DECOUPLING ELECTROMAGNETIC TORQUE OSCILLATIONS IN TWO-PHASE INDUCTION MACHINE USING SPIRAL VECTOR THEORY

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***Abstract* - This paper shows how it is possible to decouple the electromagnetic torque oscillations in two-phase induction machine using spiral vector theory. Simulations were performed to solve differential equations. The comparative graphics are also shown in the paper.**

***Keywords* – dq0 transformation, spiral vector theory, two-phase induction machine.**

I. INTRODUCTION

It is necessary to develop an adequate model that makes it possible to mathematically describe an electrical machine for an efficient control. The classic technique for modeling the symmetrical two-phase induction machine (STPIM) is the dq0 transformation [1], which is very popular.

Spiral vector theory can be used for steady state and transient analysis of AC machine and circuit [2] [3]. It has been used for analysis of induction motor, synchronous machines, and permanent magnet synchronous motors. This paper presents the transient and steady state analysis of STPIM by using spiral vector operating under unbalanced voltage. Moreover, it is possible to write an expression to electromagnetic torque where can be seen each term of positive sequence, negative sequence torque and oscillations.

This work is organized in five sections. The second section gives brief introduction to spiral vector theory. The third section use spiral vector theory to analysis the STPIM under unbalanced voltage. The fourth section gives computer simulation graphics and the fifth section the conclusion.

II. SPIRAL VECTOR INTRODUCTION

The spiral vector is the exponential function with a complex index, as given below.

$$i = Ae^{\delta t}, \quad \delta = -\eta + j\omega \tag{1}$$

Where δ is the complex frequency, η and ω are real numbers and j is the complex operator.

As the time goes on, it depicts a spiral on the complex plane. When $\eta = 0$, it becomes a circular vector on the complex plane, which corresponds to steady state alternating current quantity. When $\omega = 0$, it expresses a decaying direct current quantity. When $\delta = 0$, it expresses a steady dc quantity. Thus the spiral vector can express almost all kinds of variables which appear in electrical engineering. A spiral vector in the complex plane is shows in fig. 1.

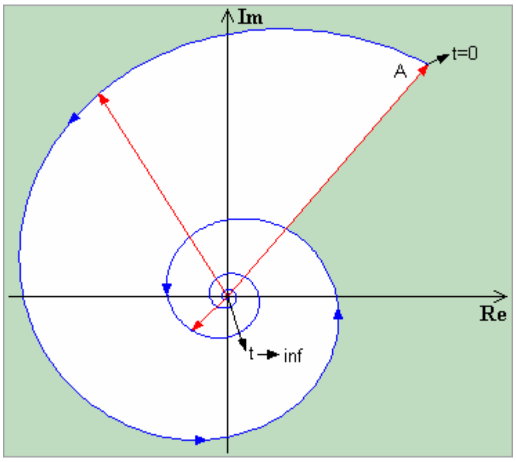

Fig. 1. Spiral vector in the complex plane.

The general solution for the lumped constant electric circuit is given by

$$i = \frac{A(p)}{B(p)} \cdot v \tag{2}$$

Where $A(p)$ and $B(p)$ are polynomials of $p = d/dt$, i and v are symbols for currents and voltages respectively and can assume any form of expressions. Let v be expressed by the following form

$$v = \sqrt{2}|V|e^{j(\omega t + \phi)} = \sqrt{2}Ve^{j\omega t} \tag{3}$$

Equation (3) is a general solution with $\eta = 0$, and where V is phasor. The general solution of (2) is obtained as

$$i = \frac{A(j\omega)}{B(j\omega)}\sqrt{2}|V|e^{j\omega t} + A_1e^{\delta_1 t} + A_2e^{\delta_2 t} + \ldots + A_ne^{\delta_n t} \tag{4}$$

Where $\delta_1, \delta_2 \ldots \delta_n$ are characteristic roots of $B(p)=0$, which is a characteristic equation. All the terms of solution in (4) are spiral vectors.

III. ANALYSIS OF STPIM USING SPIRAL VECTOR THEORY

A STPIM and its model for analysis are shown in fig.2. For phase ‘a’ of the stator and phase ‘r’ of rotor, voltage equations are given in (5) and (6), respectively. In the same way flux linkages equations are given in (7) and (8).

