STATE FEEDBACK CONTROL FOR POLE LOCATION AND OPTIMAL REJECTION OF DISTURBANCES APPLIED TO POWER ELECTRONICS

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***Abstract*— This paper provides a control design tool suitable to compute the state feedback control matrix gain for multiple-input multiple-output systems of arbitrary dimension ensuring: i) the stability by means of the location of all the closed-loop poles in a specific region in the complex plane and ii) the optimization of the disturbance rejection for the closed-loop system. The region for pole location is defined by the control designer by the choice of three parameters that provide bounds for the natural frequency, settling time and dumping factor of all the transient responses of the closed-loop system. The control design tool given here is written as a convex optimization problem, which brings the great advantage of providing the global optimal controller within a finite and previously estimated computational time, thus avoiding iterative design procedures which do not have any guarantee of convergence to the global optimal solution in a finite time. The investigation of tradeoffs between the pole location and the rejection of disturbances and also the problem of non-fragility of the controller are addressed. An application of the design tool to synthesize a proportional-integral controller to the regulation of velocity of an induction motor illustrates the efficiency of the results given in the paper.**

***Keywords* – Optimal control; Convex optimization; $\mathcal{H}_\infty$ control; Pole location; Non-fragile control; PI regulator.**

I. INTRODUCTION

The computation of a state feedback controller which assigns all the poles of a liner time-invariant system at desired places in the complex plane is undoubtedly an important control design problem, since it allows to shape the closed-loop transient response by determining parameters as overshoot, natural frequency and settling time [1, 2, 5, 12]. Although it is known that when the system is controllable, one can assign the closed-loop poles at the desired places using standard pole placement techniques, the determination of the state feedback controller which ensures the desired pole location and, simultaneously, provides the optimal rejection of disturbances can be a difficult task to be handled using classical control techniques, which usually involve iterative procedures without any guaranteed of convergence to the optimal controller.

This paper provides a systematic solution for the above control design problem for multiple-input multiple-output (MIMO) systems of arbitrary dimension. The control problem is written as a convex optimization problem with linear matrix inequality (LMI – [4]) constraints, as in [6], and is equivalent to the problem of determining the state feedback control matrix gain which minimizes the $\mathcal{H}_\infty$ norm of the closed-loop system (i.e. optimizes the rejection of energy bounded disturbances) under a prescribed pole location specification. The pole location specification is chosen *a priori* by the control designer, allowing to impose the desired bounds for natural frequency, settling time and dumping factor for all the closed-loop transient responses. The formulation of the design problem as a convex optimization problem with LMI constraints provides guarantee of finding, in a previously estimated computational time, a controller which ensures the global optimal rejection of disturbances respecting the pole location specifications, which is a great advantage when compared to methods which search the controller using discretization in the space of the controller and also when compared to methods based on the use of more advanced heuristics, as for instance genetic algorithms, to search the control matrix gain, but which do not have any guarantee of finding the global optimizing controller in a finite computational time and also are very sensitive to initialization. The result is extended to cope with the investigation of tradeoffs between the pole location specifications and the level of rejection of disturbances and also to cope with the problem of non-fragile control [7, 9]. As an application to power electronics, the proposed design conditions are employed to compute the gains of a proportional-integral (PI) controller used for velocity

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regulation of an induction motor, illustrating the efficiency of the control design tool given in the paper.

II. PROBLEM FORMULATION

Consider the linear time-invariant MIMO system given by

$$\dot{x} = Ax + B_w w + B_u u \quad (1)$$

$$y = Cx + D_w w + D_u u \quad (2)$$

where $x \in \mathbb{R}^n$ is the state vector, $w \in \mathbb{R}^q$ is the vector of disturbance inputs, $u \in \mathbb{R}^m$ is the vector of control inputs and $y \in \mathbb{R}^p$ is the vector of controlled outputs. Matrices A, B_w, B_u, C, D_w and D_u are real valued matrices of appropriate dimension.

