

SIMPLIFIED MODELS OF KALMAN FILTER FOR FUNDAMENTAL FREQUENCY, AMPLITUDE AND PHASE ANGLE DETECTION

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Abstract – This paper discusses two different applications of the discrete-time Kalman Filter (KF) for the identification of the amplitude, angular frequency and phase angle of the fundamental voltage of a power system. Modeling the fundamental voltage (first of a single-phase and then of a three-phase system) as an output of a linear time-invariant system with random perturbations and measurement noise, it is possible to construct a KF that estimates the state of the model. Such estimates are then used to obtain the fundamental frequency. Simulations are in force to show that the filter converges to the real values even if the noise measurement and perturbation variables are not wide-sense stationary white noise (lack of optimality). Practical results of the KF and the frequency estimation algorithm are obtained by means of their implementation on a digital acquisition and processing system.

Keywords - Kalman filter, fundamental frequency detection, fundamental amplitude detection, fundamental wave identification, synchronization, phase angle detection.

I. INTRODUCTION

The requirement of synchronization of several electronic devices (such as active power filters, uninterruptible power suppliers, dynamic voltage restorers, distributed generators, etc.) has been motivating the development of different algorithms to detect the amplitude, frequency and phase angle of the power grid fundamental voltage.

No matter how complex the detection algorithm is, there is always a compromise between precision (or in a wide sense, robustness) and rapidity of the response. This means that we have to sacrifice one despite the other. Besides, the computational complexity can be a major concern, as in real-time analysis and control.

As far as rapidity and precision of response are concerned, the Kalman Filter offers a nice compromise in many applications. Introduced in 1960 by R. E. Kalman [1], this filter gives the best estimate for the state of a linear-time invariant system in the presence of wide-sense stationary white noise process perturbations and measurement noise.

The list of successful applications of KF is ever growing (see, for example, [2,3] for applications in navigations, radars [2,4], telephony [3], demographics [2] and control systems [2]). In the specific areas of power systems and power

electronics, the use of KF are relatively recent, (see for example, [5-8] for interesting applications).

In this paper, the voltages of a power system are modeled as outputs of linear time-invariant systems (of discrete-time, as the KF are going to be implemented in a digital signal processor) subjected to perturbations and noises. A KF will be constructed to estimate the state of the model and such estimates will be used to calculate phase angles and frequencies. This algorithm is an improvement compared to the more elaborate techniques like those presented in [5,6] or even the simple PLL approach discussed in [9].

A major difference of this algorithm is that voltage distortions (harmonics and noise) will be treated as measurement noise. Despite this assumption destroys the optimality of the filter, it will be shown by simulations that convergence is nevertheless achieved. Such basic consideration reduces dramatically the implementation complexity of the KF estimation when compared with classical approaches.

Initially, an algorithm to estimate the frequency and phase of a single-phase system will be presented and discussed and then, a three-phase version of the KF will be proposed.

II. SINGLE-PHASE KALMAN FILTER

As proposed in [1] and discussed in [7,8], consider a discrete-time linear time-invariant model in state-space form:

$$\begin{cases} \mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{w}_{k-1} \\ \mathbf{y}_k = \mathbf{B}\mathbf{x}_k + \mathbf{z}_k \end{cases}, \quad (1)$$

where:

- k is the step,
- \mathbf{x}_k is a $n \times 1$ state vector of the system in the step k ,
- \mathbf{y}_k is a $m \times 1$ vector of the measurement in the step k ,
- \mathbf{A} is a square matrix $n \times n$, which should be adjusted in case of frequency deviations of the input signals,
- \mathbf{B} is a constant $m \times n$ matrix,
- \mathbf{w}_k is a $n \times 1$ vector representing process noise (due to perturbations and inaccuracy of the dynamic model),
- \mathbf{z}_k is a $m \times 1$ vector representing measurement noise (due to the inaccuracy of transducers and signal conditioning circuits) on the signals to be digitalized.

Defining:

- \mathbf{Q} – process noise covariance,
- \mathbf{R} – measurement noise covariance,

one can define a Kalman Filter as an state estimator for the system (1) where:

- $x_k - \hat{x}_{k/k-1}$ – initial estimation error,
- $P_{k/k-1}$ – estimation covariance error,
- $x_k - \hat{x}_k$ – final estimation error,
- P_k – final estimation error covariance and
- K_k – Kalman gain.

