MODELING AND SIMULATION OF A WIND PLANT CONTROLLED BY QUADRATIC LINEAR REGULATOR CONNECTED TO ELECTRIC DISTRIBUTION SYSTEM

V. P. Pinto, J. C. T. Campos
Federal University of Ceará- Electrical Engineering Department
Campus do Pici, P.O.Box 6001, ZIP code: 60.455-760, Fortaleza, CE - Brazil
vandilberto@yahoo.com.br, teles@ufc.br

***Abstract* – The increasing of the electric power demand in the Ceará State and the growth perspective in wide scale point out the need for new investigations in electricity generation. Due to those facts, the present work deals with the modeling and simulation of a doubly-fed induction generator (DFIG) and the power converters for interconnection to grid. The control strategy adopted uses a quadratic linear controller (LQR), where instead of using the pole positioning as design criterion, it is used the minimization of a quadratic criterion which is associated with the energy of the state-variables and the control signals designed. The project and evaluation of the proposal show the adopted controller efficiency through the appropriated choice of the *Q* and *R* weighting matrices, which resulted in an optimal solution, where the dynamic stability of the system was analyzed through the positioning of the system eigenvalues after compensation, through the computer simulations developed in the Matlab/Simulink platform.**

***Keywords* - Modeling, Simulation, Doubly Fed Induction Generator (DFIG), Dynamic Stability, Linearization, Optimal Control.**

I. INTRODUCTION

The Ceará State will receive up to 2008, about US$ 700 million for investment in wind plants, coming from the Program to use Renewable Energy Sources – Proinfa for the creation of 14 wind parks [1]. With the growth perspective in large scale of wind parks connected to the grid, there is the necessity of studies on the influence in energy quality, in order to guarantee the minimum of disturbance to grid, and thus to promote a better contact with the new model of system operation.
The increase of the power of wind turbines and the increase participation of the wind energy in the energy matrix in the some countries in the world have attracted attention of the responsible companies for the planning, operation and control of electric grids where such generators are connected.
This paper shows the modeling and simulation of the Induction Generator and of the power converter that interconnects the wind park to the grid. The computer simulations emphasize the stability analysis through the positioning of eigenvalues [2] and the efficiency of the quadratic linear controller (LQR) in way that the voltage at the Point of Common Connection (PCC) has reduced noise level [3].

II. WIND TURBINE MODEL

A wind turbine extracts a part of the kinetic energy of the wind that passes through the area swept for by the turbine blades. The turbine blades move the axis of the generator and that transforms the wind energy into electrical energy. The mechanical power is a function of the cubic of the wind speed and can be calculated according to equation (1) [4]:

$$P_m = \frac{1}{2}\cdot\rho\cdot A\cdot v^3\cdot C_p(\lambda,\beta) \qquad (1)$$

where ρ is the air density (kg/m^3), v is the wind speed (m/s), A is the cross section area of the turbine (m^2) and C_p is the power coefficient of the wind turbine.

The power developed for the wind turbine depends on the wind speed and the mechanics speed on the axis rotation. A factor sufficiently used is the relation between the tangential speed on the blade tip and the wind speed (m/s), represented for a speed ratio (λ) given by:

$$\lambda = \frac{\omega_r\cdot R}{v} \qquad (2)$$

where ω_r is the wind turbine rotor speed (rad/s) and R is the turbine rotor radius (m).
The mechanical torque T_m (N.m) of the wind turbine is the ratio of the mechanical power in relation to axis speed ω_r given by:

$$T_m = \frac{p_m}{\omega_r} \qquad (3)$$

then the mechanical torque produced for by the wind turbine on coefficient function can be expressed by the following equation [5]:

$$T_m = \frac{1}{2}\rho\cdot R^3.v^2\cdot C_q(\lambda,\beta) \qquad (4)$$

The mechanical coupling between the wind turbine and the generator adopted the model of mass only given by the following equation.

$$\dot{\omega}_r = \frac{1}{2H}(T_m - T_e - D\omega_r) \quad (5)$$

Where, T_e is the electromagnetic torque (N.m), H is the inertia constant [s] and D the damping coefficient[s] .