For the state feedback control law

$$u = Kx \ , \ K \in \mathbb{R}^{m\times n} \quad (3)$$

where the control matrix gain K is to be determined, system (1)-(2) can be rewritten in the closed-loop form as

$$\dot{x} = A_{cl}x + B_w w \ , \ A_{cl} \triangleq A + B_u K \quad (4)$$

$$y = C_{cl}x + D_w w \ , \ C_{cl} \triangleq C + D_u K \quad (5)$$

The main objective of this paper is to provide a control design tool to solve the following problem.

Problem 1: Determine the control matrix gain K for the state feedback control law (3) such that the following properties are ensured:

- i) all the eigenvalues of the closed-loop matrix A_{cl} (i.e. closed-loop poles) belong to the region $\mathcal{S}(r,\alpha,\theta)$, shown in Figure 1, where $r > 0$, $0 < \alpha < r$ and $0 < \theta \leq \pi/2$ are chosen by the control designer.

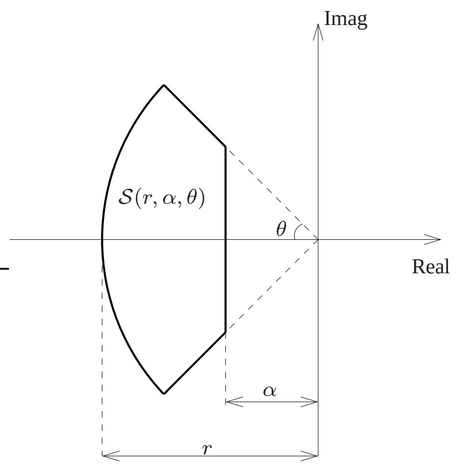

Fig. 1. Region $\mathcal{S}(r,\alpha,\theta)$, located at the open left-hand side of the complex plane, and defined by the intersection of a circle centered at the origin with radius $r > 0$, the strip at left of $-\alpha$, $0 < \alpha < r$ and the sector with angle $0 < \theta \leq \pi/2$;

- ii) the $\mathcal{H}_\infty$ norm [4] of the closed-loop system, defined as

$$\sup \frac{\|y\|_2}{\|w\|_2} \ , \ \forall w \in \mathcal{L}_2 \ , \ w \neq \mathbf{0} \ , \ x(0) = \mathbf{0} \quad (6)$$

  is minimized.

Remark 1: It is important to observe that the location of all the closed-loop poles in the region $\mathcal{S}(r,\alpha,\theta)$ (property i) in Problem 1) ensures that the transient responses of the closed-loop system will always respect prescribed bounds for natural frequency w_n, settling time τ_s and dumping factor ξ. Specifically, these bounds can be given by

$$w_n \leq r \ , \ \tau_s \leq \frac{5}{\alpha} \ , \ \cos\theta \leq \xi \leq 1 \quad (7)$$

Remark 2: It is also important to notice that the minimization of the $\mathcal{H}_\infty$ norm (property ii) in Problem 1) provides the best rejection of disturbances $w \in \mathcal{L}_2$ (i.e. energy bounded disturbances) for the closed-loop system.

Remark 3: Problem 1 focuses on the determination of the state feedback control matrix gain K that ensures, simultaneously, properties i) and ii). It is known that if the pair (A, B_u) is controllable, one can easily handle the assignment of the closed-loop poles in the region $\mathcal{S}(r,\alpha,\theta)$ by means of pole placement techniques. However, the determination of K which ensures the pole location constraints (property i) in Problem 1) and, simultaneously, optimizes globally the rejection of disturbances for the closed-loop system (property ii) in Problem 1) is a more involving problem.

III. DESIGN CONDITIONS

The next theorem, based on the results from [6], provides a solution for Problem 1 by means of a convex optimization problem with LMI constraints (see [4, 8] for details on this class of optimization problems).