The state estimation \hat{x}_k based on measurements y_k is achieved in two parts: 1) prediction step and 2) correction step, as shown in Fig. 1. The first step estimates the state ahead and gets the error covariance ahead. The second step computes the Kalman gain, update the estimation with measurement and update the error covariance.

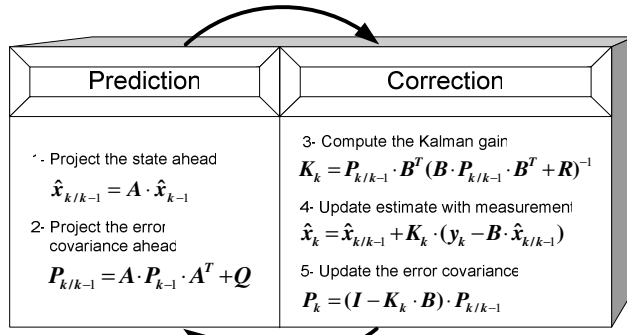


Fig. 1. KF description.

According to [5-8], a good model for the real fundamental wave of a single-phase power system can be represented by the output of a discrete-time state-variable model:

$$\begin{cases} V_1(k) = A V_1(k-1) + w_{k-1} \\ v(k) = B V_1(k) + z_k \end{cases} \quad (2)$$

where the state vector $V_1(k) = \begin{bmatrix} v_1(k) \\ v_{1\perp}(k) \end{bmatrix}$ contains the filtered signals in phase ($v_1(k)$) and orthogonal ($v_{1\perp}(k)$) to the measured signal $v(k)$, $B = \begin{bmatrix} 1 & 0 \end{bmatrix}$ is the input matrix, w_k has a dimension 2×1 and z_k , 1×1 , and

$$A = \begin{bmatrix} \cos(2\pi/N) & -\sin(2\pi/N) \\ \sin(2\pi/N) & \cos(2\pi/N) \end{bmatrix}.$$

Differently from other proposals, as [6], e.g., in this paper the harmonic components are not included in a deterministic way (that is, in the very matrix A). They will be considered perturbations and, in this sense, it will be modeled by the measurement and process noises. This assumption simplifies a lot the calculation of matrix A . Thus, the filtered signals in $V_1(k)$ presents the same amplitude of the fundamental component of $v(k)$, what means that the magnitude of the fundamental component is calculated by:

$$M \equiv \sqrt{(v_1)^2 + (v_{1\perp})^2}. \quad (3)$$

The instantaneous phase angle can be obtained from:

$$\theta(k) = \tan^{-1} \left[-\frac{v_1(k)}{v_{1\perp}(k)} \right], \quad (4)$$

and the fundamental frequency f_l can be estimated from zero-crossing detection of the signal $\theta(k)$ and improved with an average of the last four estimates [10].

Matrix A is responsible for the performance of the KF when there is a frequency variation in the input signal. As $N = f_s/f_l$, changing f_l leads to changing the elements of matrix A , if f_s is kept constant. In order to make the KF immune to these disturbs, the identification of f_l is essential.

The filtering characteristics and the convergence speed of the filter are defined by the matrices R and Q , that are the covariance matrices of the perturbation and measurement noises for what the KF are going to be optimal.

When the measurement noise is high, the trace of R and the elements of K_k will be small. So, the relative weight of y_k should be reduced in the next estimation step, what makes the convergence slow. On the other hand, when the measurement noise is small, the trace of R and the elements of K_k will be high, ensuring a better confidence in y_k and a fast dynamic response.

When the process noise is high, the trace of Q and $P_{k/k-1}$ and the elements of K_k are high, resulting in a large reliability for the measurements of y_k in the next estimation step. If the process noise is small, the trace of Q and $P_{k/k-1}$ and the K_k elements are small, resulting in a small weight for y_k and a slow convergence of the algorithm.

Therefore, as stated earlier, the KF design depends on a compromise between desired accuracy and dynamic response, what can be achieved by a proper choice of matrices Q and R , taking into account input waveform distortions and desired characteristics of final applications.