III. POWER COEFFICIENT

The aerodynamic performance of the wind turbines is calculated as a function of the power coefficient that indicates the efficiency as the wind turbine transforms the energy contained in the winds in electric power. This results that the power coefficient depends on the pitch angle β and the of speed ratio λ. The curves that depend on $C_p(\lambda, \beta)$ are supplied by the manufacturer, and obtained experimentally through tests accomplished in wind tunnels, and it can be obtained according to the mathematical model common used in the literature [4]:

$$C_p(\lambda,\beta) = C_1\left(\frac{C_2}{\lambda_i} - C_3\beta - C_4\right)e^{\frac{C_5}{\lambda_i}} + C_6\lambda \quad (6)$$

where C_1=0.5176, C_2=116, C_3=0.4, C_4=5, C_5=21 and C_6=0.0068 are constants related with the aerodynamic design of the turbine and λ_i is a parameter given by the equation below [4]:

$$\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1} \quad (7)$$

Figure 1 shows the family of curves $C_p(\lambda)$ for several values of the pitch angle β as a function of the speed ratio (λ) using the mathematical model given by the equations (6) and (7).

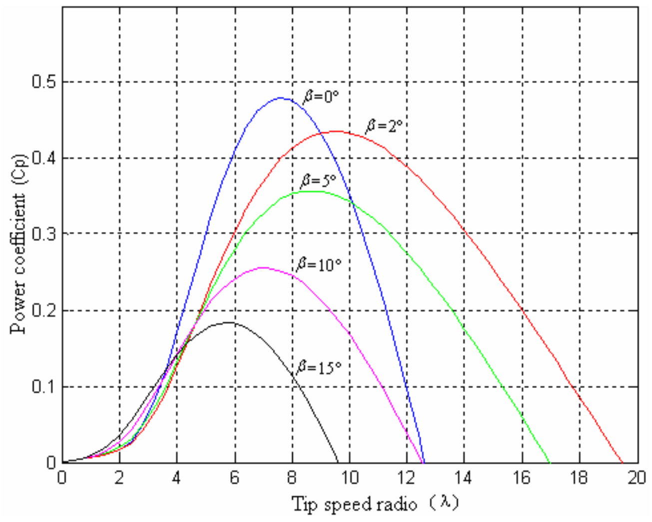

Fig .1. Power coefficient in function of the speed ratio changing the pitch angle (β).

It is observed that as the angle β increases the power coefficient is reduced the power coefficient and consequently the electric power generated for the turbine is reduced also.

IV. PITCH ANGLE CONTROLLER

The pitch angle control is a control strategy to limit the power generated whenever the nominal power of the generator is exceeded. Due to an increase of the wind speed the rotor blades are rotated around its longitudinal axis, in other words, the blades change their angle to reduce the attack angle of the wind. This reduction of the attack angle reduces the active aerodynamic forces and consequently the extraction of the mechanical power

Through the control of the pitch angle it is possible to control the value of the power coefficient C_p. That control is activated when the turbine enters in the area of constant power, in other words, for values of the wind speed over the nominal speed. For wind speeds over the nominal speed, the converter of the rotor of the generator acts in way to maintain constant speed of the rotor and activates the control, increasing the angle of the blades gradually, reducing the power coefficient systematically with the intention of not taking advantage all mechanical power extracted from the wind force.

Figure 2 shows the control of the pitch angle of the wind turbine used in this work [4] [5] [6].

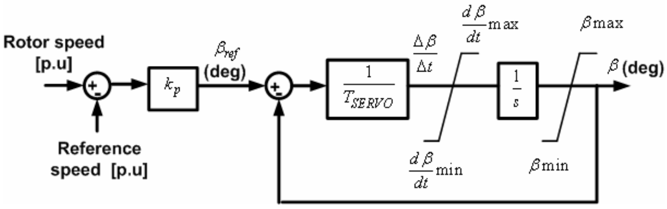

Fig. 2. Pitch angle controller model.