Theorem 1: Given $r > 0$, $0 < \alpha < r$ and $0 \leq \theta \leq \pi/2$, which define the region $\mathcal{S}(r,\alpha,\theta)$. If there exist matrices $W = W' \in \mathbb{R}^{n\times n}$ and $Z \in \mathbb{R}^{m\times n}$ and a scalar $\mu \in \mathbb{R}$ solving the following convex optimization problem

$$\mu^\star \triangleq \min_{W,Z,\mu} \mu \text{ s.t.}$$

$$AW + WA' + B_u Z + Z'B_u' + 2\alpha W < 0 \quad (8)$$

$$\begin{bmatrix} -rW & AW + B_u Z \\ WA' + Z'B_u' & -rW \end{bmatrix} < 0 \quad (9)$$

$$\begin{bmatrix} \sin\theta(T_{11}) & \cos\theta(T_{12}) \\ \cos\theta(T_{12})' & \sin\theta(T_{11}) \end{bmatrix} < 0 \quad (10)$$

$$T_{11} \triangleq AW + WA' + B_u Z + Z'B_u'$$

$$T_{12} \triangleq AW + B_u Z - WA' - Z'B_u'$$

$$\begin{bmatrix} T_{11} & B_w & WC' + Z'D_u' \\ B_w' & -\mathbf{I} & D_w' \\ CW + D_u Z & D_w & -\mu\mathbf{I} \end{bmatrix} < 0 \qquad (11)$$

then the state feedback control matrix gain

$$K = ZW^{-1} \qquad (12)$$

ensures: i) that all the closed-loop poles belong to the region $\mathcal{S}(r, \alpha, \theta)$ and ii) that the $\mathcal{H}_\infty$ norm of the closed-loop system, given by $\gamma = \sqrt{\mu^\star}$, is minimized.

Proof: If Theorem 1 has a solution, then, using the variable transformation $Z = KW$, given in [3], one recovers from (8)-(11) the LMIs given in [6], which ensure the location of the closed-loop poles in the region $\mathcal{S}(r, \alpha, \theta)$ (LMIs (8)-(10)) and, simultaneously, imposes the minimization of the $\mathcal{H}_\infty$ norm of the closed-loop system (LMI (11)). ■

Remark 4: The solution of Problem 1 by means of a convex optimization problem, as in Theorem 1, is very attractive from the computational point of view, since available algorithms as the LMI Control Toolbox from Matlab [8] provide the global optimal solution to the problem in polynomial time. Specifically, the convex optimization problem in Theorem 1 can be read as minimize μ over the variables W, Z and μ subject to the LMI constraints (8)-(11). The number of scalar variables to be determined is $\mathcal{V} = 1 + n(n+1)/2 + mn$ and the number of LMI rows is $\mathcal{R} = 6n + p + q$. Interior point based LMI solvers as [8] have guarantee of global convergence to the global optimal solution in a computational time proportional to $\mathcal{V}^3\mathcal{R}$. This represents a great advantage against solutions for Problem 1 based on exhaustive gridding procedures to search the control matrix gain K in an unbounded space ($\mathbb{R}^{m\times n}$). Approaches based on more complex heuristics to search K, as for instance as genetic algorithms, usually lead to suboptimal solutions, with no guarantee of global convergence in finite time and also can be very sensitive to initialization. The conditions in Theorem 1 overcome all these difficulties, being a very efficient solution for Problem 1.

Remark 5: Notice that Theorem 1 provides a control design tool for MIMO systems of arbitrary dimension. The control designer choose the parameters r, α and θ for pole location in $\mathcal{S}(r, \alpha, \theta)$ and also provides the matrices A, B_w, B_u, C, D_w and D_u of the system model. Then, if Theorem 1 has a solution, given by matrices W, Z and by the scalar $\mu^\star$, one has that $K = ZW^{-1}$ is the control gain ensuring the prescribed pole location with the global optimal disturbance rejection (i.e. minimum $\mathcal{H}_\infty$ norm given by $\gamma = \sqrt{\mu^\star}$).

The following corollaries extend the results from Theorem 1 to deal with the investigation of tradeoffs between pole location and rejection of disturbance, to handle decentralized control synthesis and to cope with non-fragile control.

Corollary 1: Given $r > 0$ and $0 < \alpha < r$. The solution of Theorem 1 for values of θ in the interval $0 < \theta \leq \pi/2$ allows to investigate the tradeoff between γ, which measures the system capacity of rejection of disturbances, and θ, which provides a lower bound for the dumping factor (7).

Remark 6: Tradeoffs between γ and α and between γ and r can be investigated following the ideas in Corollary 1. In general, the more stringent the pole location specification, the poorer the rejection of disturbances.