III. THREE-PHASE KALMAN FILTER

In the same spirit of the single phase model, a set of three-phase fundamental voltages can also be represented by means of a state space model [10]:

$$\begin{cases} V_1(k) = A V_1(k-1) + w_{k-1} \\ v(k) = B V_1(k) + z_k \end{cases} \quad (5)$$

where:

$$A = \begin{pmatrix} \cos\left(\frac{2\pi}{N}\right) & \frac{1}{\sqrt{3}}\sin\left(\frac{2\pi}{N}\right) & -\frac{1}{\sqrt{3}}\sin\left(\frac{2\pi}{N}\right) & 0 \\ -\frac{1}{\sqrt{3}}\sin\left(\frac{2\pi}{N}\right) & \cos\left(\frac{2\pi}{N}\right) & \frac{1}{\sqrt{3}}\sin\left(\frac{2\pi}{N}\right) & 0 \\ \frac{1}{\sqrt{3}}\sin\left(\frac{2\pi}{N}\right) & -\frac{1}{\sqrt{3}}\sin\left(\frac{2\pi}{N}\right) & \cos\left(\frac{2\pi}{N}\right) & 0 \\ \sin\left(\frac{2\pi}{N}\right) & 0 & 0 & \cos\left(\frac{2\pi}{N}\right) \end{pmatrix},$$

$$V_1(k) = \begin{pmatrix} v_{a1}(k) \\ v_{b1}(k) \\ v_{c1}(k) \\ v_{a1\perp}(k) \end{pmatrix} \text{ is a vector containing the three-phase}$$

filtered signals ($v_{a1}(k)$, $v_{b1}(k)$, $v_{c1}(k)$) and an orthogonal signal, respective to the phase a ($v_{a1\perp}(k)$), $B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$,

$\mathbf{v}(k) = \begin{pmatrix} v_a(k) \\ v_b(k) \\ v_c(k) \end{pmatrix}$ is a vector with the measured voltages, \mathbf{w}_k is

a 4×1 dimension matrix and \mathbf{z}_k , is a 3×1 dimension matrix.

The instantaneous phase angle can thus be obtained by:

$$\theta(k) = \tan^{-1} \left[-\frac{v_{a1}(k)}{v_{a1\perp}(k)} \right] \quad (6)$$

and the fundamental frequency f_i can be calculated in the same way as in the single-phase KF.

In case of unbalanced (amplitudes), but symmetrical (phase angle) input voltages, such KF model has the advantage to be able to identify the positive sequence component, in such a way that its magnitude can be calculated by:

$$V^+ \equiv \sqrt{\frac{2}{3} [(v_{a1})^2 + (v_{b1})^2 + (v_{c1})^2]}. \quad (7)$$

As well as in the single-phase model, waveform distortions will be considered by means of \mathbf{z}_k . In addition, it has been verified that, if \mathbf{Q} and \mathbf{R} were set to attenuating harmonic distortions, the dynamic convergence of the three-phase model is better than the single-phase one, since, statistically, its model has more information than the previous. However, it should be considered that its implementation complexity is also superior (4×4 systems instead of 2×2).

IV. SIMULATION RESULTS

In order to compare different KF's, some simulations were realized whose results are presented in the sequel. The sampling frequency f_s were chosen to be 12 kHz, what means that the number of samples per period N varies with f_i ($N = 200$ for $f_i = 60$ Hz) and \mathbf{P}_0 was set to \mathbf{Q} . In order to find the ideal balance between velocity of response and filtering quality, different matrices \mathbf{Q} and \mathbf{R} were tested. The best values found are, for the single-phase system:

$$\mathbf{Q} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \quad \text{e} \quad \mathbf{R} = [25], \quad (8)$$

and for the three-phase system:

$$\mathbf{Q} = \begin{bmatrix} 0.0025 & 0 & 0 & 0 \\ 0 & 0.0025 & 0 & 0 \\ 0 & 0 & 0.0025 & 0 \\ 0 & 0 & 0 & 0.0025 \end{bmatrix} \quad \text{e} \quad \mathbf{R} = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix}, \quad (9)$$

These values were found following the suggestion of [4] and references therein. Note that the entries of \mathbf{R} are much greater than the ones in \mathbf{Q} , what is the way to attenuate the harmonic distortion in \mathbf{z}_k .

It is worth mentioning that in order to guarantee the same performance, different values of these matrices must be used for different voltage levels.