The rotor speed is compared with a reference speed that produces error sign that is sent to the proportional controller, producing the reference of the pitch angle to be controlled. This reference is applied to the system, that is modeled for a delay of first order, being this delay referred the time constant (T_{SERVO}) of the servo-motor that rotates the blades using a hydraulic system. The value adopted for that time constant was 250ms [4] [5]. The maximum variation rate of the pitch angle has typical values in the range of 3 to 5 degrees /s [7]. Then the pitch angle β it is obtained through an integrator that limits β between values of *ßmax* and *ßmin*.

V. THE DOUBLY FED INDUCTION GENERATOR

The Doubly Fed Induction Generator (DFIG) is an induction machine of with wound rotor where one of the converters is connected to grid and the other one is connected to the rotor. The two converters are interconnected through a capacitive circuit and with PWM control [8].

The converter interconnected to the Generator controls the rotor speed and the reactive power injected or consumed by the generator through stator. The converter interconnected to grid controls the voltage on the DC link and the active power that the rotor changes with the grid.

In Figure 3 the converter is presented with connection “back-to-back” and modulation PWM that is a strategy extensively used in those days [7].

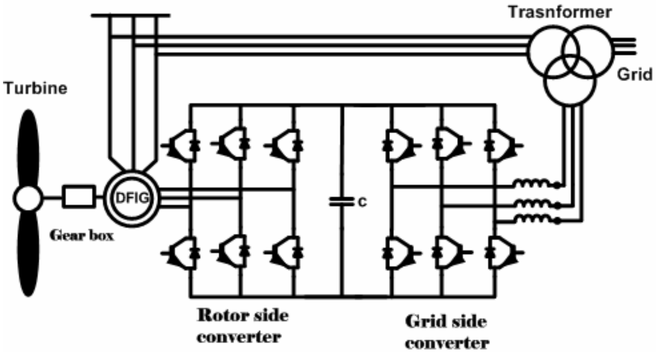

Fig. 3. Doubly-fed induction generator system.

The PWM modulation reduces the current harmonic component at the input and at the output of the system. As a consequence, it reduces the torque pulsation in the generator and improves the quality of the output power. It has become a standard option for applications in high power applications [7].

A. Induction Generator Modeling

For the modeling of the induction generator Park's transformation was used. Estator and rotor equations of the generator are expressed in components *dq* (*d* is the direct axis and *q* it is the axis in quadrature) that can be represented by equations (8-11) [2] [4].

$$V_{ds} = -R_s\ i_{ds} - \omega_s \lambda_{qs} + \frac{d\lambda_{ds}}{dt} \tag{8}$$

$$V_{qs} = -R_s\ i_{qs} + \omega_s \lambda_{ds} + \frac{d\lambda_{qs}}{dt} \tag{9}$$

$$V_{dr} = -R_r\ i_{dr} - s\omega_s \lambda_{qr} + \frac{d\lambda_{dr}}{dt} \tag{10}$$

$$V_{qr} = -R_r\ i_{qr} + s\omega_s \lambda_{dr} + \frac{d\lambda_{qr}}{dt} \tag{11}$$

B.The Linearized model

The linearization was made through the expansion in Taylor's series around the operation point, being disregarded the terms of superior order where was just considered the lineal terms. These terms should be sufficiently small, so that, the values of the variables change around the operation condition. The linearized system around an operation point is represented by the state space equation [9]:

$$\begin{bmatrix} \Delta\dot{i}_{dr} \\ \Delta\dot{i}_{qr} \\ \Delta\dot{\omega}_r \\ \Delta\dot{i}_d \\ \Delta\dot{i}_q \\ \Delta\dot{V}_{dc} \end{bmatrix} = \begin{bmatrix} \gamma & s_0\omega_0 & 0 & 0 & 0 & 0 \\ -s_0\omega_0 & \gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta & 0 \\ 0 & 0 & 0 & \rho & \xi & \psi \end{bmatrix} \begin{bmatrix} \Delta i_{dr} \\ \Delta i_{qr} \\ \Delta\omega_r \\ \Delta i_d \\ \Delta i_q \\ \Delta V_{dc} \end{bmatrix} +$$