Corollary 2: The solution of Theorem 1 for block-diagonal matrix variables W and Z yields a block-diagonal control matrix gain, which can be suitable to deal with decentralized control or static output feedback (see, for instance, [11]).

Corollary 3: Given $r > 0$, $0 < \alpha < r$, $0 < \theta \leq \pi/2$ and $0 \leq \delta < 1$. If there exists a solution for Theorem 1 with matrix Z replaced by $Z(1 \pm \delta)$ in each LMI[1] then any state feedback control matrix gain $K \in \mathcal{K}$, where[2]

$$\begin{aligned} \mathcal{K} = \{K \in \mathbb{R}^{m\times n} \ &: \\ &K = \big(\sigma_1(1-\delta) + \sigma_2(1+\delta)\big)ZW^{-1}, \\ &\sigma_i \in \mathbb{R}_+ \ , \ i = 1,2 \ , \ \sigma_1 + \sigma_2 = 1\} \end{aligned} \qquad (13)$$

ensures that the closed-loop poles belong to the region $\mathcal{S}(r, \alpha, \theta)$ and that $\gamma = \sqrt{\mu^\star}$ is an upper bound on the $\mathcal{H}_\infty$ norm of the closed-loop system, called an $\mathcal{H}_\infty$ guaranteed cost for the closed-loop system.

Proof: To prove Corollary 3, notice that replacing Z by $Z(1 \pm \delta)$ in the LMIs of Theorem 1 one has, from convexity [4], that the state feedback control matrix gain given by any convex combination of $(1-\delta)ZW^{-1}$ and $(1+\delta)ZW^{-1}$ ensures that the closed-loop poles are assigned in $\mathcal{S}(r, \alpha, \theta)$ and that $\gamma = \sqrt{\mu^\star}$ is an $\mathcal{H}_\infty$ guaranteed cost for the closed-loop system. ■

Remark 7: Corollary 3 provides a solution for the problem of robustness to a perturbation δ on the control matrix gain ($\delta = 0$ means no perturbation), which is a problem of non-fragile control [7, 9]. Such perturbation on the control matrix gain can occur in practice, for instance, due to the implementation of the controller on a platform with limited precision and also due to slow variation of the gains of the controller which may happen during

[1]The symbol $\pm$ means that each LMI in Theorem 1 is placed by two LMIs in Corollary 3: one LMI with Z replaced by $Z(1+\delta)$ and another LMI with Z replaced by $Z(1-\delta)$.

[2]The set $\mathbb{R}_+$ represents all nonnegative reals.

operation. Corollary 3 allows to the control designer to use the information on the value of the perturbation δ in the control synthesis. Notice that any of the control matrix gains in the set $\mathcal{K}$ ensures the prescribed pole location with a guaranteed rejection of disturbance.

Remark 8: The solution of Corollary 3 for vales of δ in the interval $0 \leq \delta < 1$ allows to investigate the tradeoff between γ (capacity of rejection of disturbances) and δ (perturbation on the control matrix gain).

Remark 9: Concerning practical implementation, discretization can be used to implement the continuous-time control strategy given here by means of digital platforms. Moreover, the estimated state vector, provided by state observers, can be employed in the state feedback control law.

IV. EXAMPLE

This section illustrates the application of the control design conditions provided previously to synthesize the gains of an optimal PI controller applied to the regulation of velocity of an induction motor whose parameters are given in [10, Chapter 3]. The closed-loop control system is given in Figure 2, where ρ is a constant reference input, e is the regulation error, u_c is the output of the PI controller, w is an energy bounded disturbance input, u is the control input which drives the plant and y is the plant controlled output (velocity of the induction motor). The transfer functions of the controller and of the plant

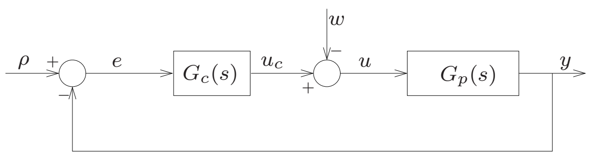

Fig. 2. PI controller (G_c) applied to the velocity regulation of an induction motor (G_p).

are given, respectively, by

$$G_c(s) = k_p + \frac{k_i}{s} \quad , \quad G_p(s) = \frac{b_0}{s + a_0} \tag{14}$$

where k_p and k_i are real parameters to be determined (gains of the controller) and a_0 and b_0 are the parameters of the plant.