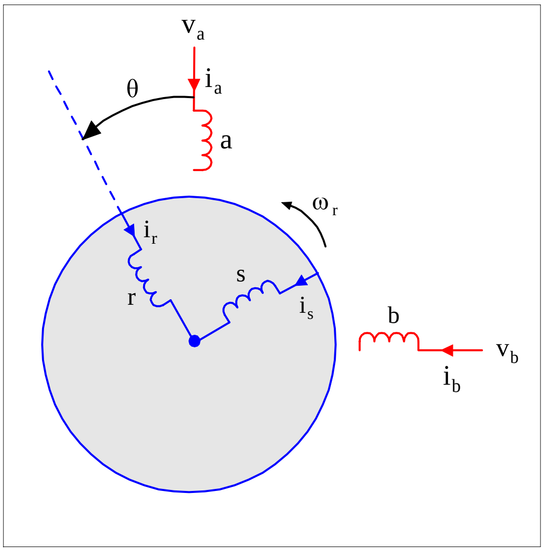

Fig. 2. Symmetrical two-phase induction machine model.

$$v_a = R_1 \cdot i_a + l_1 \cdot pi_a + p\lambda_{ga} \quad (5)$$

$$0 = R_2 \cdot i_r + l_2 \cdot pi_r + p\lambda_{gr} \quad (6)$$

$$\lambda_{ga} = M\left(i_a + i_r \cos\theta + i_s sen\theta\right) \quad (7)$$

$$\lambda_{gr} = M\left(i_r + i_a \cos\theta - i_b sen\theta\right) \quad (8)$$

Under steady state conditions i_a, i_b, i_r and i_s are given

$$i_a = \sqrt{2}\left|\dot{I}_a\right| e^{j(\omega_1 t+\phi_1)} \quad i_b = \sqrt{2}\left|\dot{I}_a\right| e^{j(\omega_1 t+\phi_1+\pi/2)} \quad (9)$$

$$i_r = \sqrt{2}\left|\dot{I}_r\right| e^{j(\omega_2 t+\phi_2)} \quad i_s = \sqrt{2}\left|\dot{I}_r\right| e^{j(\omega_2 t+\phi_2+\pi/2)} \quad (10)$$

Since the all currents are spiral vectors (11) and (12) are

$$i_b = j \cdot i_a \quad (11)$$

$$i_s = j \cdot i_r \quad (12)$$

Using the equations above it is possible to find (13) and (14) bellow.

$$v_a = R_1 \cdot i_a + \left(l_1 + M\right) \cdot pi_a + M \cdot pi_r' \quad (13)$$

$$0 = R_2 \cdot i_r' + \left(l_2 + M\right) \cdot \left(p - j \cdot \omega_r\right) \cdot i_r' + M\left(p - j \cdot \omega_r\right) \cdot i_a \quad (14)$$

Where ω_r is the electrical rotor speed, R_1 is the stator resistance of phase ‘a’, R_2 is the rotor resistance of phase ‘r’, l_1 is the stator leakage of phase ‘a’, l_2 is the rotor leakage of phase ‘r’ and M is the mutual inductance.

It should be noticed that here that the voltage and currents in (13) and (14) are of phases ‘a’ and ‘r’ only, the other phases of the primary and secondary two phases being left out. Phases ‘a’ and ‘r’ are thus segregated from the other phases. This is called phase segregation method. This means that one only of the two phases is sufficient to write the circuit equation of the two-phase machine. Equation (13) and (14) are valid for both steady state and transient state operations. Replacing the stator and rotor phase subscripts ‘a’ and ‘r’ by ‘1’ and ‘2’, respectively, (13) and (14) can be expressed by the following matrix form [2], [3].

$$\begin{bmatrix} v_1 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + \left(l_1 + M\right)p & Mp \\ M\left(p - j\omega_r\right) & R_2 + \left(l_2 + M\right)\left(p - j\omega_r\right) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (15)$$

Under symmetrical (or balanced) operation state variables of two phases are symmetrical. Under asymmetrical operation asymmetrical state variables in (15) are transformed by the spiral vector symmetrical component method into symmetrical spiral vectors as bellow.

$$\begin{bmatrix} v_1^+ \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + \left(l_1 + M\right)p & Mp \\ M\left(p - j\omega_r\right) & R_2 + \left(l_2 + M\right)\left(p - j\omega_r\right) \end{bmatrix} \begin{bmatrix} i_1^+ \\ i_2^+ \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} v_1^- \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + \left(l_1 + M\right)p & Mp \\ M\left(p + j\omega_r\right) & R_2 + \left(l_2 + M\right)\left(p + j\omega_r\right) \end{bmatrix} \begin{bmatrix} i_1^- \\ i_2^- \end{bmatrix} \quad (17)$$