$$+ \begin{bmatrix} \mu & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2H} & 0 & 0 \\ 0 & 0 & 0 & \kappa & 0 \\ 0 & 0 & 0 & 0 & \kappa \\ 0 & 0 & 0 & -\frac{i_{d0}}{C\ V_{dc0}} & \eta \end{bmatrix} \begin{bmatrix} \Delta v_{qr} \\ \Delta v_{dr} \\ \Delta T_m \\ \Delta v_d \\ \Delta v_q \end{bmatrix} \tag{12}$$

$$\gamma = \frac{R_r}{L_{rr}\sigma}, \tag{13}$$

$$\varepsilon = -\frac{\frac{1}{2}\cdot\rho\cdot A\cdot v^3\cdot C_p(\lambda,\beta)}{2H\omega^2{}_r} + \frac{\frac{1}{2}\cdot\rho\cdot A\cdot v^3}{2H\omega_r}\cdot\frac{d\ C_p(\lambda,\beta)}{d\lambda}\cdot\frac{R}{v}, \tag{14}$$

$$\delta = -\frac{R_S}{L_S}, \tag{15}$$

$$\rho = -\frac{v_{d0}}{C\ V_{dc0}}, \tag{16}$$

$$\xi = -\frac{v_{q0}}{C\ V_{dc0}}, \tag{17}$$

$$\psi = -\frac{v_{d0}\ i_{d0} + v_{q0} i_{q0}}{C\ V^2{}_{dc0}}, \tag{18}$$

$$\mu = \frac{1}{\mathrm{L}_{\mathrm{rr}}\sigma}, \tag{19}$$

$$\kappa = \frac{1}{L_S}, \tag{20}$$

$$\eta = -\frac{i_{q0}}{C\ V_{dc0}}. \tag{21}$$

VI. THE CONTROL STRATEGY

The control strategy adopted in this work has as objective to substitute proportional and integral controllers (PI), adjusted for attempt and error, for a quadratic linear controller (LQR), as Figure 4. The system performance was compared resulting in the performance improvement of the dynamic stability of the wind system integrated with the grid in relation to strategies from control with the classic controllers [10]. Having been compared also by [11] providing a better performance in the dynamic behavior and a faster damping of the transitory oscillations for the terminal voltage of the machine through the design of optimal control in relation the conventional controllers.

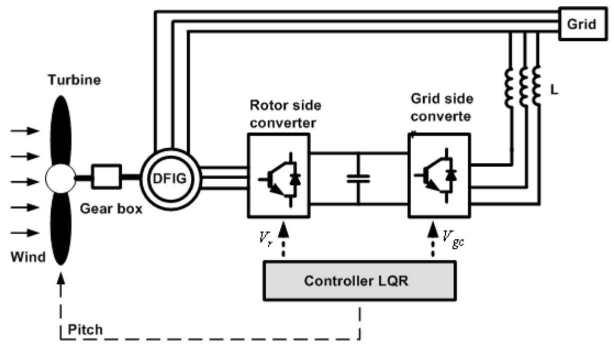

Fig .4. Induction generator diagram with quadratic linear controller

The energy of a scalar signal is defined as:

$$\int_0^\infty s^2(t).dt \tag{22}$$

$$\int_0^\infty \sum_0^\infty x_i^2(t).dt = \int_0^\infty x'(t).x(t).dt \tag{23}$$