Observe that the transfer function from the reference ρ to the output y, represented by $G_{y\rho}(s)$, is such that $G_{y\rho}(0) = 1$, thus ensuring zero steady state error to any constant reference input. Moreover, defining $\int e \triangleq \int_0^t e(\beta)d\beta$, one can write the control system from Figure 2 in the following state space representation

$$\dot{\varepsilon} = A\varepsilon + B_w w + B_{u_c} u_c + B_\rho \rho \tag{15}$$

$$y = C\varepsilon + D_w w + D_{u_c} u_c + D_\rho \rho \tag{16}$$

where

$$\varepsilon = \begin{bmatrix} \int e \\ e \end{bmatrix} , \; A = \begin{bmatrix} 0 & 1 \\ 0 & -a_0 \end{bmatrix} , \; B_w = \begin{bmatrix} 0 \\ b_0 \end{bmatrix} ,$$
$$B_{u_c} = \begin{bmatrix} 0 \\ -b_0 \end{bmatrix} , \; B_\rho = \begin{bmatrix} 0 \\ a_0 \end{bmatrix} , \; C = \begin{bmatrix} 0 & -1 \end{bmatrix} ,$$
$$D_w = 0 \; , \; D_{u_c} = 0 \; , \; D_\rho = 1$$

It is very important to notice that the problem of determining the gains of the PI controller which drives the error vector ε of system (15)-(16) to zero with a prescribed dynamic and, simultaneously, ensures the optimal rejection of the disturbance w can be tackled as the problem of computing the gains k_p and k_i of the state feedback control law

$$u_c = \begin{bmatrix} k_i & k_p \end{bmatrix} \begin{bmatrix} \int e \\ e \end{bmatrix} \tag{17}$$

for system (15)-(16), with $\rho = 0$, such that the poles of the closed-loop system belong to a prescribed region $\mathcal{S}(r, \alpha, \theta)$ and the $\mathcal{H}_\infty$ norm of the closed-loop system is minimized. Theorem 1 is precisely a design tool which solves this problem. Using the parameters for the induction motor from [10, Chapter 3], one has $a_0 = 0.1$ and $b_0 = 100$, which define the matrices A, B_w, B_{u_c}, C, D_w and D_{u_c} for system (15)-(16). The parameters for the region $\mathcal{S}(r, \alpha, \theta)$ are chosen as $r = 200$, $\alpha = 20$ and $\theta = \pi/12$. In this case, the Theorem 1 provides as solution

$$K = \begin{bmatrix} k_i & k_p \end{bmatrix} = \begin{bmatrix} 40.6517 & 2.1957 \end{bmatrix} \tag{18}$$

and

$$\gamma = 0.5424$$

The eigenvalues of $A + B_{u_c}K$ (i.e. closed-loop poles) are given by -20.3999 and -199.2744, thus belonging to the region $\mathcal{S}(r, \alpha, \theta)$.

It is worth to mention that the control gains (18) were obtained solving Theorem 1 using the LMI Control Toolbox from Matlab running in a notebook with a $1.66\ GHz$ Core Duo processor and with $1\ GB$ of RAM, spending a computational time of 0.14 seconds, which shows the rapid convergence of the design tool provided here to the global optimal controller, without using any complex heuristic or exhaustive computational procedure to search the control gains.

To illustrate the good quality of the results, some dynamic simulation of the closed-loop system are carried out. For instance, the response of the closed-loop system with $w = 0$ and with a constant reference given equal to $45\ rad/s$ is given in Figure 3. Notice the fast transient response, provided by the closed-loop poles inside chosen region, and no steady state error, as expected.