A. Single-Phase KF

In the figures that follow, four simulation results will be discussed (4 different cases):

Case 1 – Sinusoidal voltage with 127 V_{rms}, 60 Hz and phase angle equals to 60° (Fig. 2);

Case 2 – Voltage with 5% of harmonic distortion in the 3rd, 5th and 7th harmonic, but with the remaining parameters values like in the last example (Fig. 3);

Case 3 – Voltage like in case 2, but with a sag of 50% (Fig. 4);

Case 4 – Voltage like in case 2, but with an abrupt transition in f_i from 60 Hz to 59 Hz (Fig. 5).

In Fig. 2, one can see the influence of the initial state of the filter (that must be set by the user). In this case, values were chosen such that the initial phase of the signal is 0°. In the uppermost graphic, the sinusoidal voltage v (output of the KF) along with their in-phase (v_1) and orthogonal ($v_{1\perp}$) components are presented. Note that the convergence is achieved in less than 6 cycles. In the central graphic, the corresponding phase θ of signal v_1 is presented. In the lowest graphic, the fundamental frequency f_i calculated as indicated earlier and with initial value equals to 60 Hz. The observed transient comes from the difference between the real state of the system and the one fixed in the algorithm.

The good filtering quality of KF-1 ϕ can be observed in Fig. 3, where case 2 are in force. The Total Harmonic Distortion (THD), which would be 8,66% in the modeled signal, was reduced to 1,41%.

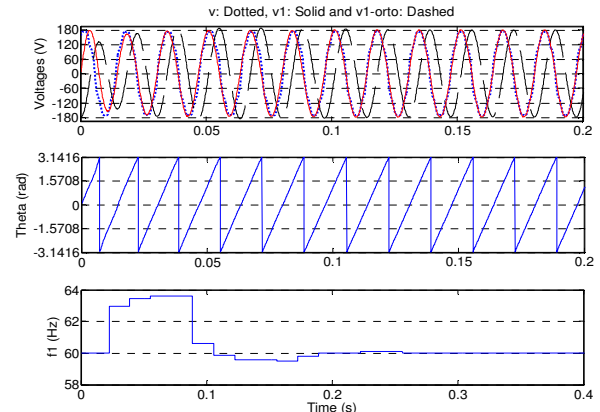


Fig. 2. KF-1 ϕ : Input Voltages (127 V_{rms}, 60 Hz, 60°) and filter outputs: θ e f_i .

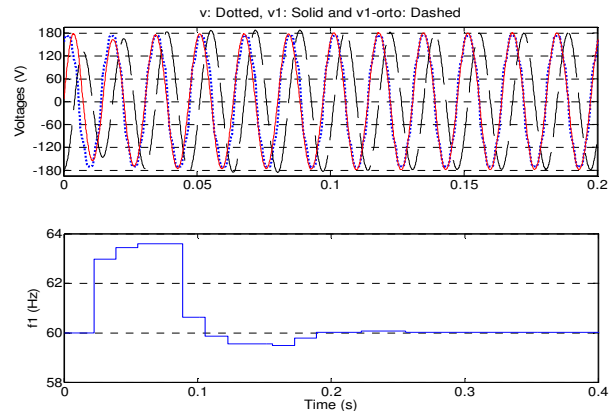


Fig. 3. KF-1 ϕ : Input Voltage (127 V_{rms}, 60 Hz, 60°), with 5% of distortion in 3rd, 5th and 7th harmonics and filter output f_i .

Fig. 4 e 5 show the performance of KF-1 ϕ when there are abrupt transitions in amplitude and frequency. In the first case, estimated voltages converge in 2 cycles while fundamental frequency estimate practically does not change. In the second case, v_1 e $v_{1\perp}$ follow v while f_1 stabilizes in 9 cycles. Little ripple in frequency (58,98 Hz -59,04 Hz) can be observed as the ratio f_s/f_1 is no longer an integer number.

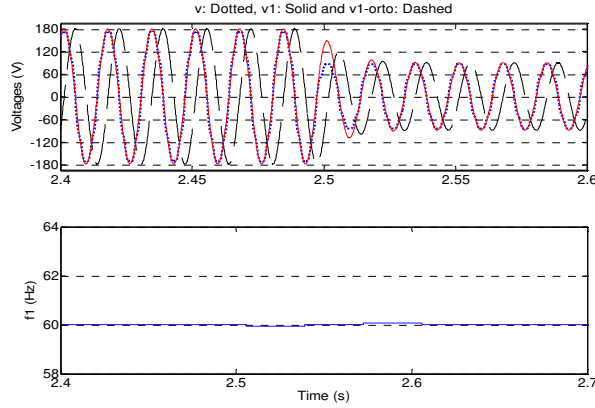


Fig. 4. KF-1 ϕ : Filter Input and Output Voltages and f_1 , after a voltage sag.