The basic idea of LQR design in this work is to establish a commitment between the energies of the state vector *x(t)* and of the control vector *u(t)*, through the following cost function to be minimized [12]:

$$J = \min_{u(t)} \int x'(t)\ Q(t)x(t) + u'(t)R(t)u(t)]dt \tag{24}$$

Where Q and R are matrices positives definite, $Q > 0$ and $R > 0$.
Supposing that the system be stabilizable or be controllable, the control law that stabilizes the system and minimizes the criterion is:

$$u(t) = -k.x(t) \tag{25}$$

where:

$$k = R^{-1}.B'.P \tag{26}$$

The matrix P is positive definite and it is the solution of following Ricatti's equation:

$$A'.P + P.A - P.B.R^{-1}.B'.P + Q = 0 \tag{27}$$

A. Weighting matrices Q e R

One of the great challenges is the choice of the weighting matrices Q and R, Q is a weighting matrix associated with the system states and R is a weighting matrix associated with the control variables. The choice difficulty is due to the fact that does not exist a systematic method for such selection, being normally adopted the diagonal form for matrices Q and R and having the choice done through some simulations using trial and error method, in way that the values selected for weighting matrices satisfy the criteria established by the designer. The criterion for the choice of the weighting matrices is to evaluate the system performance mainly in stabilization time and an overshoot level.
In this work it will be used the methodology presented in [13] for the choice of the weighting matrices that characterize the performance of the control system. This methodology is known in literature as *Bryson's* method.

B. Bryson's Method

The Bryson's Method, also known as squared of the inverse has as basic idea to normalize the output and the control term in the index function from quadratic performance, where the weighting matrices Q and R are defined as:

$$Q = \begin{bmatrix} q_1 & 0 & \cdots & 0 \\ 0 & q_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{ne} \end{bmatrix} \quad e \quad R = \begin{bmatrix} r_1 & 0 & \cdots & 0 \\ 0 & r_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{na} \end{bmatrix} \tag{28}$$

where "*ne*" is the number of states and "*na*" it is the number of actuators in the control system. The desired performance of the system is obtained for the adjustment of the weighting matrices chosen through the following equation [14]:

$$q_i = \frac{1}{(\Delta x_i)^2} \quad \text{e} \quad r_i = \frac{1}{(\Delta u_i)^2} \tag{29}$$

The values of Δu_i are based on the control maximum effort or operation maximum value operation of the actuators. Already the values of Δx_i are based on the operation range of the states. Figure 5 it is presented the program flowchart implemented for the Bryson's method, as [13].

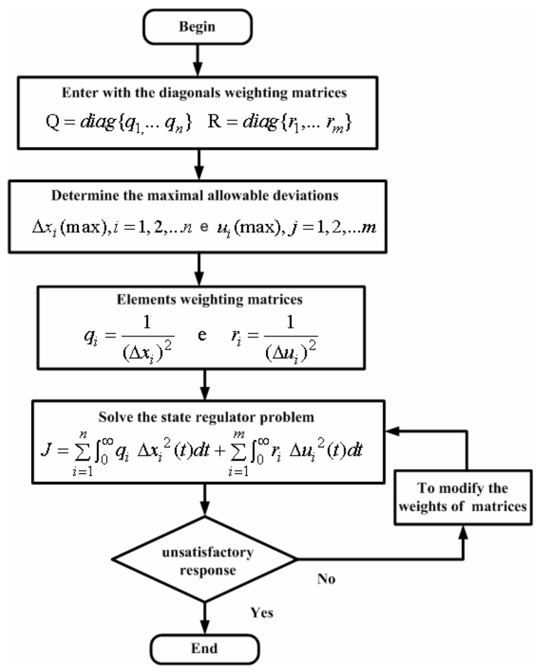

Fig .5. Algorithm of the Bryson's Method.

VII. SIMULATIONS

The developed simulations have an objective to evaluate the controller performance in the dynamic stability study of a wind plant the system performance through the quadratic lineal controller (LQR), mainly in stabilization time and an overshoot level. The choices of the weighting matrices were

chosen according to [13] [14] so that the performance of the control law is optimized.
Table I shows displays the matrix eigenvalues of the non compensated system. As they possess positive real parts, the system is unstable. Already the eigenvalues of the feedback matrix, in other words, of the compensated system, possesses all the eigenvalues with real part in the left semi-plane, consequently the system was stabilized.