STPIM modeling using spiral vector and symmetrical component concept makes it possible to decompose the electromagnetic torque into its positive and negative components, as well as torque oscillations as can be seen in (18).

$$\begin{aligned} T_{el}^{sv} &= z_p \cdot M\left\{ Im\left\{ i_1^+ \cdot \left(i_2^+\right)^* \right\} - Im\left\{ i_1^- \cdot \left(i_2^-\right)^* \right\}\right\} \\ &+ z_p \cdot M\left\{ Im\left\{ i_1^+ \cdot i_2^- \right\} - Im\left\{ i_1^- \cdot i_2^+ \right\}\right\} \end{aligned} \quad (18)$$

T_{el}^{sv} is the electromagnetic torque equation from spiral vector theory.

STPIM modeling using dq0 transformation in the stationary reference frame is given by (19) and (20) as follows [1].

$$\begin{bmatrix} v_{sd}^{s} \\ v_{sq}^{s} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + L_1 p & 0 & Mp & 0 \\ 0 & R_1 + L_1 p & 0 & Mp \\ Mp & M\omega_r & R_2 + L_2 p & L_2\omega_r \\ -M\omega_r & Mp & -L_2\omega_r & R_2 + L_2 p \end{bmatrix} \begin{bmatrix} i_{sd}^{s} \\ i_{sq}^{s} \\ i_{rd}^{'s} \\ i_{rq}^{'s} \end{bmatrix} \tag{19}$$

$$T_{el}^{dq} = z_p \cdot M \cdot \left(i_{sq}^{s} i_{rd}^{'s} - i_{sd}^{s} i_{rq}^{'s} \right) \tag{20}$$

T_{el}^{dq} is the electromagnetic torque equation from dq0 transformation theory.

The mechanical equation is obtained as follows.

$$\frac{J}{z_p} p(\omega_r) = T_{el} - T_L - \frac{K_f}{z_p}\omega_r \tag{21}$$

Where J is the inertia of the rotor and the connected load, z_p is the par of poles, K_f is the friction losses coefficient and T_L is the load torque.

Probably the reader must to be thinking: what the relation between i_1^+, i_1^-, i_2^+, i_2^-, i^s_{sq}, i^s_{sd}, i'^s_{rq} and i'^s_{rd} to make possible write (18)? The answer is given bellow.

$$\begin{aligned} i_{sd}^{s} &= Re\left\{i_1^+\right\} + Re\left\{i_1^-\right\} \\ i_{sq}^{s} &= Re\left\{-j \cdot i_1^+\right\} + Re\left\{j \cdot i_1^-\right\} \\ i_{rd}^{'s} &= Re\left\{i_2^+\right\} + Re\left\{i_2^-\right\} \\ i_{rq}^{'s} &= Re\left\{-j \cdot i_2^+\right\} + Re\left\{j \cdot i_2^-\right\} \end{aligned} \tag{22}$$

Using the equations above it is possible to find (22) as bellow.

$$\begin{aligned} i_{sq}^{s} i_{rd}^{'s} - i_{sd}^{s} i_{rq}^{'s} = &\left(Re\left\{-j \cdot i_1^+\right\}Re\left\{i_2^+\right\} - Re\left\{i_1^+\right\}Re\left\{-j \cdot i_2^+\right\}\right) \\ &+ \left(Re\left\{-j \cdot i_1^+\right\}Re\left\{i_2^-\right\} - Re\left\{i_2^+\right\}Re\left\{j \cdot i_2^-\right\}\right) \\ &+ \left(Re\left\{j \cdot i_1^-\right\}Re\left\{i_2^+\right\} - Re\left\{i_1^-\right\}Re\left\{-j \cdot i_2^+\right\}\right) \\ &+ \left(Re\left\{j \cdot i_1^-\right\}Re\left\{i_2^-\right\} - Re\left\{i_1^-\right\}Re\left\{j \cdot i_2^-\right\}\right) \end{aligned} \tag{23}$$

In the other hand it is necessary to remember three relations from complex variables theory as follows.