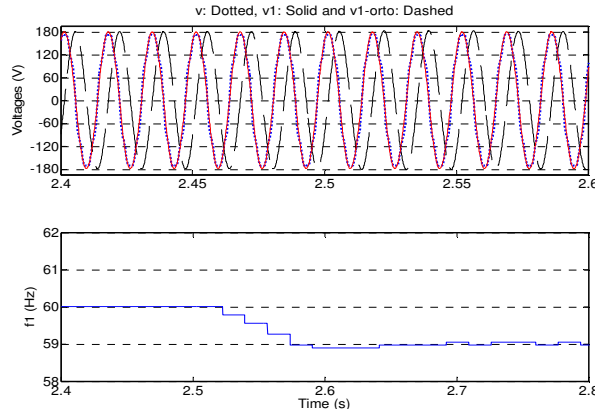


Fig. 5. KF-1 ϕ : Filter Input and Output f_1 , after a frequency transient.

B. Three-Phase KF

In the following figures, one shows simulation results for the next cases:

Case 1 – Balanced 127 V_{rms}, 60 Hz sinusoidal three-phase voltages with 60° of initial phase angle in phase *a* (Fig. 6);

Case 2 – Unbalanced voltages (70% in phase *b* and 85% in phase *c*), with 5% of 3rd, 5th and 7th harmonic distortion, with all the remaining parameters like in case 1 (Fig. 7);

Case 3 – Balanced Voltages with distortion like in case 2, but with a 50% voltage sag (Fig. 8).

Case 4 – Voltages like in case 3, but with an abrupt transition in f_1 (from 60 Hz to 59 Hz (Fig. 9)).

In Fig. 6, again one can see the influence of the initial state assumption in the performance of the system (phase *a*

with a phase angle of 0°). In the uppermost graphic, the sinusoidal voltage v_a and the in-phase voltage, that comes from the filtering (v_{a1}) are put together. One can see the convergence in six cycles. In the middle graphic, the phase angle θ of v_{a1} are presented and in the inferior graphic, one presents the calculated fundamental frequency f_1 (the initial frequency assumed to be 60 Hz). Again, the difference between the adopted initial state and the real one can explain the transient.

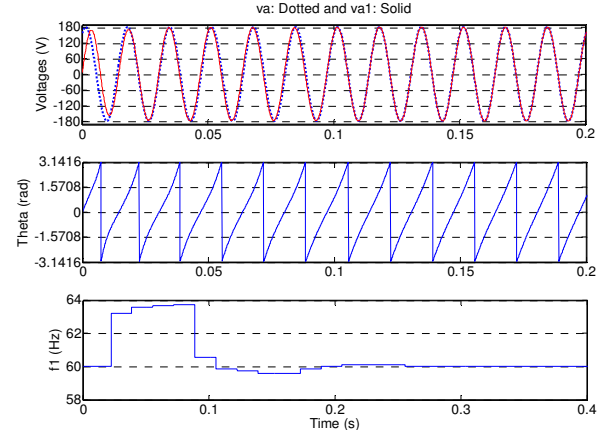


Fig. 6. KF-3 ϕ : Balanced input voltages (127 V_{rms}, 60 Hz, 60° - phase *a*), and outputs v_{a1} , θ e f_1 .

In Fig. 7, a case with unbalanced three-phase voltages with 8,66% of DHT is analyzed. Once more, one can see the convergence in six cycles (with the same initial state in the KF) and a distortion reduction to 0,79% of DHT, what shows the good filtering capability of the algorithm. Besides that, the amplitudes of v_{a1} and of the others resulting signals are $179,6 \times (1 + 0,7 + 0,85)/3 \approx 152,66$ V, what is the mean value of the measured amplitudes that, in this case, coincides with the positive sequence.