Table I
Eigenvalues of the system.

Eigenvalues	
DFIG - No-compensated	**DFIG – compensated**
0.7188	-39.4294
-0.0723 + 0.3333i	-55.7447
-0.0723 - 0.3333i	-21.9508
-0.0320 + 1.0000i	-17.6856
-0.0320 - 1.0000i	-13.3703 + 2.6033
-0.1924	-13.3703 - 2.6033i

For the simulations were considered the parameters of the induction generator in agreement with the work [3] [7]. With the used strategy, the system was excited by a unit step. Figures 6, 7 and 8 present the dynamic behavior of the system.

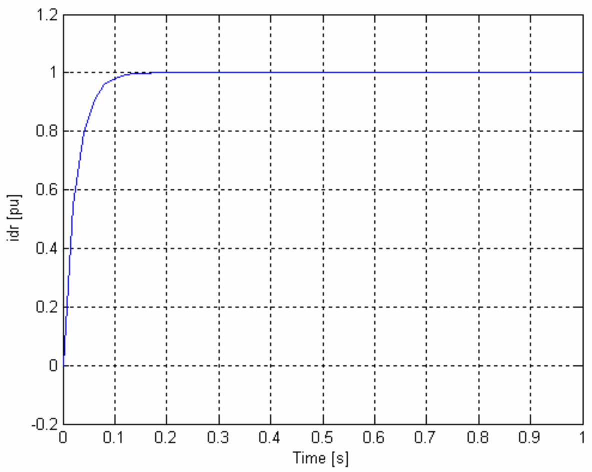

Fig .6. Current of the rotor of the direct axis excited by a unit step.

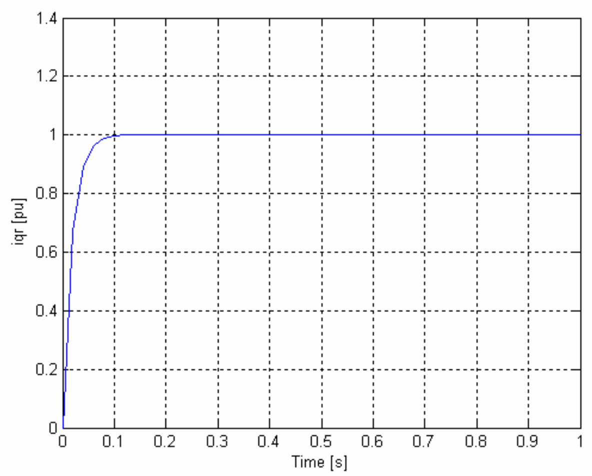

Fig .7. Current of the rotor of the quadrature axis excited by a unit step.

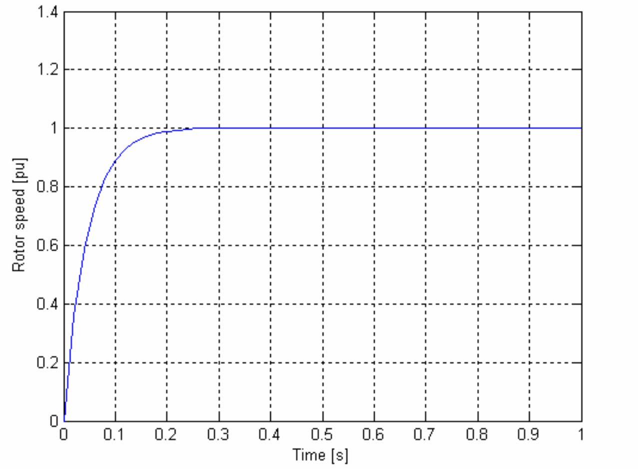

Fig. 8. Speed of the rotor excited by a unit step.