$$Re\{Z_1 Z_2\} = Re\{Z_1\}Re\{Z_2\} - Im\{Z_1\} Im\, Re\{Z_2\} \tag{24}$$

$$Im\{Z_1\} = \frac{\left(Z_1 - Z_1^*\right)}{2j} = \frac{-j}{2}\left(Z_1 - Z_1^*\right) \tag{25}$$

$$Re\{Z_1\} = \frac{\left(Z_1 + Z_1^*\right)}{2} \tag{26}$$

Using (24) and (25) in the first and last terms from (23) and multiply the result by $z_p.M$ it is possible seen positive and negative electromagnetic torque components.

$$\left(T_{el}^{sv}\right)_{-}^{+} = z_p \cdot M \left\{ Im\left\{ i_1^+ \cdot \left(i_2^+\right)^* \right\} - Im\left\{ i_1^- \cdot \left(i_2^-\right)^* \right\} \right\} \tag{27}$$

$\left(T_{el}^{sv}\right)_{-}^{+}$ is the sum of positive and negative electromagnetic torque components. The first term in (27) is the positive and second term is the negative electromagnetic torque component.

In the same way using (24), (25) and (26) in the second and third terms from (23) it is possible to write the electromagnetic torque oscillations as follows.

$$\left(T_{el}^{sv}\right)^{osc} = z_p \cdot M \left\{ Im\left\{ i_1^+ \cdot i_2^- \right\} - Im\left\{ i_1^- \cdot i_2^+ \right\} \right\} \tag{28}$$

$\left(T_{el}^{sv}\right)^{osc}$ is the electromagnetic torque oscillations.

It is clear in (27) that the first term it is responsible for useful electromagnetic torque and the second term is responsible to cause an opposite effect in the useful electromagnetic torque. In the other hand (28) shows that the electromagnetic torque oscillations it is a result from interaction between positive and negative current components.

IV. COMPUTER SIMULATION

To compare both types of modeling under unbalanced voltage, simulations were performed to solve differential equations. The results were obtained from a symmetrical two-phase, four poles, 1 HP induction machine. The voltage applied in the stator winding ‘a’ is two time grader than the applied voltage in the stator winding ‘b’. Fig. 3 shows the electromagnetic torque using dq0 transformation and spiral vector. Fig. 4 shows the speed using dq0 transformation and spiral vector theory; fig. 5 shows positive and negative electromagnetic torque without oscillations from spiral vector theory and electromagnetic torque from dq0 transformation. Fig 6 and 7 shows mechanical speed from dq0 transformation and spiral vector theory without electromagnetic torque terms oscillations from equation (18).