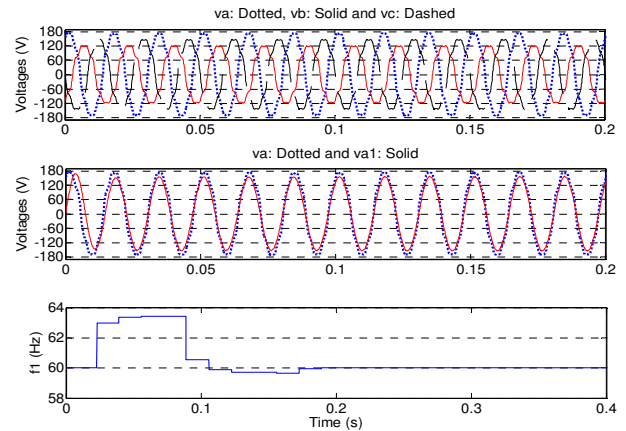


Fig. 7. KF-3 ϕ : filter response in a situation with unbalanced and distorted voltages.

Fig. 8 and 9 show the performance of the KF-3 ϕ when the power system voltage signals present an abrupt amplitude and frequency variations. In the case of amplitude variation (Fig. 8), the filtered voltages converge in 1 cycle while the frequency practically does not alter. In case of frequency

variation, f_i stabilizes in about 9 cycles. One can see, in this case, a little ripple (58,98 Hz -59,04 Hz), what comes from the fact that f_s/f_i is no longer a integer number.

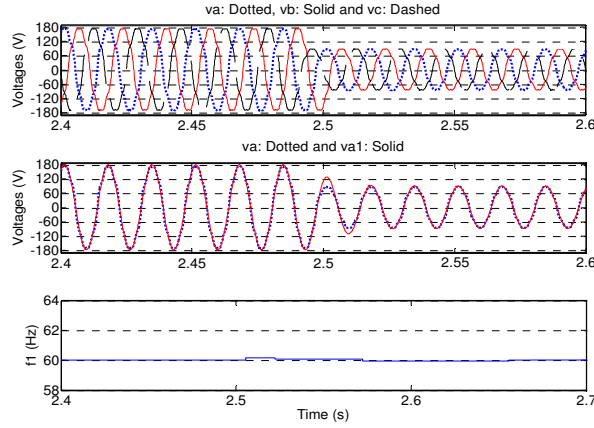


Fig. 8. KF-3φ with distorted voltages and voltage sags.

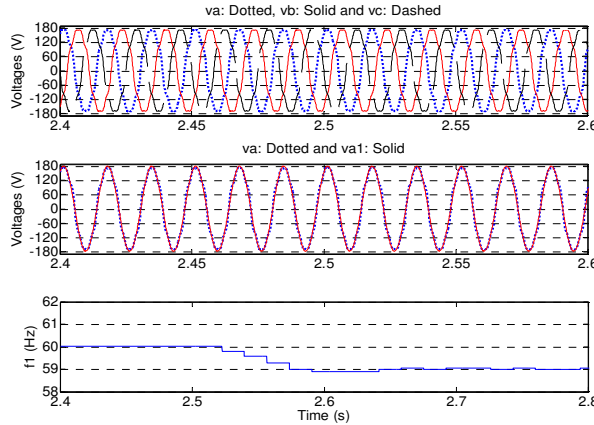


Fig. 9. KF-3φ with distorted voltages and abrupt frequency changes.

V. EXPERIMENTAL RESULTS

The single-phase algorithm was tested in a data acquisition and processing system based on the concept of virtual instrumentation [11]. The experimental apparatus was composed by an eight-channel simultaneous acquisition board with 16-bit AD converter (PCI-6143 from *National Instruments* - NI) with maximum frequency of 200 kHz. The analog signals were measured by current and voltage Hall-effect sensors (LV-25P and LA-55P from *LEM*) and the computational part was implemented in a Pentium 4 desktop with LabView 7.1 platform (from NI). The voltages were generated by a programmable three-phase generator from *California Instruments*, model 4500iL.

Initially, the KF-1φ algorithm was tested for different voltage conditions. In the uppermost graphic of Fig. 10, one has a sinusoidal 127 V_{rms}, 60 Hz input voltage v with 10% of 3rd, 5th and 7th harmonic distortion. In the middle graphic, one has the estimated and filtered voltage v_1 . In the lowest

graphic, one shows the estimated fundamental frequency f_i [Hz].