The simulations results show the optimal performance of the system excited by a unit step. It can be noted that the system behaves as a first-order system, and has a accommodation time lower than 0.3 seconds for all excitations.
The control of the rotor speed is made through the rotor currents, controlling the currents between the converter and the grid and the DC link voltage. In this way, the active and reactive power can be obtained directly due the vectorial control.
Figures 9 and 10 show the of the active and reactive power of the rotor. In figure 9 the machine is in the sub-synchronous speed , in other words, when the rotor speed is smaller than the synchronous speed of the machine, the slip is positive (s >0). That ($P_r < 0$) it indicates that the rotor is absorbing grid power. When the machine is in the super-synchronous speed , that is, when the rotor speed is larger than the synchronous speed of the machine, the slip is negative, (s < 0), and ($P_r > 0$) indicating that the power flows from the machine to the grid, according to in Figure 10. Therefore, the active power can be supplied or delivered through the converter connected to the rotor and in both cases (super-synchronous and sub-synchronous speed) the estator supplies active power to the grid.

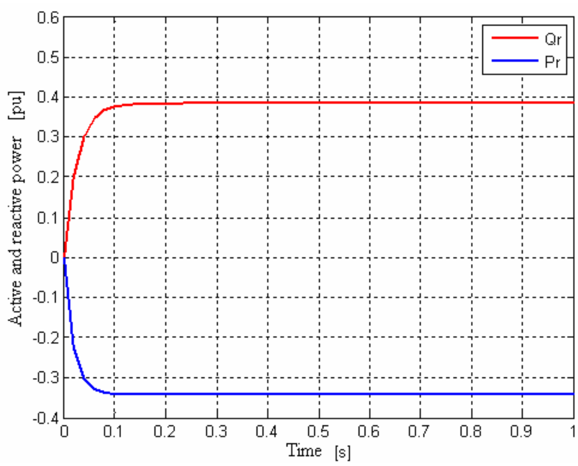

Fig. 9. Active and reactive power of the rotor in the sub-synchronous speed.

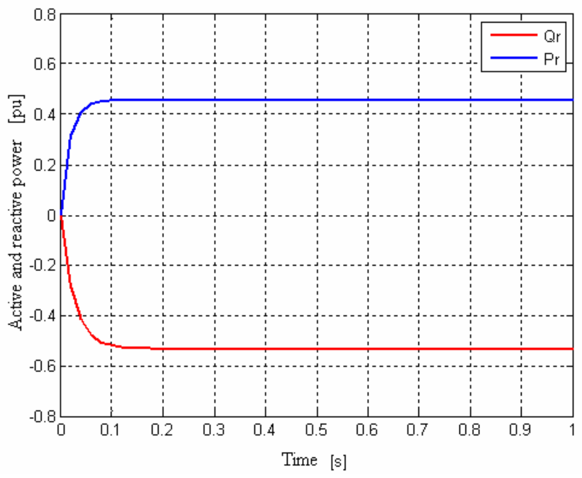

Fig. 10. Active and reactive power of the rotor in the super-synchronous speed.

VIII. CONCLUSION

This paper presented a design methodology for power flow control of the Doubly Fed Induction Generator, using controller LQR. The controller LQR applied to the converter corresponds, to a solution, through the appropriate choice of the weighing matrices, that improves the performance of the dynamic stability of the wind system integrated to electric grid. The method presented itself as effective for the control of the power converters, where the performance of the control law is considered excellent, with the overshoot level and stabilization time being minimum. In this way, the voltage at the PCC has reduced noise level. The method presents disadvantages as, simple implementation, robustness and stability. However, it presents the disadvantage to require measurements of the state variables for the control signal feedback. Such fact can be easily overcame by adding one observer to the design LQR, known as Kalman's filter, the combined result is a Linear. Quadratic Gaussian (LQG) controller. Meanwhile the inclusion Kalman's filter can result in the robustness property degradation of the LQR that can be recovered with the methodology of robust control LQG/LTR.

ACKNOWLEDGEMENTS

The authors want to thank the Fundação Cearense de Apoio ao Desenvolvimento Científico e Tecnológico (Funcap), for the financial support of this research and .Prof. Phd. Fernando Luiz Macelo Antunes for detailed advice and helpful suggestions.

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