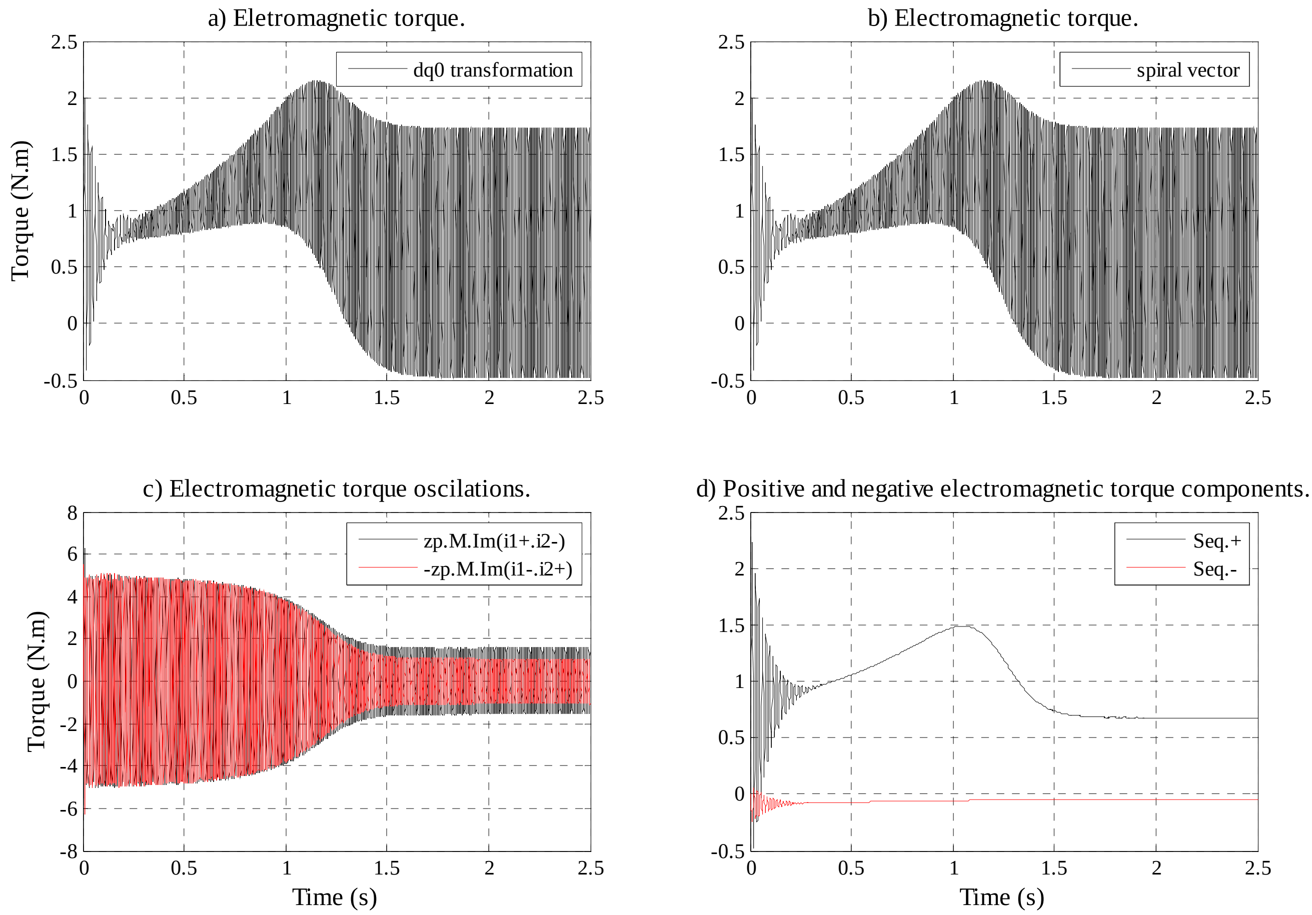

Fig. 3. Electromagnetic torque. a) dq0 transformation, b) Spiral vector, c) Torque oscillations, d) Positive and negative components

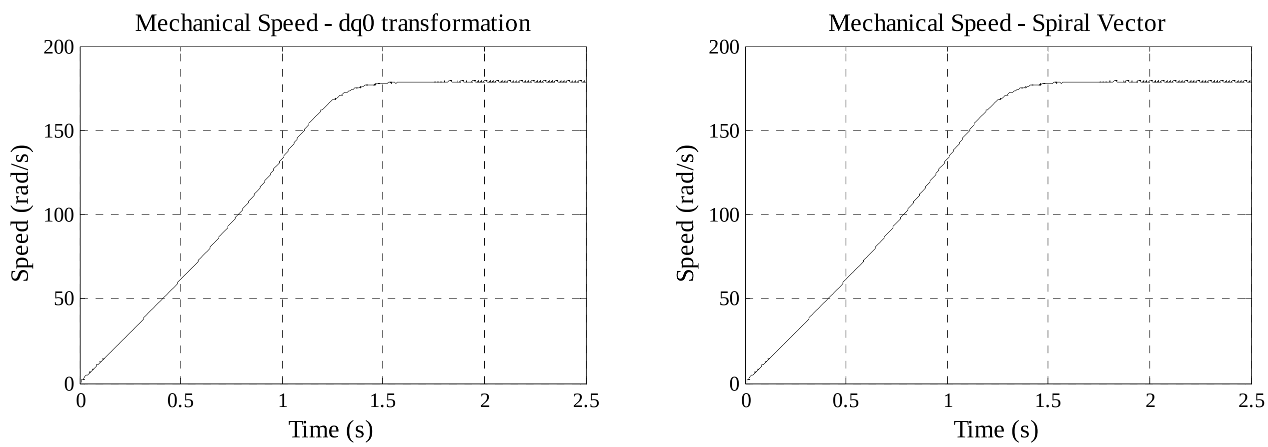

Fig. 4. Mechanical speed using dq0 transformation and spiral vector theory.