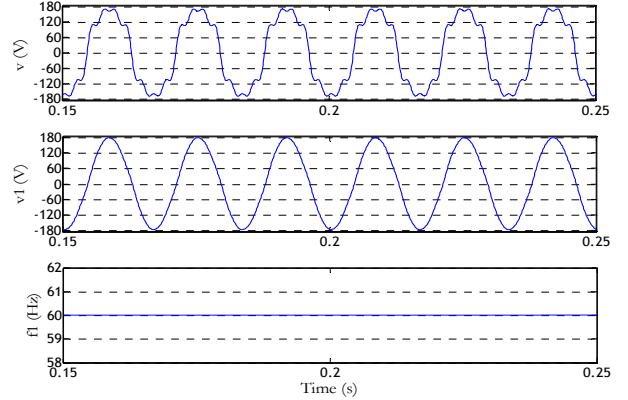


Fig. 10. KF-1φ: Input voltage (127 V_{rms}, 60 Hz), with 10% of 3rd, 5th and 7th harmonic distortion and influence in the frequency.

In Fig. 11, the input voltage suffers a voltage sag of 50%, but despite that, the algorithm manages to converge in about one cycle. In Fig. 12, the amplitude remains constant but there is an abrupt change in the fundamental frequency from 60 to 59 Hz. There is no change in the filtered voltage, but the frequency stabilizes in about 10 cycles.

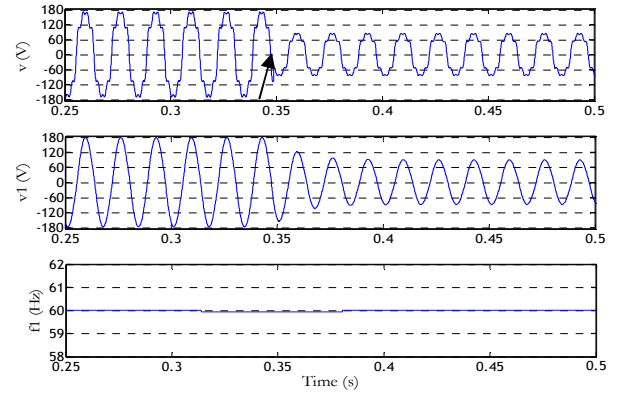


Fig. 11. KF-1φ: Sag of 50% in the input voltage.

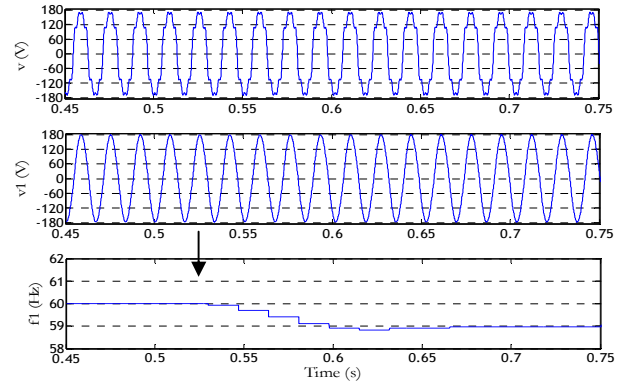


Fig. 12. KF-1φ: Abrupt change in f_i from 60 Hz to 59 Hz.

VI. CONCLUSION

The Kalman Filter was applied in this work as a way to estimate the fundamental component from a distorted signal like those present in a power system. One could then estimate the fundamental frequency of the signal f_j . The filter was designed by adopting a stochastic state-space model for the power system voltage, what is a very reasonable hypothesis. The performance of the zero-crossing detection method, based on the estimated voltage, was shown to be very effective as a way to calculate the fundamental frequency, even in the presence of harmonic distortion.

Complex matrix calculations are not a concern here because their dimensions never exceed two, in the single-phase case (and four, in the three-phase case). It makes possible to do calculations elementwise. In the three-phase case, the algorithm also estimates the mean-amplitude in the presence of unbalanced voltages, what could be used, in some cases, in place of a positive-sequence detector [10]. It was also shown that, setting adequate values to \mathbf{Q} and \mathbf{R} , the performance of the three-phase could be better than the single-phase case.

Comparing with other synchronization algorithms, like those in [9,10] and/or those in [5-8,12-14], as to say the PLL-based and the DFT-based, one can say that the proposed Kalman filter based algorithm is 1) so precise in steady-state as the others two; 2) as fast and sensible to voltage distortions as the PLL-based and 3) computationally simpler than other Kalman-based algorithms presented in literature.

Therefore, this algorithm, along with the PLL-based and the DFT-based, are interesting alternatives in applications that need synchronization with the power system as well as in power quality analysis.

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