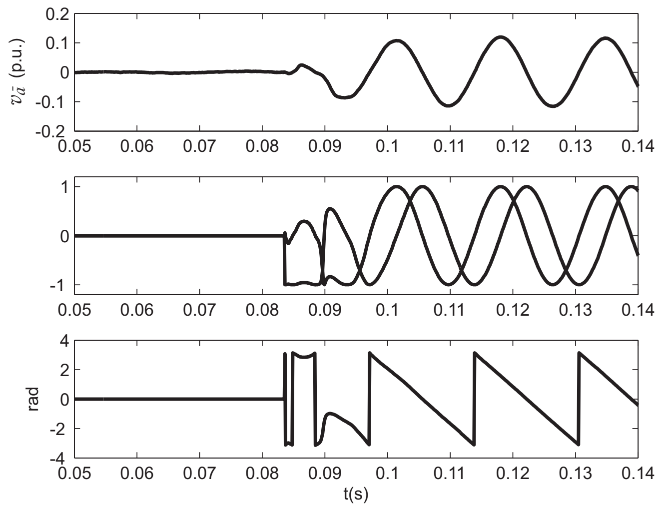


Fig. 8. Synchronism signals with negative sequence and instantaneous phase.

the fundamental voltages, respectively, with their associated instantaneous phases.

Fig. 9 depicts the THD of the phase a voltage evaluated in real-time. It also shows the identified grid frequency, that due the normalized signal used to drive de identifier, is practically imune to transients in the grid voltages.

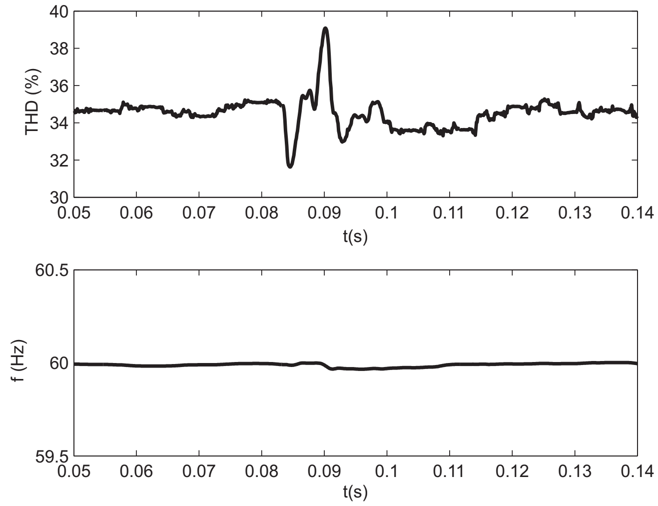


Fig. 9. THD and identified frequency.

Finally, Fig. 10 shows the transient behavior of the identified grid frequency for different values for the gain $K_u$.

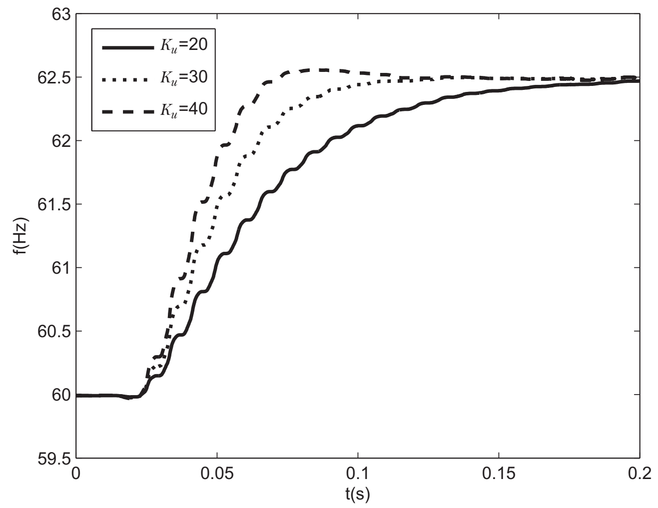


Fig. 10. Frequency identification.

## VII. Conclusions

This paper proposed an algorithm based on optimum filtering theory for power quality evaluation. The proposed method is capable to extract several important features of the grid voltages in real-time. The extracted features are: fundamental component, harmonics, amplitude of those components, positive, negative and zero sequence components with their amplitudes, synchronization signals, instantaneous phase and frequency of the grid voltages. It was also shown that the extraction of these features can also be extended to any signal provided by the Kalman filter. Thus, the proposed scheme is a powerful tool on the analysis of the power system grid voltages.

The tuning of the proposed method is based on three parameters: marices $Q$, $R$ and the gain $K_u$. The last one is responsible by the speed of convergence of the frequency identifier while matrices $Q$ and $R$ will define the behavior of the Kalman filter. That is, they will define, optimally, the transient behavior (speed of convergence) versus the rejection of measurement noise and process perturbations. Hence, if the measured signals has small noise perturbation as in the results presented, the filter will present a high speed of convergence. Higher noise level will imply in small speed of convergence to ensure noise rejection.

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