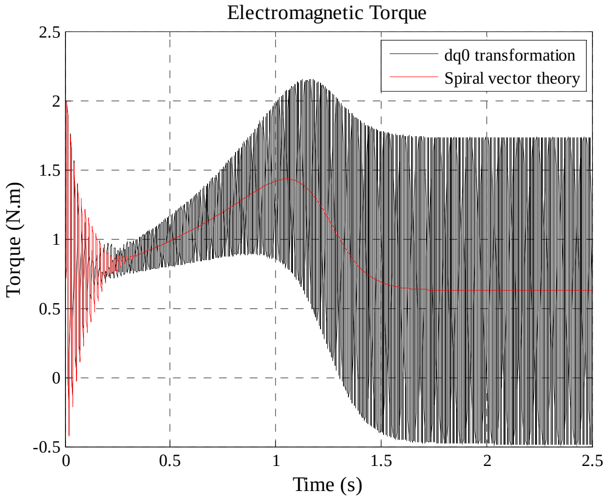

Fig. 5. Positive and negative electromagnetic torque without oscillations from spiral vector theory and dq0 transformation.

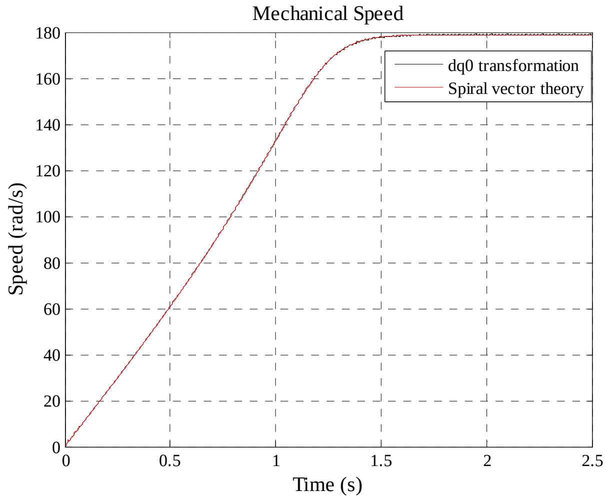

Fig. 6. Mechanical speed without oscillations from spiral vector theory and dq0 transformation.

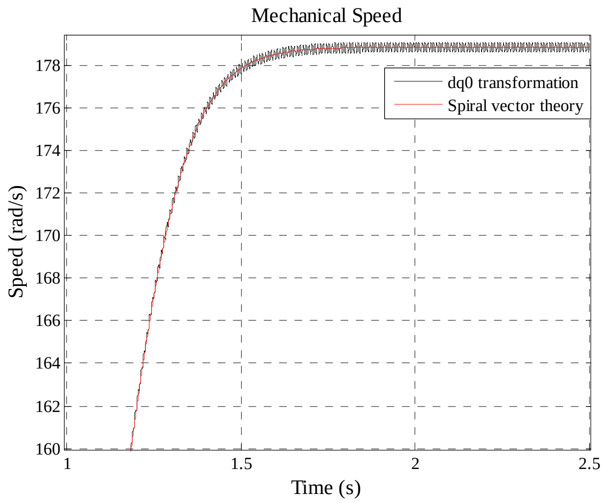

Fig. 7. Mechanical speed without oscillations from spiral vector theory and dq0 transformation, zoom in fig 6.

V. CONCLUSION

STPIM modeling using spiral vector theory leads to the same results obtained using dq0 transformation as can be seen in the a) and b) from fig 3 and fig 4. However, spiral vector and symmetrical component concept makes it possible to decompose the electromagnetic torque into its positive and negative components, as well as torque oscillations. Each component is valid in steady and transient states as can be seen in the c) and d) in fig. 3.

Moreover, it is possible to write an expression to electromagnetic torque where can be seen each term of positive sequence, negative sequence torque and oscillations. Such results can not be achieved using dq0 transformation, as the symmetrical component concept can be used only in steady state analysis because in dq0 transformation all currents are considered real values. The electromagnetic torque expression (18) obtained using spiral vector is a novel result in symmetrical two-phase induction machine analysis and can be applied in asymmetrical two-phase machine.

When electromagnetic oscillations from (28) is not consider to solve differential equations the spiral vector theory leads the electromagnetic torque and speed in the middle value to electromagnetic torque and speed from dq0 transformation as can be seen in fig 5, 6 and 7. In fact the positive component is responsible for dynamic response even if the unbalanced voltage in the stator winding ‘a’ is two time grater than voltage in stator winding ‘b’.

STPIM modeling using spiral vector theory suggest new ideas to study unsymmetrical two-phase induction machine.

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