Figure 4 shows the closed-loop system response to a disturbance w, measured in Nm, applied from $3 \leq t \leq 5$

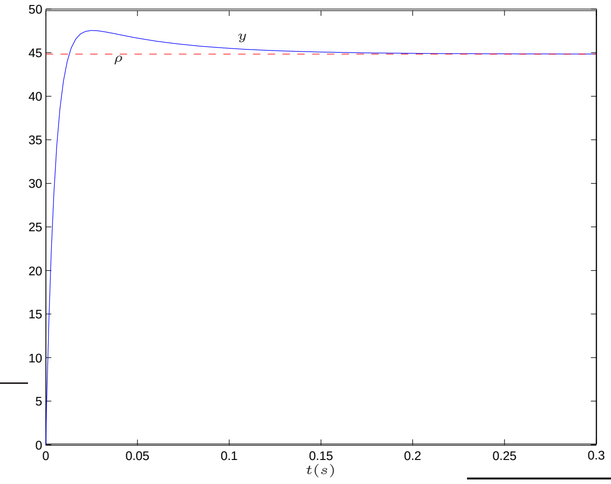

Fig. 3. Transient response in the start of the system with the gains of the PI controller given by (18). y and ρ are measured in rad/s.

seconds. Again, one can notice the fast transient response in the recover from the disturbance action, with a good rejection of the disturbance provided by the design based on the minimization of the $\mathcal{H}_\infty$ norm. For a comparison, one has that the response using the proposed design condition for the PI controller, given in Figure 4, exhibits faster transients, with smaller deviations from the reference than those in the response provided by the PI adaptive controller given in [10, Chapter 3].

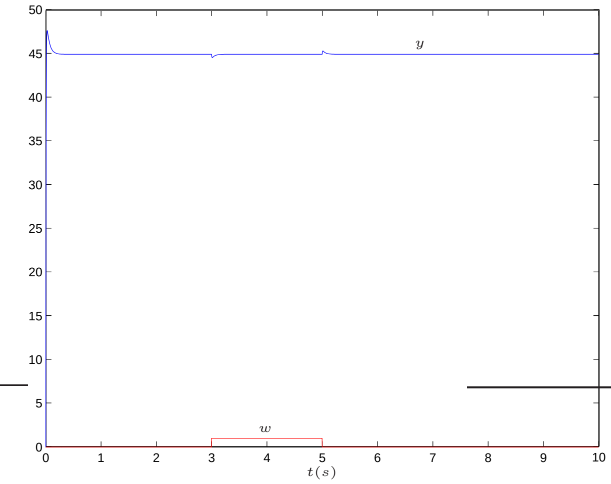

Fig. 4. Response of the closed-loop system with PI controller given by (18) for an energy bounded disturbance w, measured in Nm. y is measured in rad/s.

Figure 5 provides a detail on the regulation error for the simulation of the closed-loop system with PI controller given by (18) with reference $\rho = 0$ and for the same energy bounded disturbance used in the simulation from Figure 4, which leads to $\|e\|_2/\|w\|_2 = 0.1272 < \gamma = 0.5424$, thus corroborating the rejection of disturbance provided by the $\mathcal{H}_\infty$ norm.

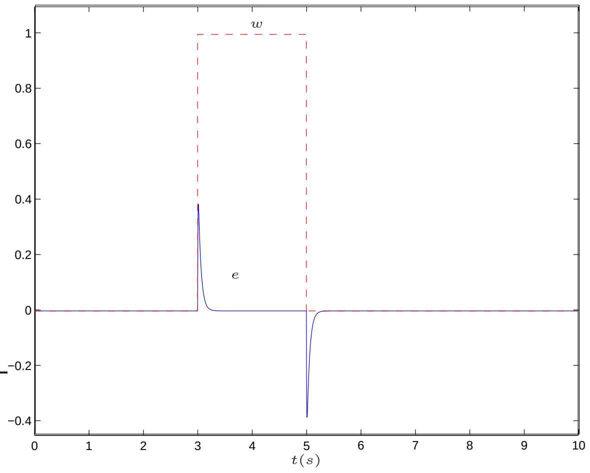

Fig. 5. Detail on the regulation error in the interval of application of the disturbance, for reference equals to zero. e is measured in rad/s and w is measured in Nm.

Finally, to have an evaluation of non-fragility of the controller obtained with the design techniques given here, Figure 6 shows the tradeoff between the perturbation on the control gains, δ, and the value of the $\mathcal{H}_\infty$ guaranteed cost (upper bound on the $\mathcal{H}_\infty$ norm) of the closed-loop system provided by Corollary 3. The point marked with $\diamond$

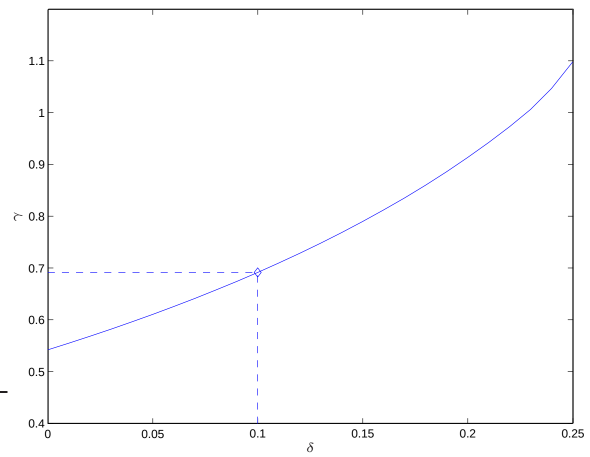

Fig. 6. Tradeoff between the perturbation δ on the control gains and the value of the $\mathcal{H}_\infty$ guaranteed cost of the closed-loop system.

on the curve of Figure 6 is obtained solving Corollary 3 for $r = 200$, $\alpha = 20$, $\theta = \pi/12$ and $\delta = .1$ (i.e. a perturbation of $\pm 10\%$ on the control gains). In this case, Corollary 3 yields as solution the matrices

$$W = \begin{bmatrix} 0.0256 & -0.8308 \\ -0.8308 & 84.0773 \end{bmatrix}$$

and

$$Z = \begin{bmatrix} -0.7290 & 137.8418 \end{bmatrix}$$

allowing to obtain the set of control gains

$$\begin{aligned} \mathcal{K} = \{ K \in \mathbb{R}^{1\times 2} : K = (\sigma_1(1-\delta) + \sigma_2(1+\delta))ZW^{-1} \\ = (\sigma_1(1-\delta) + \sigma_2(1+\delta)) \begin{bmatrix} 36.3841 & 1.9990 \end{bmatrix} , \\ \sigma_i \in \mathbb{R}_+ \ , \ i = 1,2 \ , \ \sigma_1 + \sigma_2 = 1 \ , \ \delta = 0.1 \} \end{aligned} \quad (19)$$

which ensures the above described pole location and an $\mathcal{H}_\infty$ guaranteed cost given by $\gamma = 0.6921$. In other words, any of the gains $K = [k_i \ k_p]$ in the set $\mathcal{K}$ ensures to the closed-loop system the pole location in the region $\mathcal{S}(r, \alpha, \theta)$, with $r = 200$, $\alpha = 20$, $\theta = \pi/12$ and also ensures that $\|e\|_2 < 0.6921\|w\|_2$, $\forall w \in \mathcal{L}_2$, $w \neq \mathbf{0}$. This illustrates operation with guaranteed performance for the closed-loop system under a perturbation δ affecting the control gains.

V. CONCLUSION

This paper provides a tool (Theorem 1) to synthesize the control matrix gain for state feedback control laws applied to MIMO systems of arbitrary dimension. The design tool is given by means of a convex optimization problem whose solution yields the control matrix gain which ensures the pole location inside a region $\mathcal{S}(r, \alpha, \theta)$ chosen *a priori* by the control designer and also optimizes the rejection of energy bounded disturbances for the closed-loop system. The choice of the parameters of the region for pole location allows to impose bounds on the natural frequency, settling time and dumping factor for all transient responses of the closed-loop system. Extensions to deal with the investigation of tradeoffs between pole location and rejection of disturbance, to handle decentralized control and also to cope with non-fragile control are provided (corollaries 1 to 3, respectively). The main advantage of the conditions given here is the formulation based on convex optimization with LMI constraints for which there exists globally convergent algorithms that provide the global optimal solution in a finite and previously estimated computational time. The efficiency of the conditions provided in the paper is illustrated by means of an example of application to the design of a global optimal PI controller applied to the regulation of velocity of an induction motor.

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